Computer Graphics

- Light Transport BRDFs & Shading -

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Overview

- **Last time**
  - Radiance
  - Light sources
  - Rendering Equation & Formal Solutions
- **Today**
  - Bidirectional Reflectance Distribution Function (BRDF)
  - Reflection models
  - Projection onto spherical basis functions
  - Shading
- **Next lecture**
  - Varying (reflection) properties over object surface: texturing

Reflection Equation - Reflectance

- **Reflection equation**

$$L_o(\underline{x}, \underline{\omega}_o) = \int_{\Omega_+} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o)\, L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

- **BRDF**
  - Ratio of reflected radiance to incident irradiance

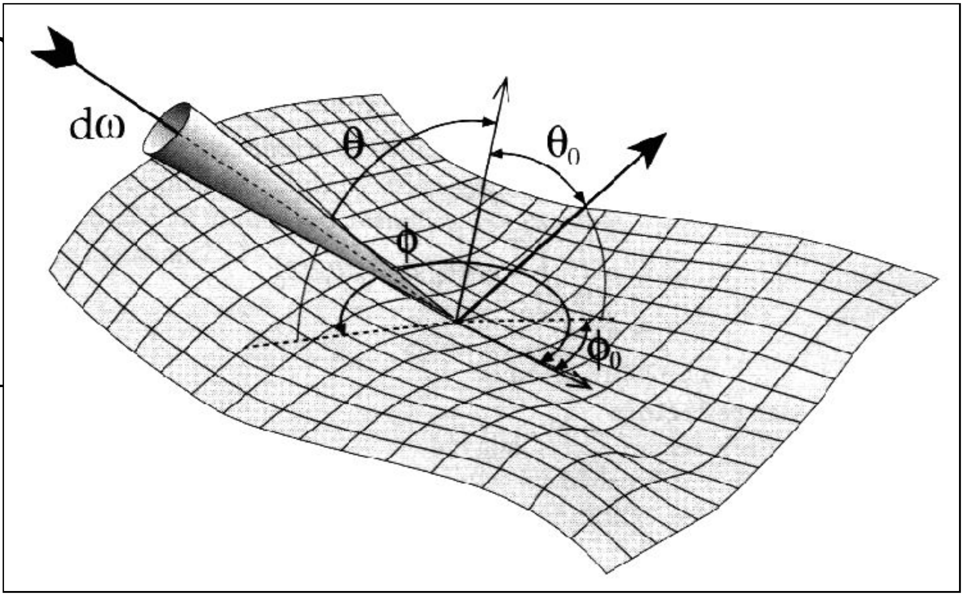
$$f_r(\omega_o, x, \omega_i) = \frac{L_o(x, \omega_o)}{dE_i(x, \omega_i)}$$

Bidirectional Reflectance Distribution Function

- **BRDF describes surface reflection for light incident from direction $\boldsymbol{\omega}_i(\theta_i,\varphi_i)$ observed from direction $\boldsymbol{\omega}_o(\theta_o,\varphi_o)$**
- **Bidirectional**
  - Depends on two directions and position (6 degrees of freedom (DOF))
  - Determines the density ("fraction") of all photons hitting a surface from a specific direction ω_i to be reflected into the outgoing direction ω_o
- **Distribution function**
  - Can be infinite, for ideal reflection
- **Unit [1/sr]**

$$f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = \frac{L_o(\underline{x}, \underline{\omega}_o)}{dE_i(\underline{x}, \underline{\omega}_i)}$$

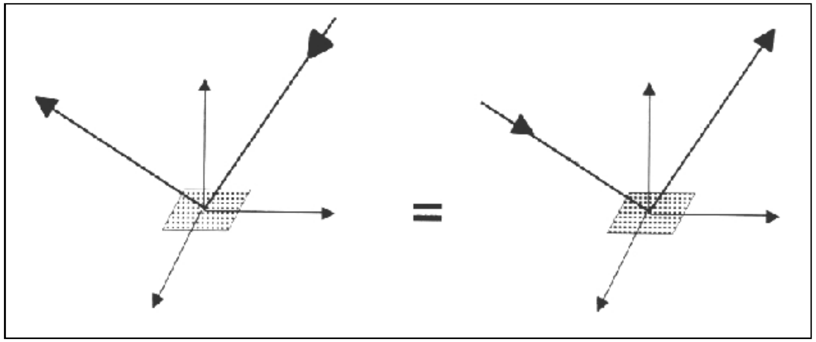
$$= \frac{L_o(\underline{x}, \underline{\omega}_o)}{L_i(\underline{x}, \underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i}$$



BRDF Properties

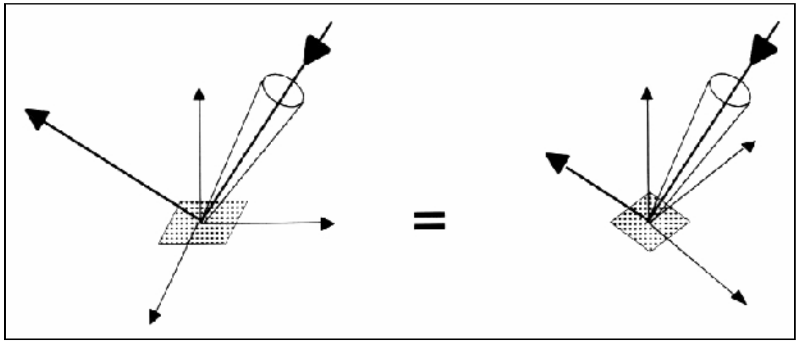
- **Helmholtz reciprocity principle**
  - BRDF remains unchanged if incident and reflected directions are interchanged

$$f_r(\omega_o, x, \omega_i) = f_r(\omega_i, x, \omega_o)$$



- **Smooth surface: isotropic BRDF**
  - Reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(\theta_i, \underline{x}, \theta_o, \varphi_o - \varphi_i)$$



BRDF Properties

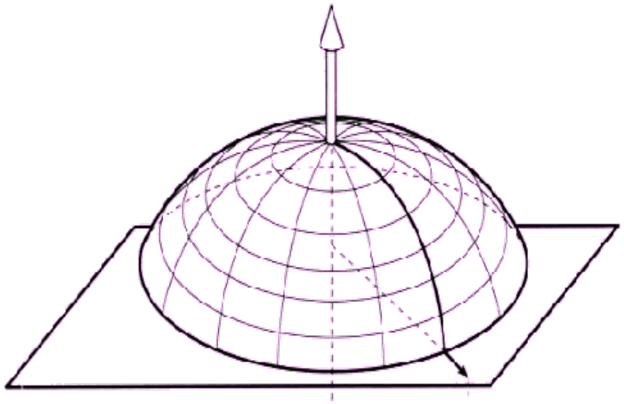
- **Characteristics**
  - BRDF units $[sr^{-1}]$
  - Range of values:
    - From 0 (absorption) to ∞ (perfect reflection, δ -function)
  - Energy conservation law
    - No self-emission
    - Possible absorption

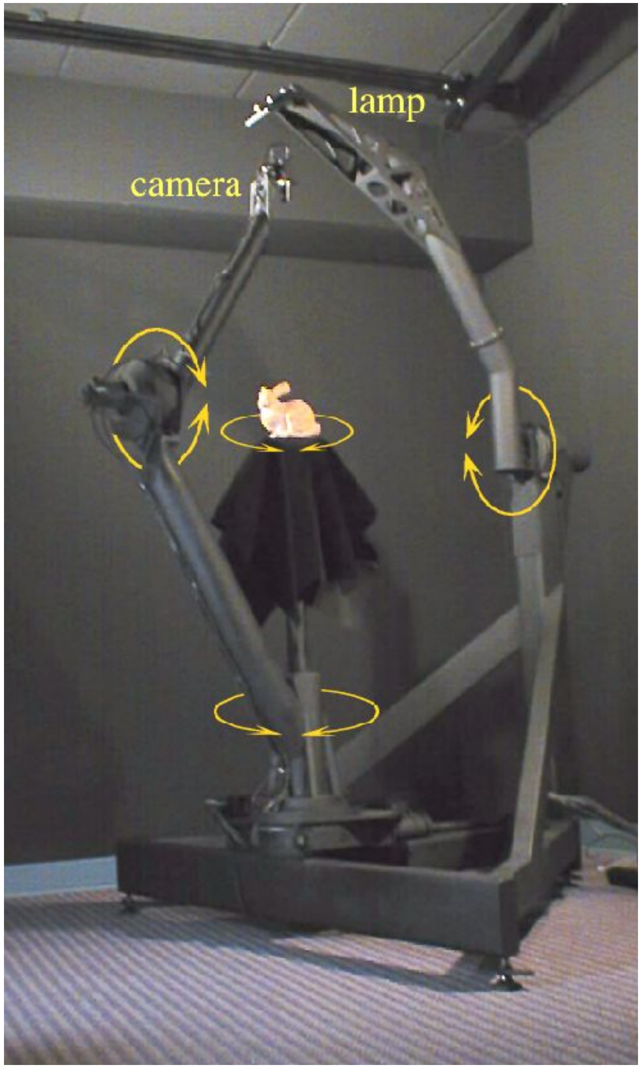
$$\int_{\Omega} f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) \cos\theta_o \, d\omega_o \leq 1 \quad \forall \, \theta, \varphi$$

  - Reflection only at the point of entry ($x_i = x_o$)
    - No subsurface scattering
    - Cannot handle skin, marble, etc.

BRDF Measurement

- **Gonio-Reflectometer**
- **BRDF measurement**
  - Point light source position (θ_i, φ_i)
  - Light detector position (θ_o, φ_o)
- **BRDF representation**
  - m incident direction samples
  - n outgoing direction samples
  - $m*n$ reflectance values (large!!!)

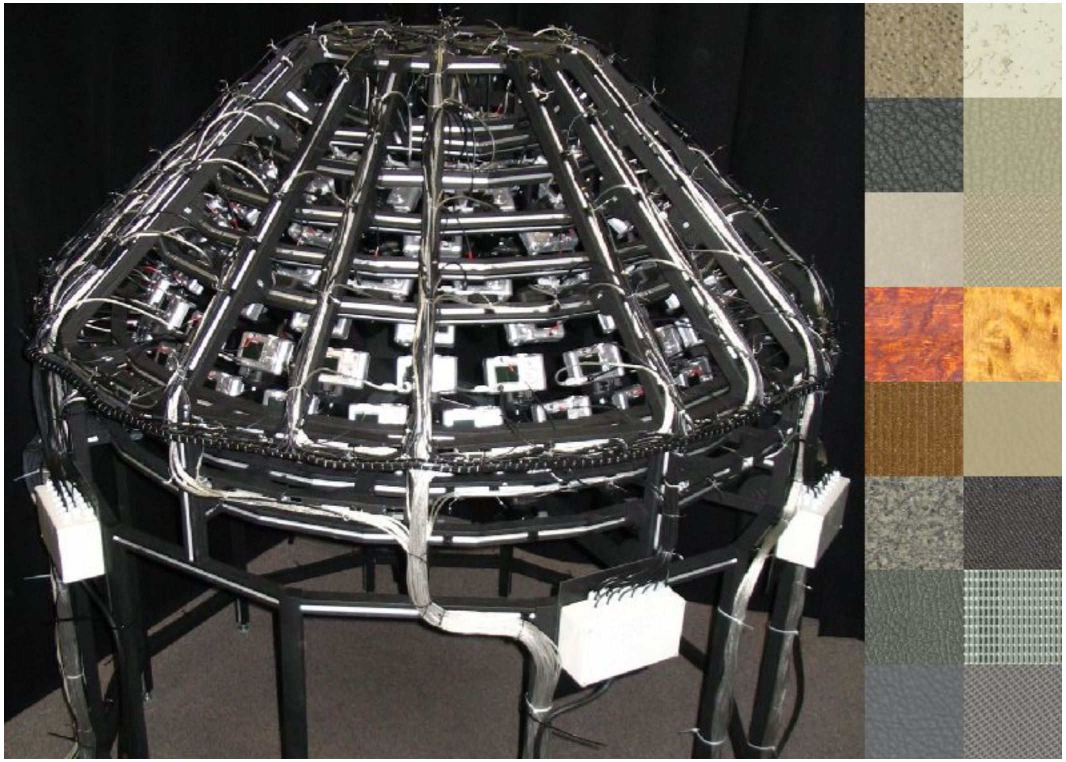




Stanford light gantry

Extension to BTFs

- **Full BRDFs needed also for entire surface areas**
  - Extension to *Bidirectional Texture Function* (BTF)
- **BTF measurement device (Bonn University)**
  - 151 digital cameras in a hemicube arrangement
  - Simultaneously take images of a probe in the center (12x12 cm)
    - One camera flashes
  - Iterate over all cameras
    - 151 x 151 images
  - Must be compressed

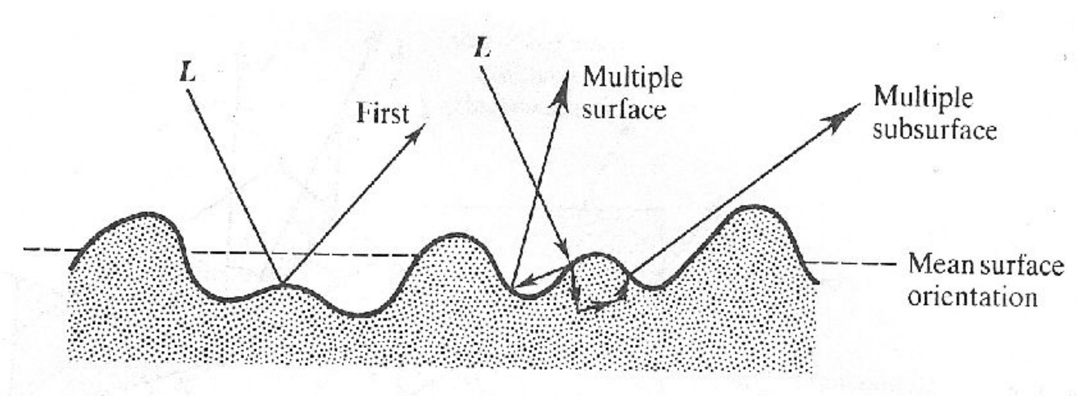


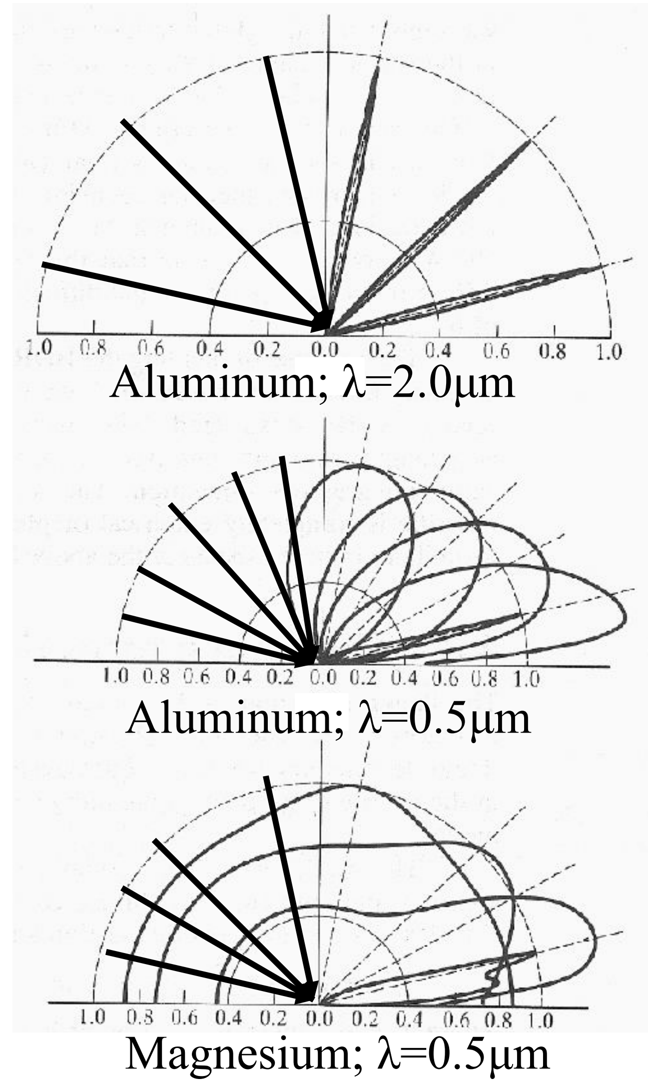
Rendering from Measured BRDF

- **Linearity, superposition principle**
  - Complex illumination: integrating light distribution against BRDF
  - Sampled BRDF: superimposed point light sources
- **Interpolation**
  - Look-up during rendering
  - Sampled BRDF must be filtered
- **BRDF Modeling**
  - Fit parameterized BRDF model to measured data
  - Continuous function
  - No interpolation
  - Fast evaluation
- **Representation in spherical harmonics basis**
  - Hierarchical basis functions on the sphere (a la Fourier)
  - Mathematically elegant filtering, illumination-BRDF integration

Reflectance

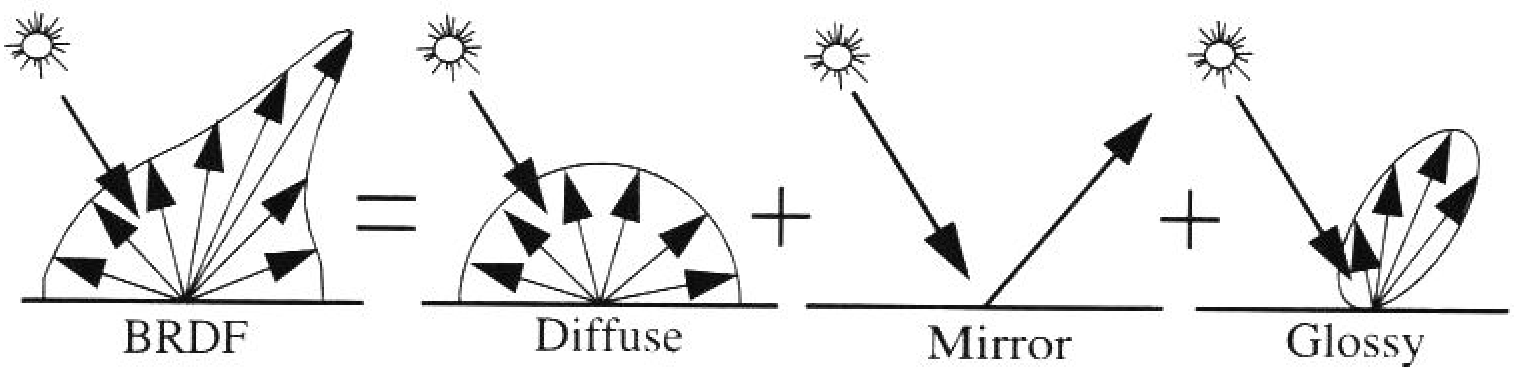
- **Reflectance may vary with**
  - Illumination angle
  - Viewing angle
  - Wavelength
  - (Polarization, ...)
- **Variations due to**
  - Surface micro-geometry
  - Index of refraction / dielectric constant
  - Scattering in the material
  - Absorption





BRDF Modeling

- **Phenomenological approach**
  - Description of visual surface appearance
- **Ideal specular reflection**
  - Reflection law
  - Mirror
- **Glossy reflection**
  - Directional diffuse
  - Shiny surfaces
- **Ideal diffuse reflection**
  - Lambert's law
  - Matte surfaces

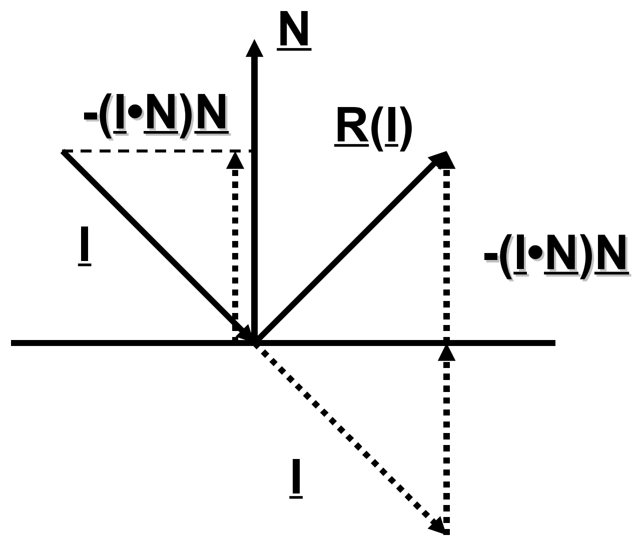


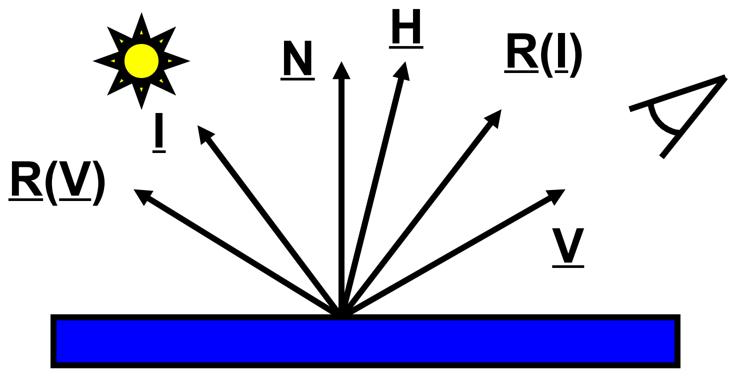
Reflection Geometry

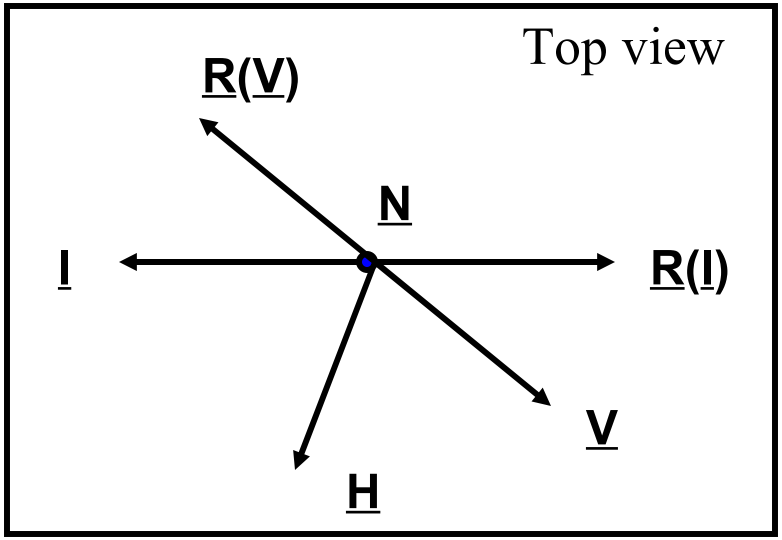
- **Direction vectors (normalize):**
  - N: surface normal
  - I: vector to the light source
  - V: viewpoint direction vector
  - H: halfway vector

    $\underline{H} = (\underline{I} + \underline{V}) / |\underline{I} + \underline{V}|$
  - R(I): reflection vector

    $\underline{R}(\underline{I}) = \underline{I} - 2(\underline{I} \cdot \underline{N})\underline{N}$
  - Tangential surface: local plane





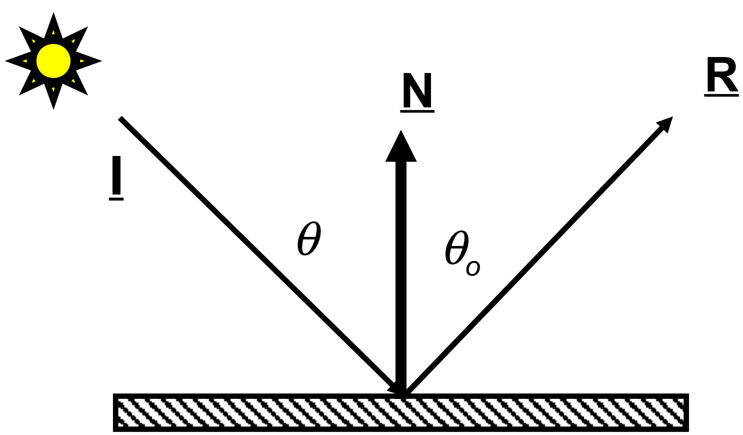


Ideal Specular Reflection

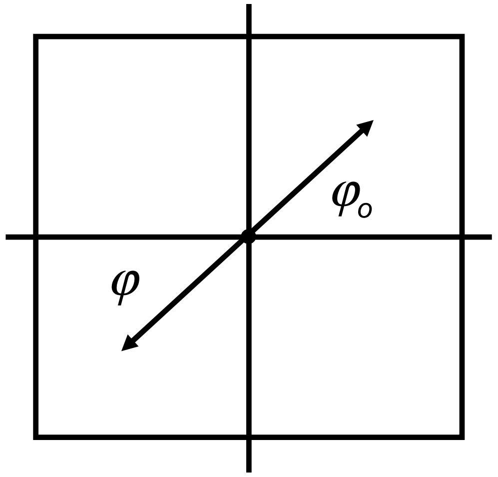
- **Angle of reflectance equal to angle of incidence**
- **Reflected vector in a plane with incident ray and surface normal vector**

$$\underline{\mathbf{R}}+(-\underline{\mathbf{I}}) = 2\cos\theta\,\underline{\mathbf{N}} = -2(\underline{\mathbf{I}}\cdot\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$

$$\underline{\mathbf{R}}(\underline{\mathbf{I}}) = \underline{\mathbf{I}} - 2(\underline{\mathbf{I}}\cdot\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$



$\theta = \theta_o$



$\varphi = \varphi_o + 180°$

Ideal Specular Reflection

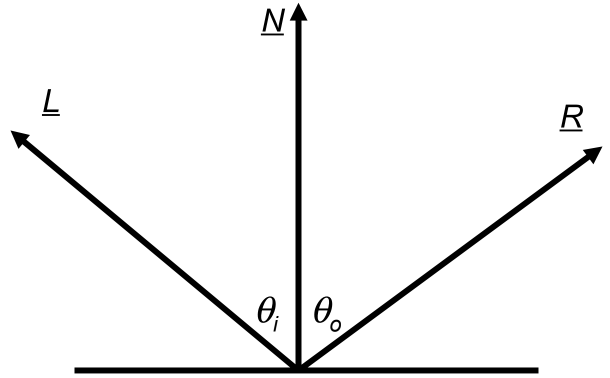
- **Dirac Delta function *δ(x)***
  - *δ(x)* : zero everywhere except at $x=0$
  - Unit integral iff integration domain contains zero (zero otherwise)
- **Specular Reflection:**
  - All light is ideally reflected

$$f_{r,m}(\omega_o, x, \omega_i) = \rho_s(\theta_i) \cdot \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \cdot \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x,\omega_o) = \int_{\Omega_+} f_{r,m}(\omega_o, x, \omega_i) L_i(\theta_i, \varphi_i) \cos\theta_i \, d\underline{\omega}_i = \rho_s(\theta_i) L_i(\theta_o, \varphi_o \pm \pi)$$

- **Specular reflectance ρ_s**
  - Ratio of reflected radiance in specular direction versus incoming radiance
  - Dimensionless quantity between *0* and *1*

$$\rho_s(\theta_i) = \frac{L_o(\theta_o)}{L_i(\theta_i)}$$



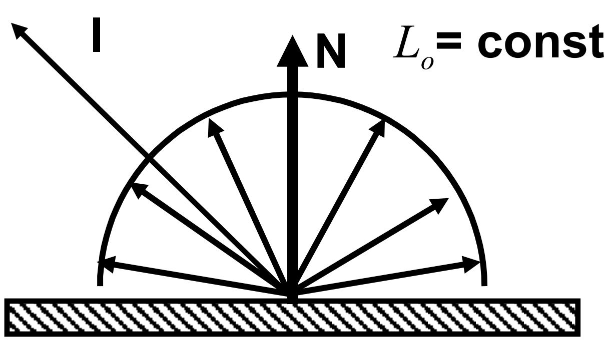
Diffuse Reflection

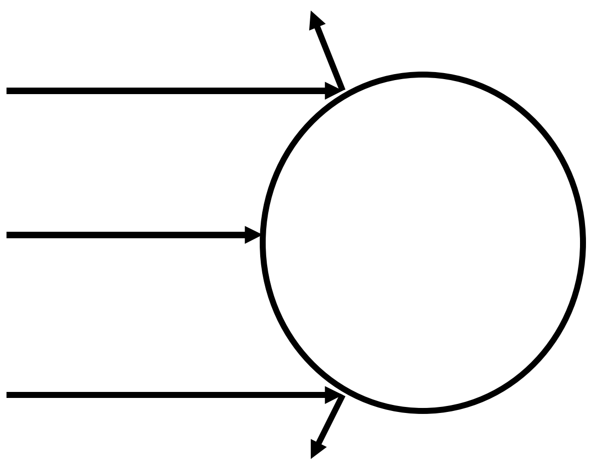
- **Light equally likely to be reflected in any output direction (independent of input direction)**
- **Constant BRDF**

$$f_{r,d}(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = k_d = \text{const}$$

$$L_o(\underline{x}, \underline{\omega}_o) = \int_\Omega k_d L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d \int_\Omega L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d E$$

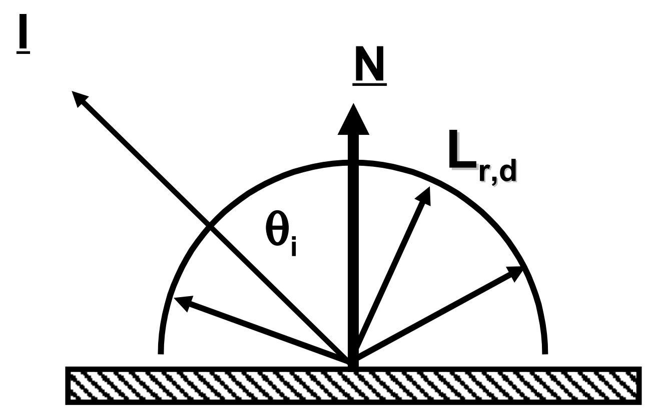
  - k_d: diffuse coefficient, material property [1/sr]





Lambertian Diffuse Reflection

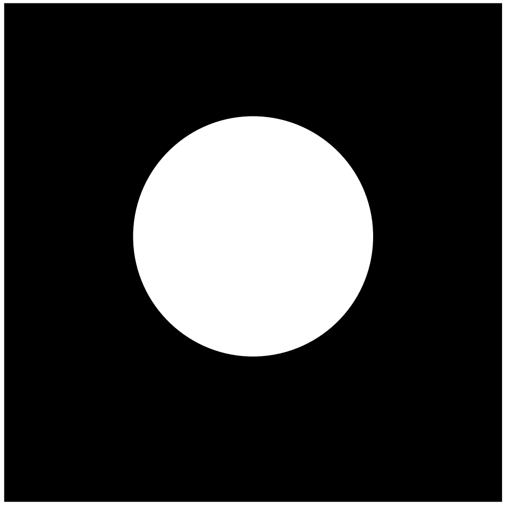
- **Radiosity** $$B = \int_\Omega L_o(\underline{x}, \underline{\omega}_o) \cos\theta_o \, d\underline{\omega}_o = L_o \int_\Omega \cos\theta_o \, d\underline{\omega}_o = \pi L_o$$
- **Diffuse Reflectance** $$\rho_d = \frac{B}{E} = \pi k_d$$
- **Lambert's Cosine Law** $$B = \rho_d E = \rho_d E_i \cos\theta_i$$
- **For each light source**
  - $L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (\underline{I} \cdot \underline{N})$



Lambertian Objects

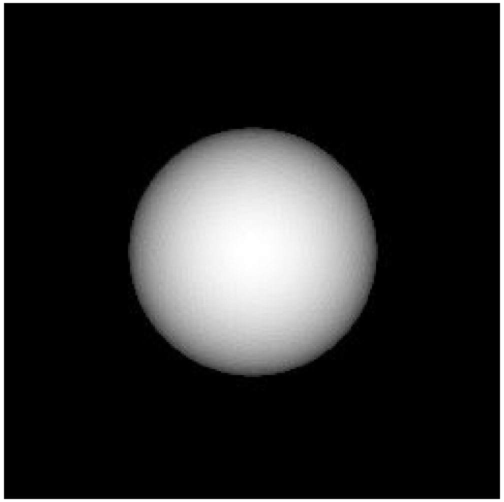
Self-Luminous spherical Lambertian Light Source

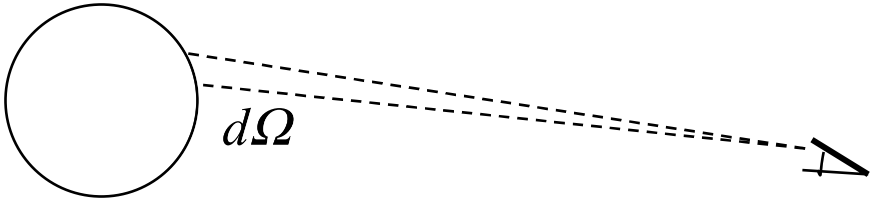
$$\Phi_0 \propto L_0 \cdot d\Omega$$

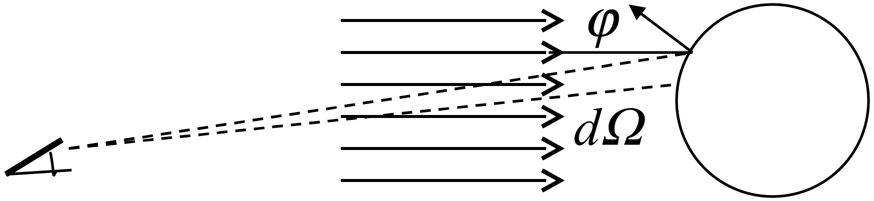


Eye-light illuminated Spherical Lambertian Reflector

$$\Phi_1 \propto L_0 \cdot \cos\varphi \cdot d\Omega$$

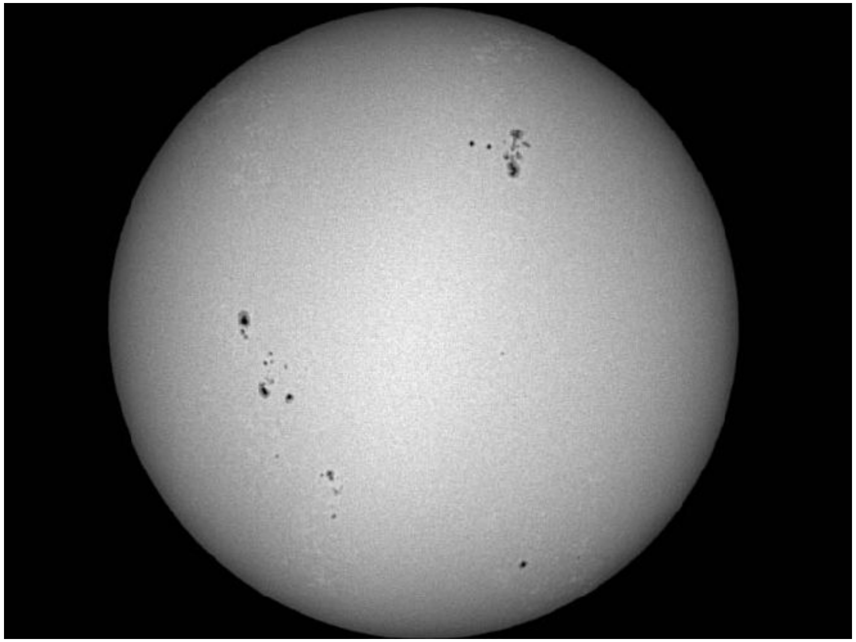






Lambertian Objects II

The Sun



The Moon

- Absorption in photosphere
- Path length through photosphere longer from the Sun’s rim

- Surface covered with fine dust
- Dust on TV visible best from slanted viewing angle

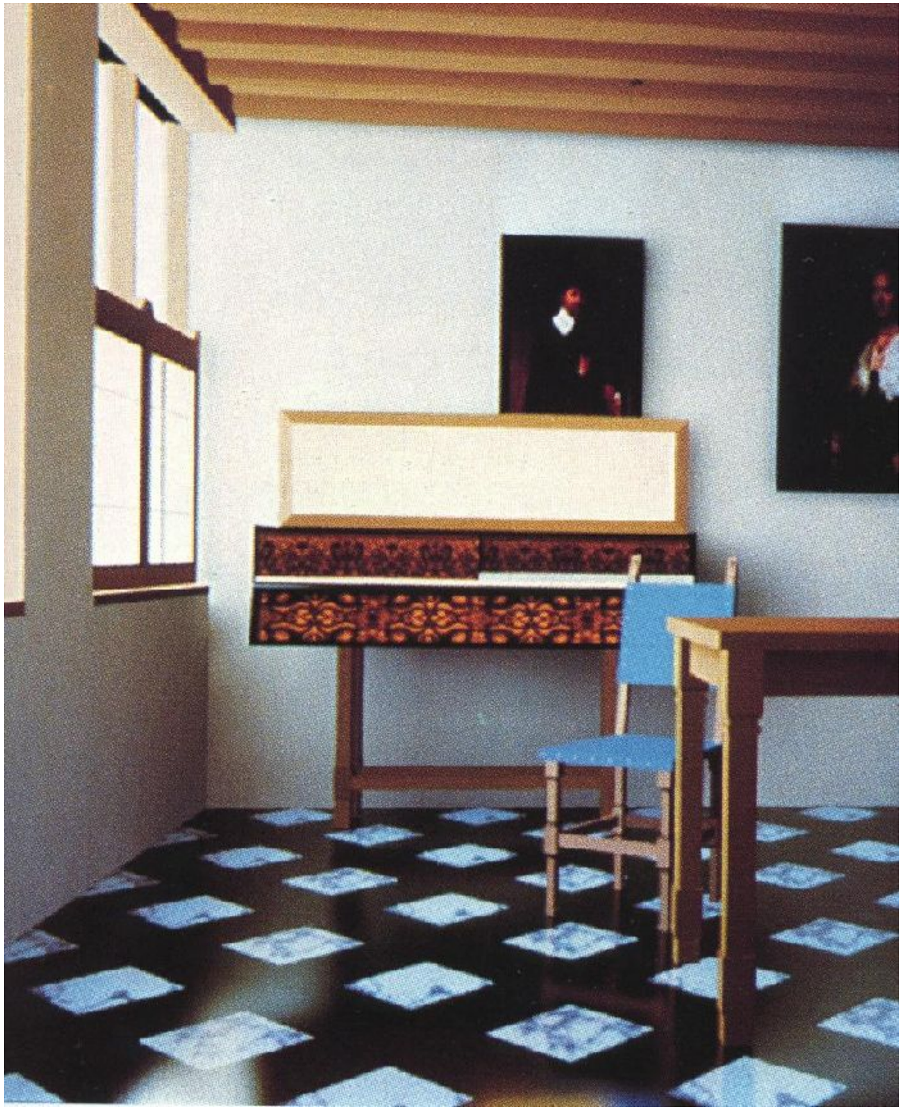
⇒ Neither the Sun nor the Moon are Lambertian

“Diffuse” Reflection

- **Theoretical explanation**
  - Multiple scattering
- **Experimental realization**
  - Pressed magnesium oxide powder
  - Almost never valid at high angles of incidence

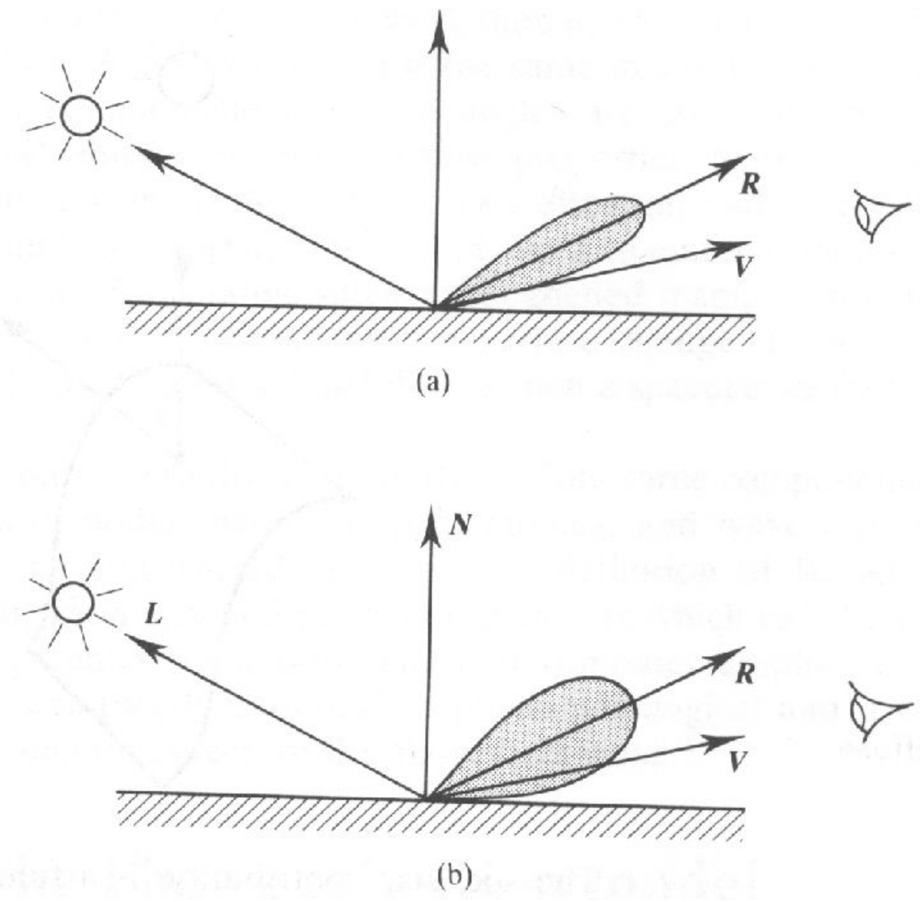
Paint manufacturers attempt to create ideal diffuse paints

Glossy Reflection



Glossy Reflection

- **Due to surface roughness**
- **Empirical models**
  - Phong
  - Blinn-Phong
- **Physical models**
  - Blinn
  - Cook & Torrance

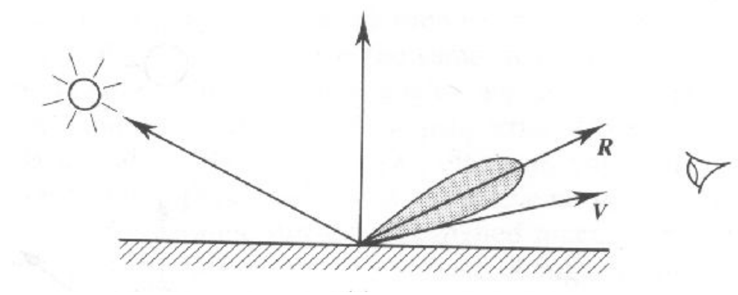


Phong Reflection Model

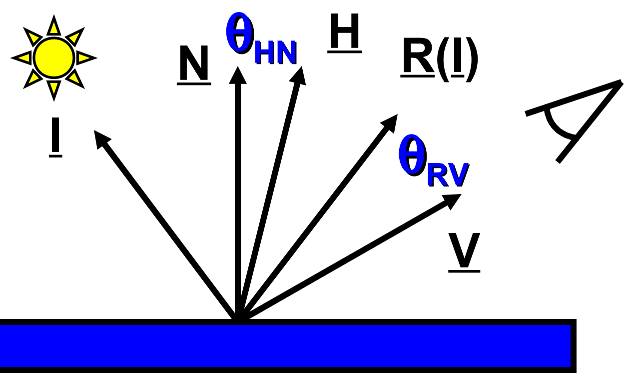
- **Cosine power lobe**

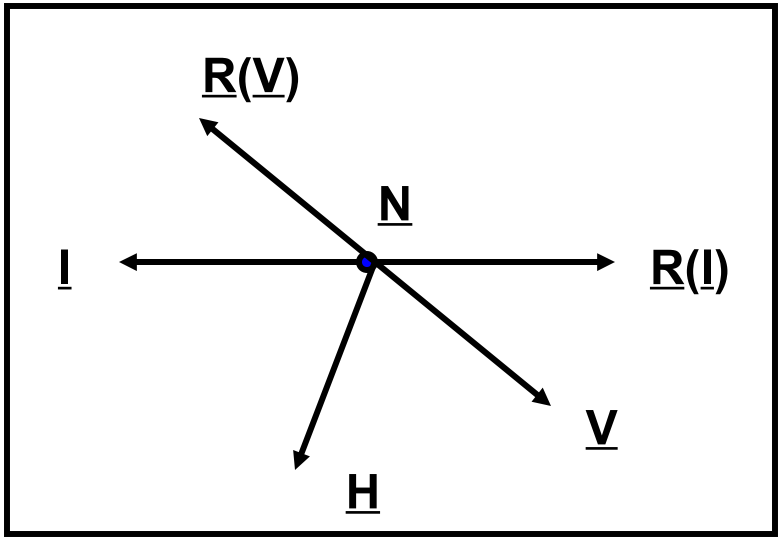
$$f_r(\omega_o, x, \omega_i) = k_s\left(\underline{R}(\underline{I}) \cdot \underline{V}\right)^{k_e}$$

  - $L_{r,s} = L_i\, k_s \cos^{ke} \theta_{RV}$



- **Dot product & power**
- **Not energy conserving/reciprocal**
- **Plastic-like appearance**

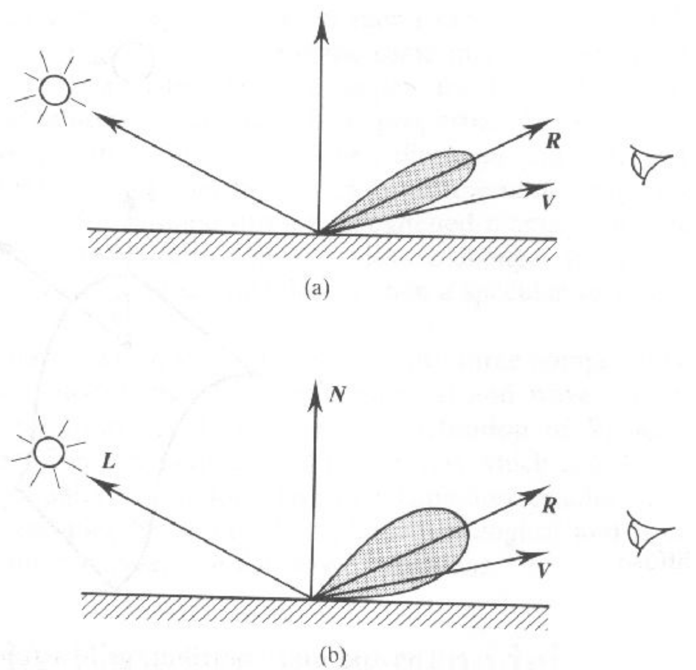


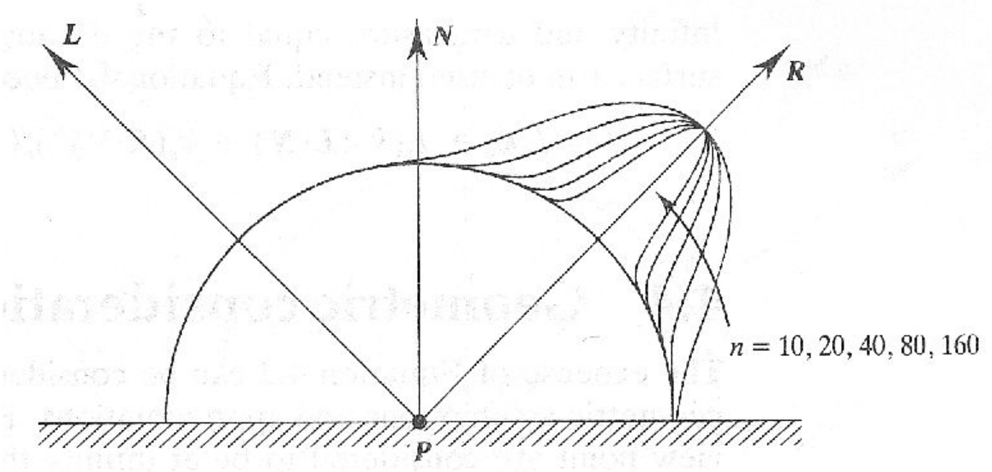


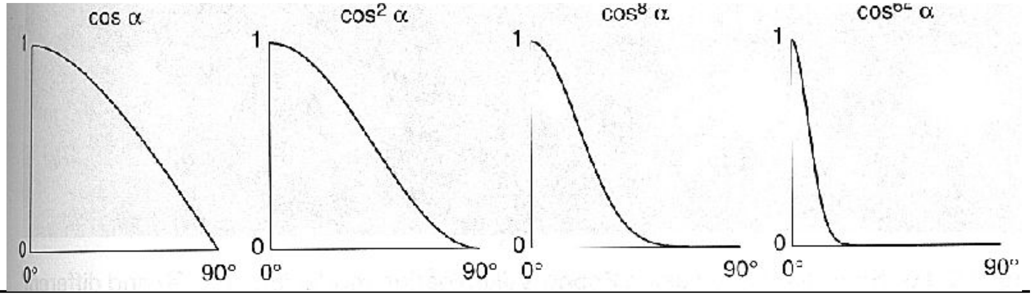
Phong Exponent k_e

$$f_r(\omega_o, x, \omega_i) = k_s\left(\underline{R}(\underline{I}) \cdot \underline{V}\right)^{k_e}$$

- **Determines size of highlight**







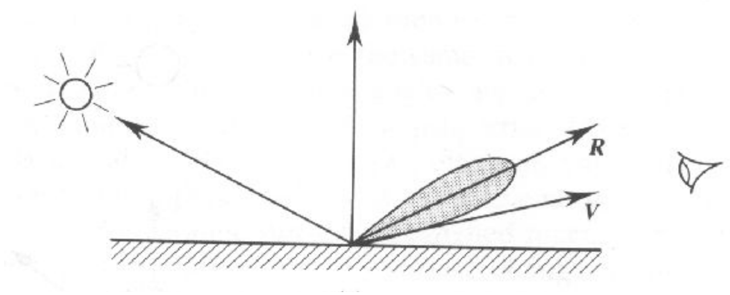
Blinn-Phong Reflection Model

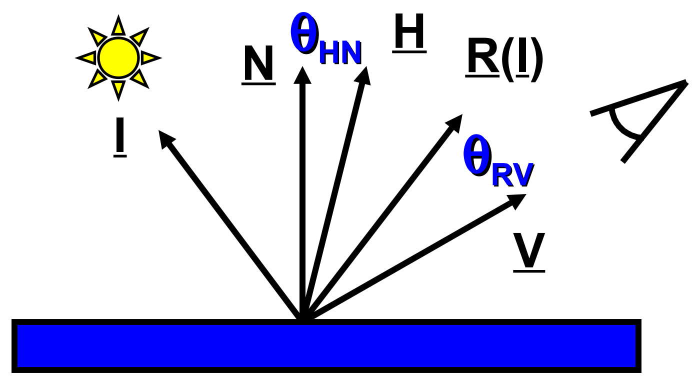
- **Blinn-Phong reflection model**

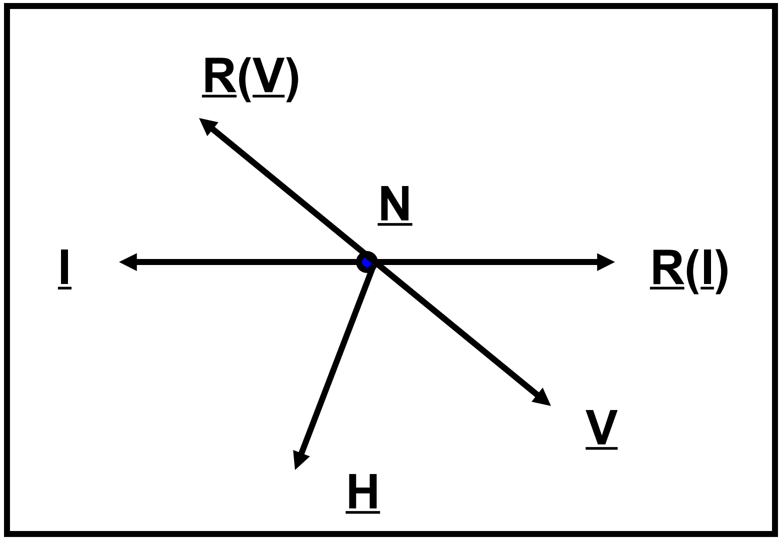
$$f_r(\omega_o, x, \omega_i) = k_s (H \cdot N)^{k_e}$$

  - $L_{r,s} = L_i \, k_s \cos^{ke} \theta_{HN}$
  - $\theta_{RV} \Rightarrow \theta_{HN}$
  - Light source, viewer far away
  - I, R constant: H constant

    θ_{HN} less expensive to compute







Phong Illumination Model

- **Extended light sources: *l* point light sources**

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(Phong)}$$

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(Blinn)}$$

- **Color of specular reflection equal to light source**
- **Heuristic model**
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- **Often: light sources & viewer assumed to be far away**