Computer Graphics

- Anti-Aliasing & Super-Sampling -

Philipp Slusallek

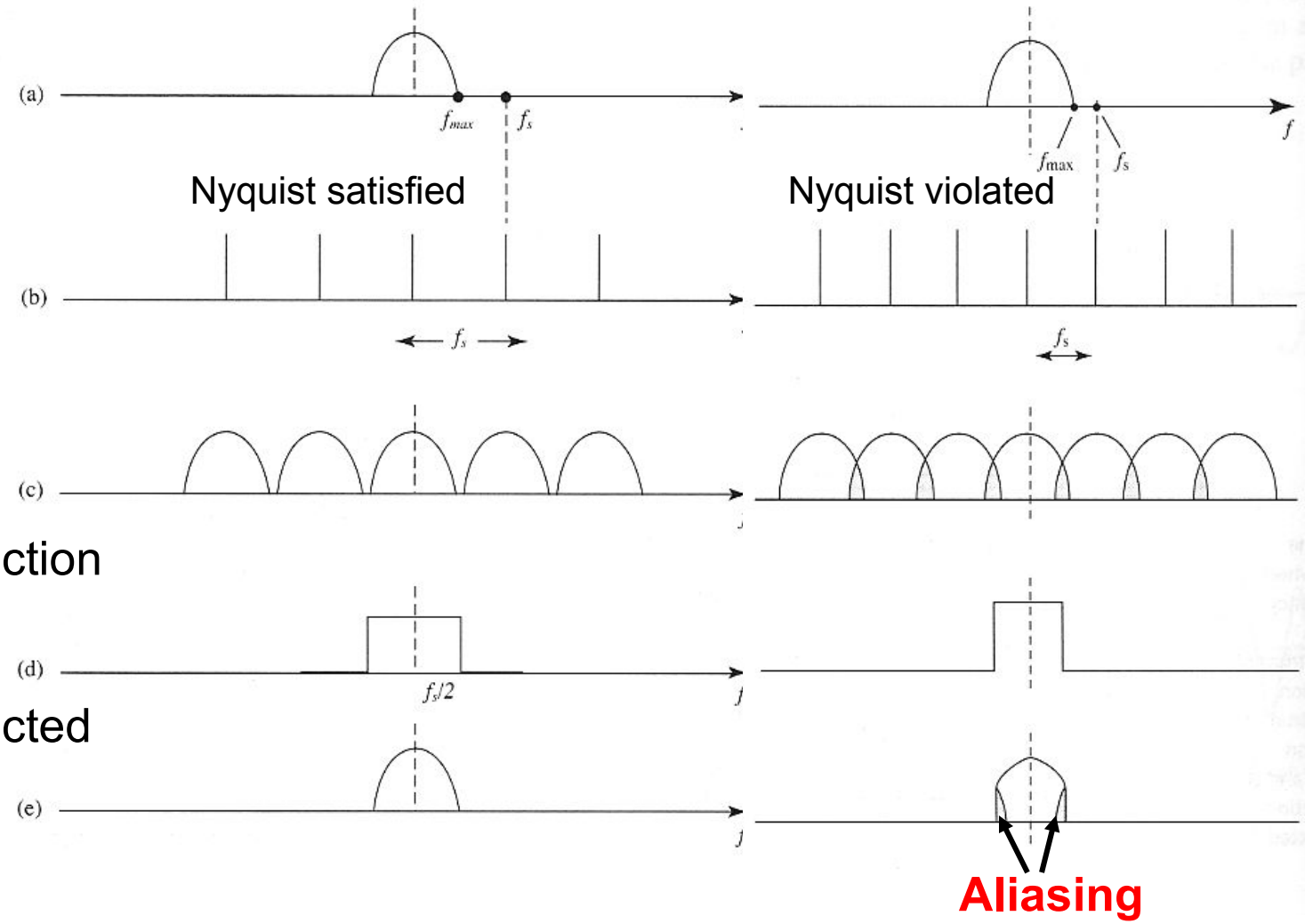
Overview

- **Last time**
  - Signal processing
- **Today**
  - Anti-aliasing & supersampling
- **Next lecture**
  - The Human Visual System

Aliasing

- **In Fourier space**

- Original spectrum
- Sampling comb
- Resulting spectrum
- Reconstruction Filter
- Reconstructed spectrum

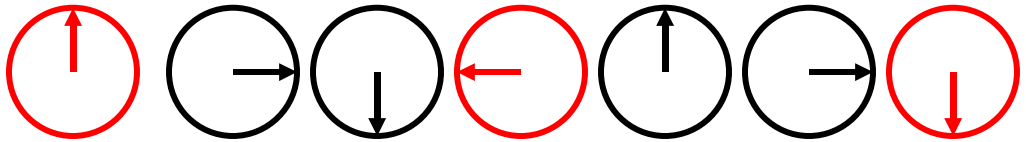


Sampling Artifacts

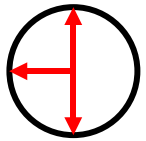
- **Spatial aliasing:**
  - Stair cases, Moiré patterns, etc.
- **Solutions:**
  - Increasing the sampling rate
    - Ok, but infinite frequencies at sharp edges
  - Post-filtering (after reconstruction)
    - Does not work - only leads to blurred stair cases
  - Pre-filtering (Blurring) of sharp geometry features
    - Slowly make geometry transparent at the edges
    - Correct solution in principal
    - Analytic low-pass filtering hard to implement
    - Super-sampling

Sampling Artifacts

- **Temporal Aliasing**
  - Car wheels, ...



- **Solutions**
  - Increasing the frame rate
    - OK
  - Pre-filtering (Motion Blur)
    - Yes, possible for simple geometry (e.g., Cartoons)
    - Problems with texture, etc.
  - Post-filtering (Averaging several frames)
    - Does not work – only multiple detail



- **Important**
  - Distinction between aliasing errors and reconstruction errors

Aliasing

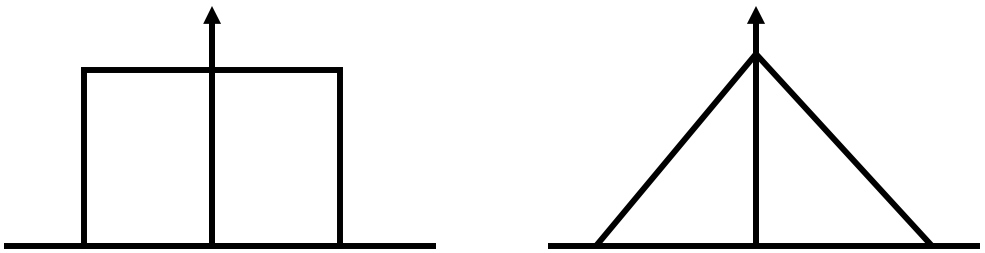
- **It all comes from sampling at discrete points**
  - Multiplied with comb function, no smoothly weighted filters
  - Comb function: repeats frequency spectrum
- **Or, from using non band limited primitives**
  - Hard edges $\Rightarrow$ infinitely high frequencies
- **In reality, integration over finite region necessary**
  - E.g., finite CCD pixel size
- **Computer: Analytic integration often not possible**
  - No analytic description of radiance or visible geometry available
- **Only way: numerical integration**
  - Estimate integral by taking multiple point samples, average
    - Leads to aliasing
  - Computationally expensive
  - Approximate

Sources of High Frequencies

- **Geometry**
  - Edges, vertices, sharp boundaries
  - Silhouettes (view dependent)
  - …
- **Texture**
  - E.g., checkerboard pattern, other discontinuities, …
- **Illumination**
  - Shadows, lighting effects, projections, …

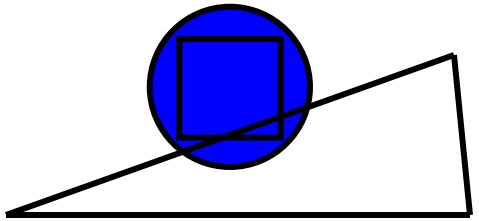
➔**Analytic filtering almost impossible**

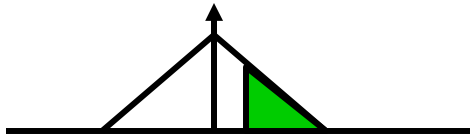
  - Even with the most simple filters



Comparison

- **Analytic low-pass filtering**
  - Ideally eliminates aliasing completely
  - Hard to implement
    - Weighted or unweighted area sampling
    - Compute distance from pixel to a line
    - Filter values can be stored in look-up tables
      - Possibly taking into account slope
      - Distance correction
      - Non rotationally symmetric filters
    - Does not work at corners
- **Over-/Super-sampling**
  - Very easy to implement
  - Does not eliminate aliasing completely
    - Sharp edges contain *infinitely* high frequencies

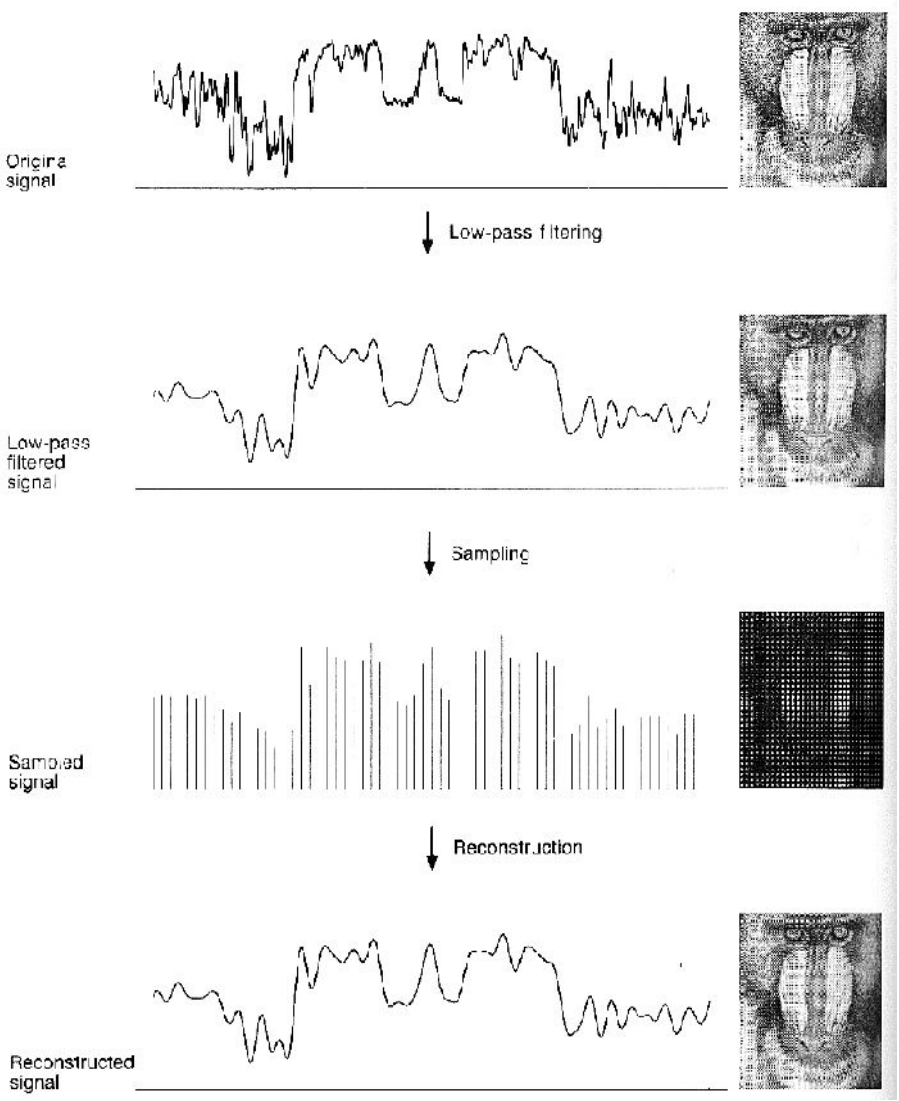




Antialiasing by Pre-Filtering

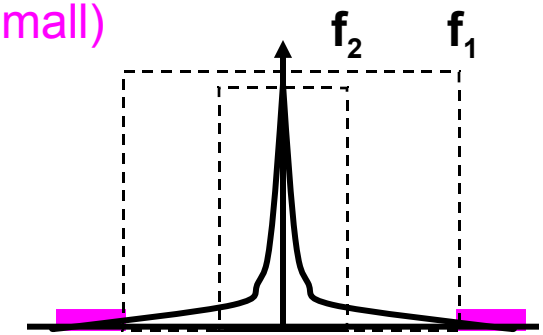
- **Filtering before sampling**
  - Band-limiting signal
  - Analog/analytic or
  - Reduce Nyquist frequency for chosen sampling-rate
- **Ideal reconstruction**
  - Convolution with sinc
- **Practical reconstruction**
  - Convolution with
    - Box filter, Bartlett (Tent)

  → Reconstruction error



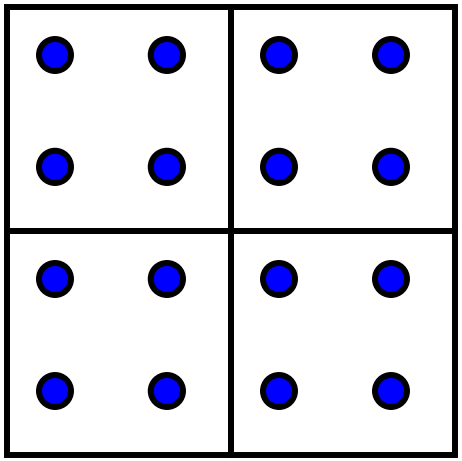
Re-Sampling Pipeline

- **Assumption**
  - Energy in high frequencies decreases quickly
  - Reduced aliasing by intermediate sampling with higher frequencies
- **Algorithm**
  - Super-sampling
    - Sample continuous signal with high frequency f_1
    - Aliasing with energy beyond f_1 (assumed to be small)
  - Reconstruction of signal
    - Filtering with $g_1(x)$: e.g. convolution with sinc_f_1
    - Exact representation with sampled values !!
  - Analytic low-pass filtering of signal
    - Filtering with filter $g_2(x)$ with $f_2 << f_1$
    - Signal is now band limited w.r.t. f_2
  - Re-sampling with a sampling frequency that is compatible with f_2
    - No additional aliasing
  - Filters $g_1(x)$ and $g_2(x)$ can be combined



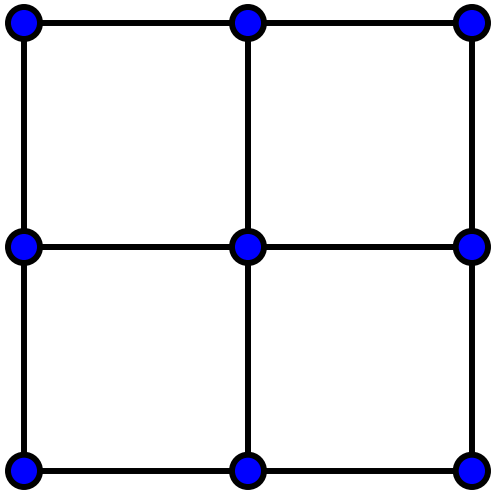
Super-Sampling in Practice

- **Regular super-sampling**
  - Averaging of N samples per pixel on a grid
  - N:
    - 4 quite good
    - 16 almost always sufficient
  - Samples
    - Rays, z-buffer, reflection, motion, ...
  - Filter Weights
    - Box filter
    - Others: B-spline, Pyramid (Bartlett), Hexagonal, ...
  - Regular super-sampling
    - Nyquist frequency for aliasing only shifted
      - ➔ Irregular sampling patterns



Super-Sampling Caveats

- **Popular mistake**
  - Sampling at the corners of every pixel
  - Pixel color by averaging
  - Free super-sampling ???
- **Problem**
  - Wrong reconstruction filter !!!
  - Same sampling frequency,
    but post-filtering with a hat function
  - Blurring: Loss of information



- **Post-Reconstruction Blur**

1x1 Sampling, 3x3 Blur

1x1 Sampling, 7x7 Blur

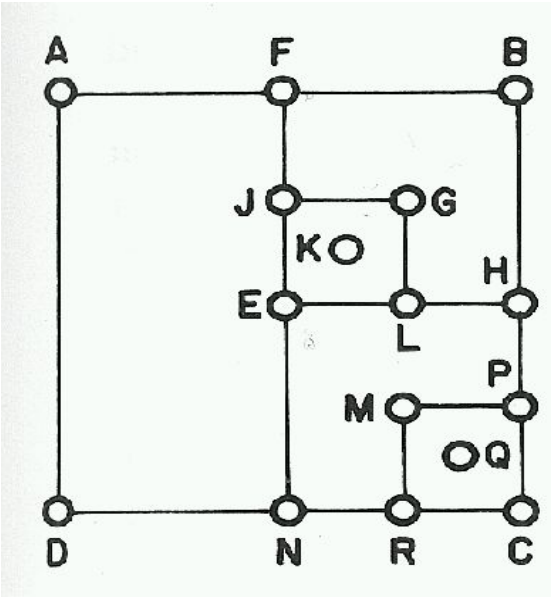
➔**„Super-sampling“ does not come for free**

Adaptive Super-Sampling

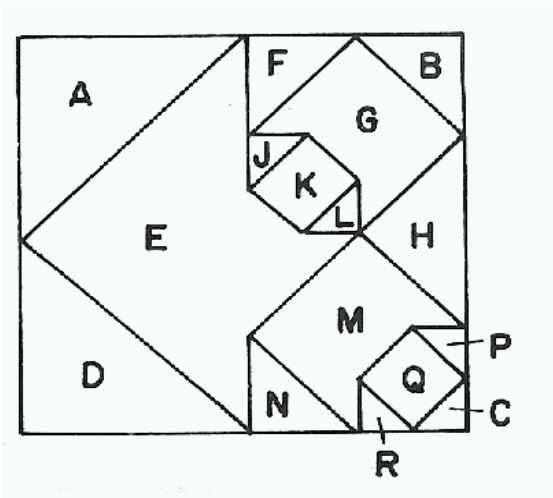
- **Adaptive super-sampling**
  - Idea: locally adapt sampling density
    - Slowly varying signal: low sampling rate
    - Strong changes: high sampling rate
  - Decide sampling density locally
  - Decision criterion needed
    - Differences of pixel values
    - Contrast (relative difference)
      - |A-B| / |A|+|B|

Adaptive Super-Sampling

- **Algorithm**
  - Sampling at corners and mid points
  - Recursive subdivision of each quadrant
  - Decision criterion
    - Differences, contrast, object-IDs, ray trees, ...
  - Filtering with weighted averaging
    - ¼ from each quadrant
    - Quadrant: ½ (midpoint + corner)
      - Recursion



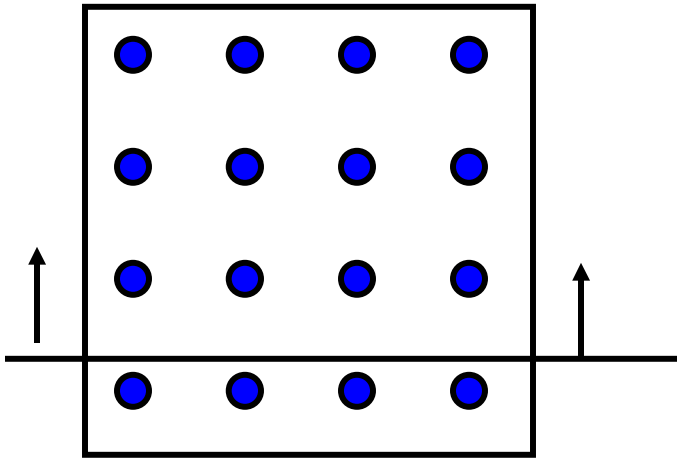
$$\frac{1}{4}\left(\frac{A+E}{2}+\frac{D+E}{2}+\frac{1}{4}\left[\frac{F+G}{2}+\frac{B+G}{2}+\frac{H+G}{2}+\frac{1}{4}\left\{\frac{J+K}{2}+\frac{G+K}{2}+\frac{L+K}{2}+\frac{E+K}{2}\right\}\right]\right.$$
$$\left.+\frac{1}{4}\left[\frac{E+M}{2}+\frac{H+M}{2}+\frac{N+M}{2}+\frac{1}{4}\left\{\frac{M+Q}{2}+\frac{P+Q}{2}+\frac{C+Q}{2}+\frac{R+Q}{2}\right\}\right]\right)$$



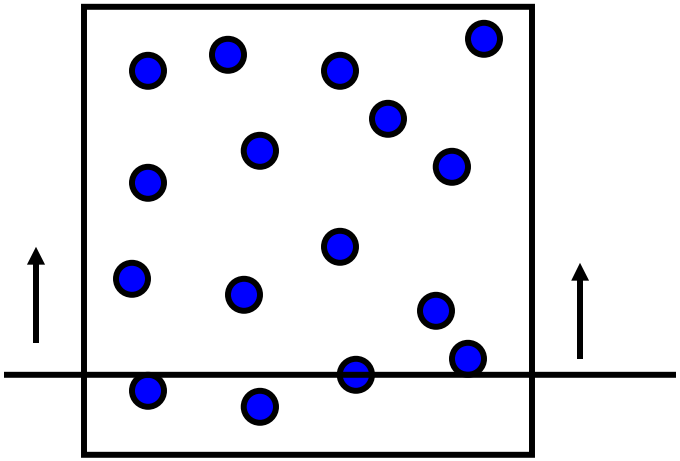
- **Extension**
  - Jittering of sample points

Super-Sampling in Practice

- **Problems with regular super-sampling**
  - Expensive: 4-fold to 16-fold effort
  - Non-adaptive: Same effort everywhere
  - Too regular: Apparent reduction of number of levels
- **Introduce irregular sampling pattern**



0 → 4/16 → 8/16 → 12/16 → 16/16



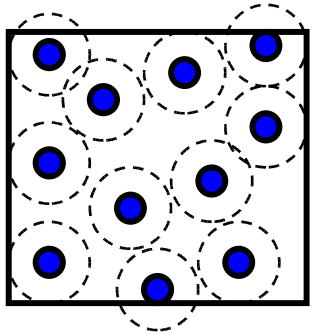
Better, but noisy

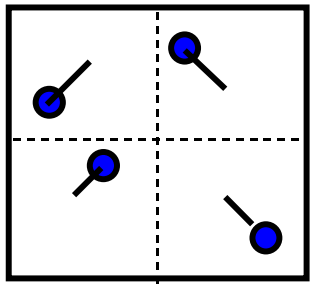
→ Stochastic super-sampling

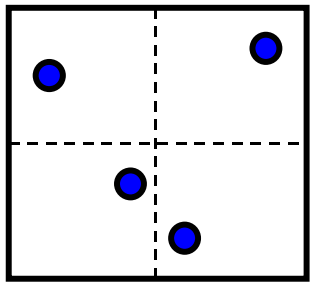
- Or analytic computation of pixel coverage and pixel mask

Stochastic Sampling

- **Requirements**
  - Even distribution
  - Little correlation between samples
  - Incremental generation
- **Generation of samples**
  - Poisson-disk sampling
    - Fixes a minimum distance between samples
    - Random generation of samples
      - Rejection, if too close to other samples
  - Jittered sampling
    - Random perturbation from regular positions
  - Stratified Sampling
    - Subdivision into areas with one random sample each
    - Improves even distribution
  - Quasi-random numbers (Quasi-Monte Carlo)
    - E.g. Halton Sequence
    - Advanced feature

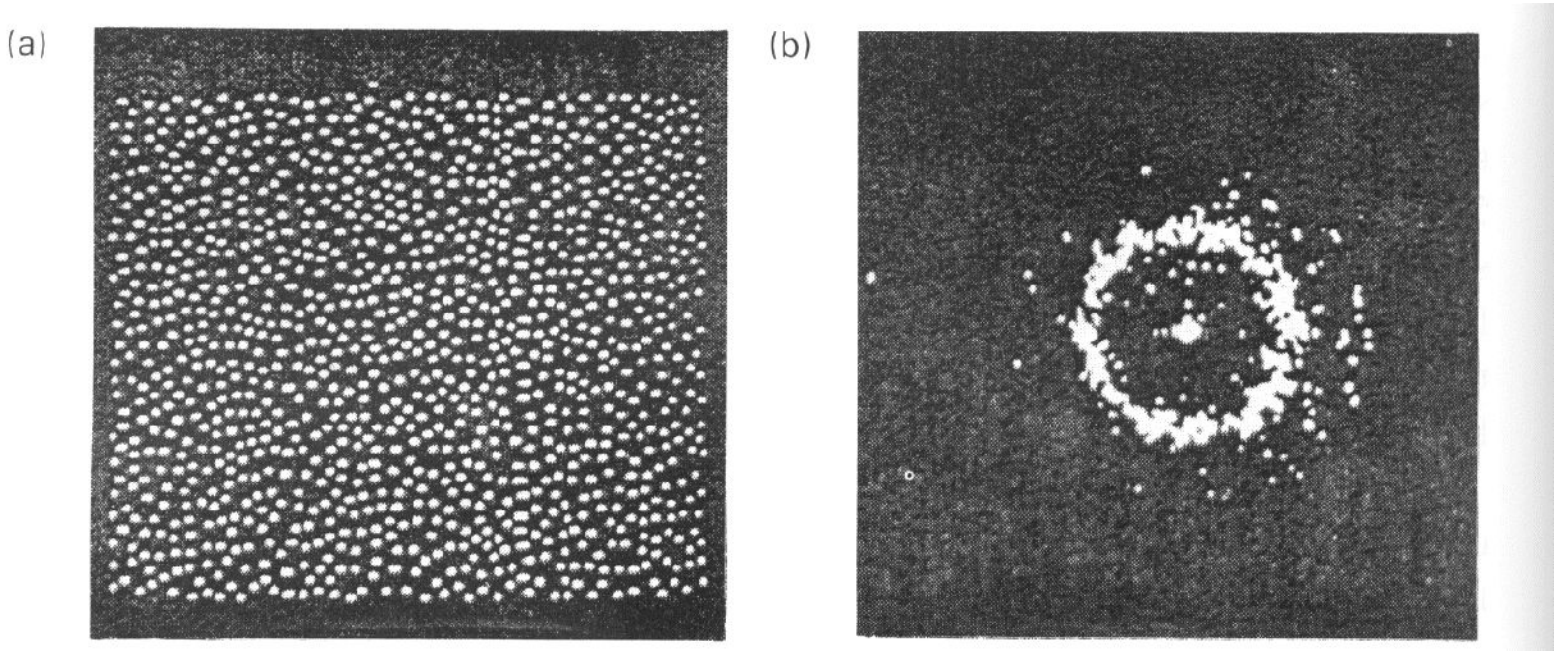






Poisson-Disk Sample Distribution

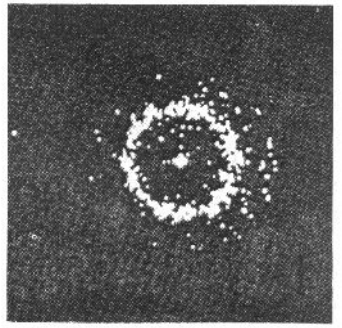
- **Motivation**
  - Distribution of the optical receptors on the retina (here: ape)

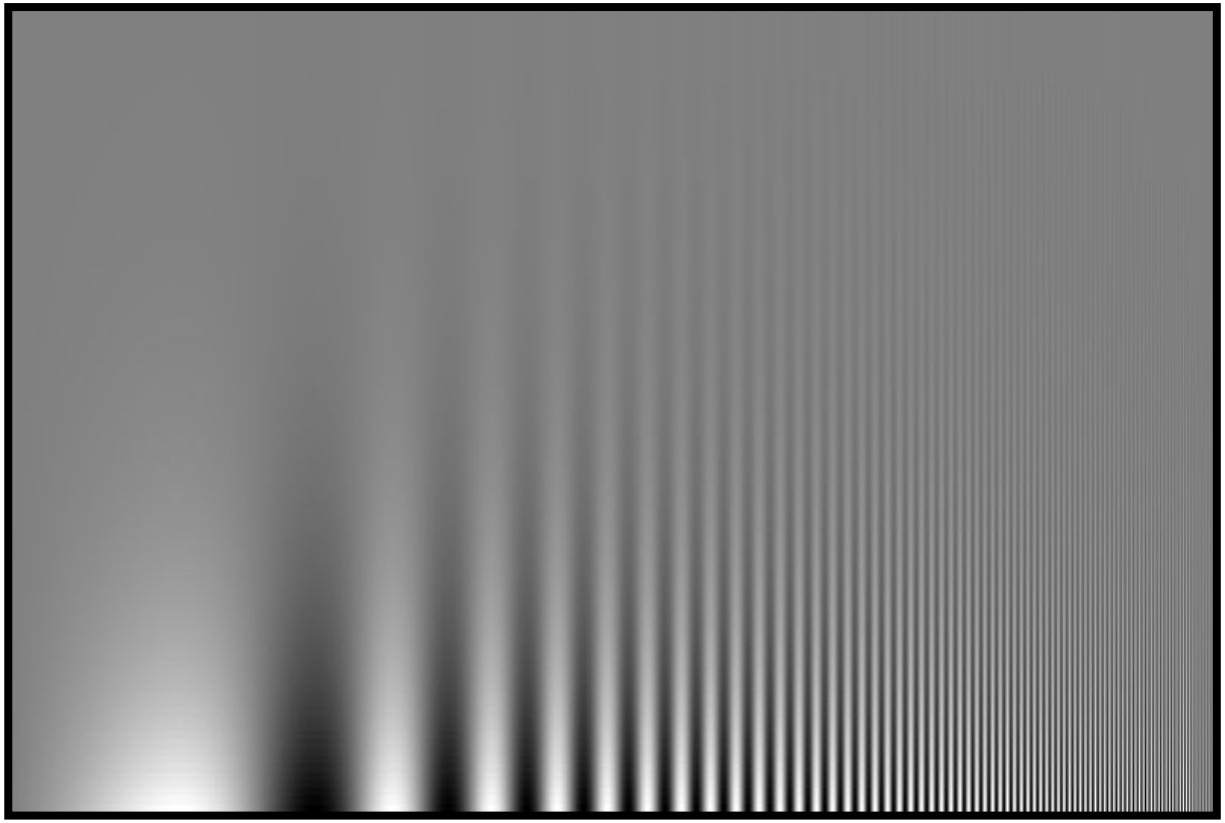


Distribution of the receptors **Fourier analysis**

HVS: Poisson Disk Experiment

- **Human Perception**
  - Very sensitive to regular structures
  - Insensitive against (high frequency) noise

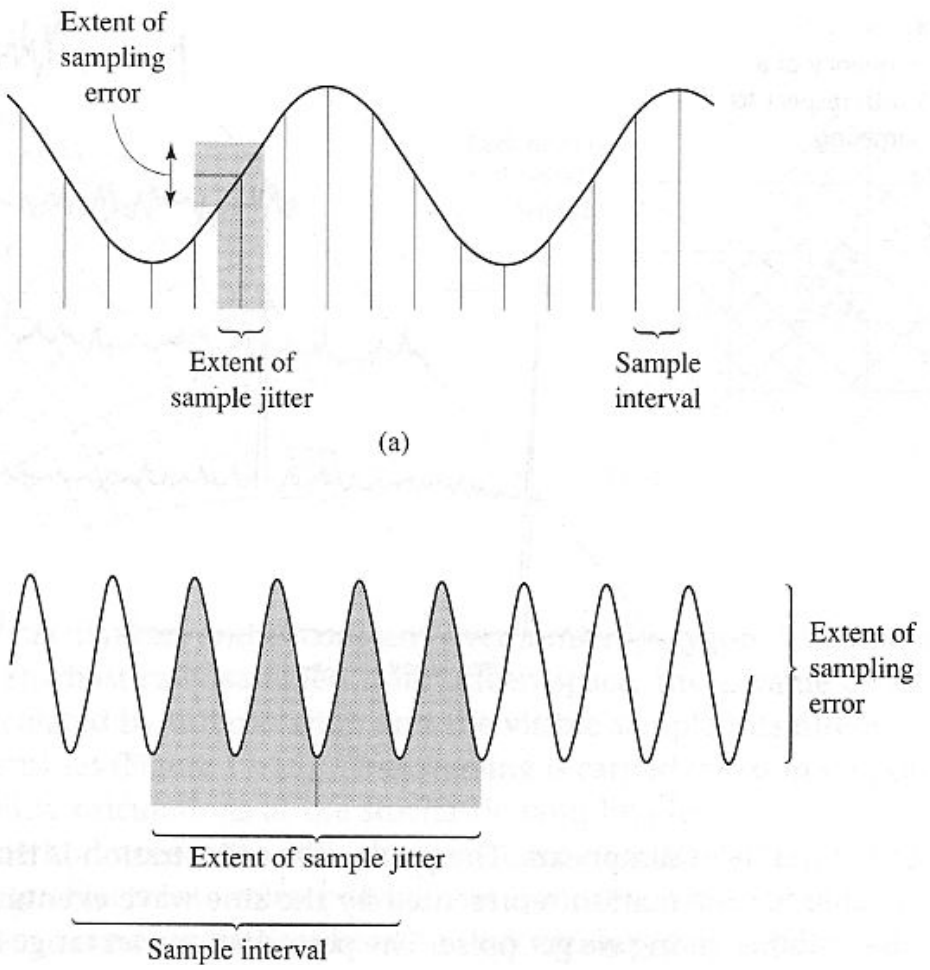




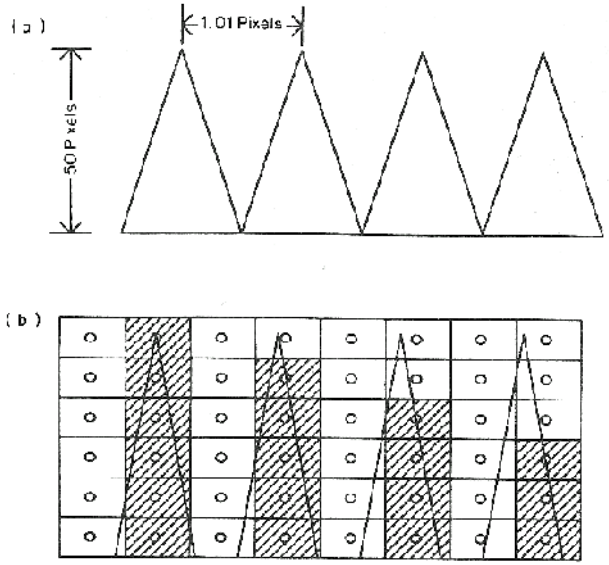
Campbell-Robson contrast sensitivity chart

Stochastic Sampling

- **Stochastic Sampling**
  - Transforms energy in high frequency bands into noise
  - Low variation in sample domain
    - Closely reconstructs target value
  - High variation
    - Reconstructs average value

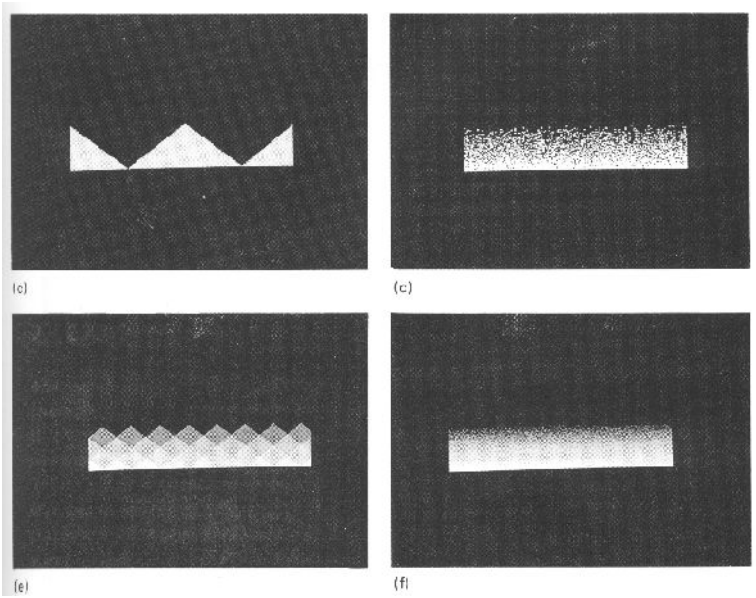


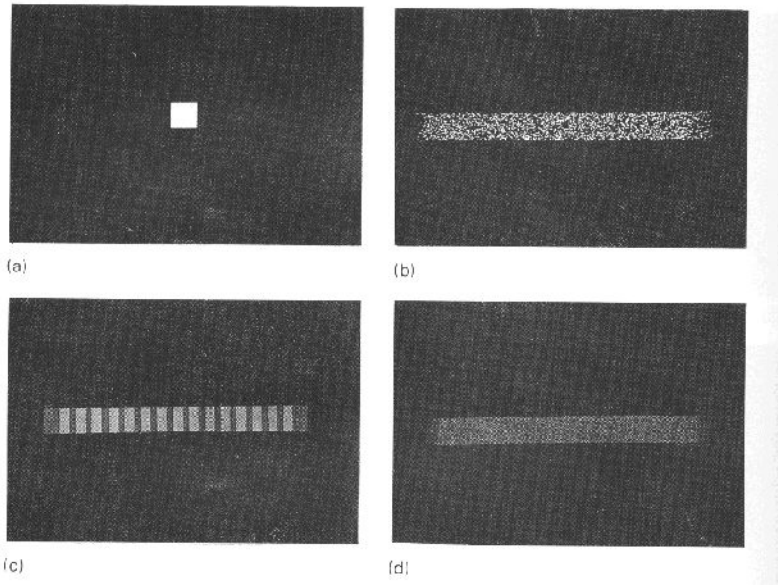
Examples



Triangle comb:
(Width: 1.01 pix, Heigth: 50 pix):
1 sample, no jittering
1 sample, jittering
16 samples, no jittering
16 samples, jittering

Motion Blur:
1 sample, no jittering
1 sample, jittering
16 samples, no jittering
16 samples, jittering





Comparison

Regular, 1x1

Regular 3x3

Regular, 7x7

Jittered, 3x3

Jittered, 7x7