
Computer Graphics

- Light Transport -

Philipp Slusallek

Overview

- **So far**
 - Nuts and bolts of ray tracing
- **Today**
 - Physics behind ray tracing
 - Physical light quantities
 - Perception of light
 - Light sources
 - Light transport simulation
- **Next lecture**
 - Light-matter interaction
 - Reflectance function
 - Reflection models

What is Light ?

- **Ray**
 - Linear propagation
 - ⇒ **Geometrical optics**
- **Vector**
 - Polarization
 - ⇒ **Jones Calculus**: matrix representation
- **Wave**
 - Diffraction, Interference
 - ⇒ **Maxwell equations**: propagation of light
- **Particle**
 - Light comes in discrete energy quanta: photons
 - ⇒ **Quantum theory**: interaction of light with matter
- **Field**
 - Electromagnetic force: exchange of virtual photons
 - ⇒ **Quantum Electrodynamics (QED)**: interaction between particles

What is Light ?

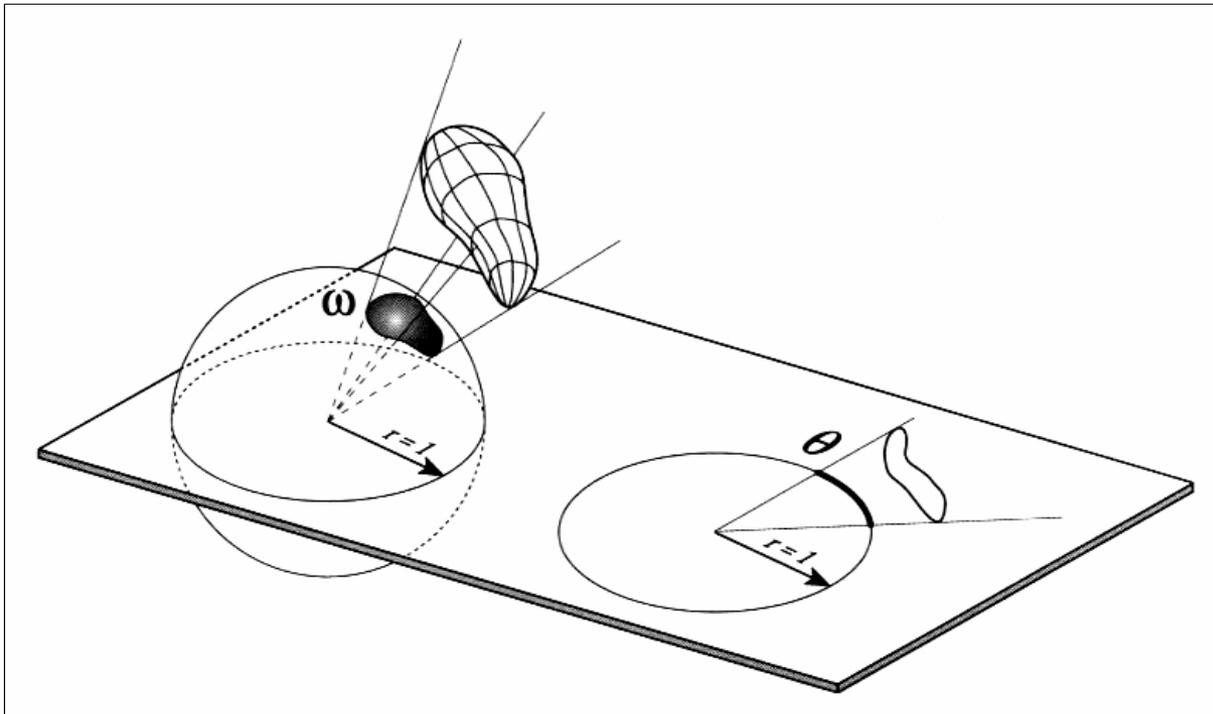
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Light in Computer Graphics

- **Based on human visual perception**
 - Macroscopic geometry
 - Tristimulus color model
 - Psycho-physics: tone mapping, compression, ...
- **Ray optics**
 - Light: scalar, real-valued quantity
 - Linear propagation
 - Macroscopic objects
 - Incoherent light
 - Superposition principle: light contributions add up linearly
 - No attenuation in free space
- **Limitations**
 - Microscopic structures ($\approx \lambda$)
 - Diffraction, Interference
 - Polarization
 - Dispersion

Angle and Solid Angle

- θ the **angle** subtended by a curve in the plane, is the length of the corresponding arc on the unit circle.
- $\Omega, d\omega$ the **solid angle** subtended by an object, is the surface area of its projection onto the unit sphere,
Units for measuring solid angle: steradians [sr]



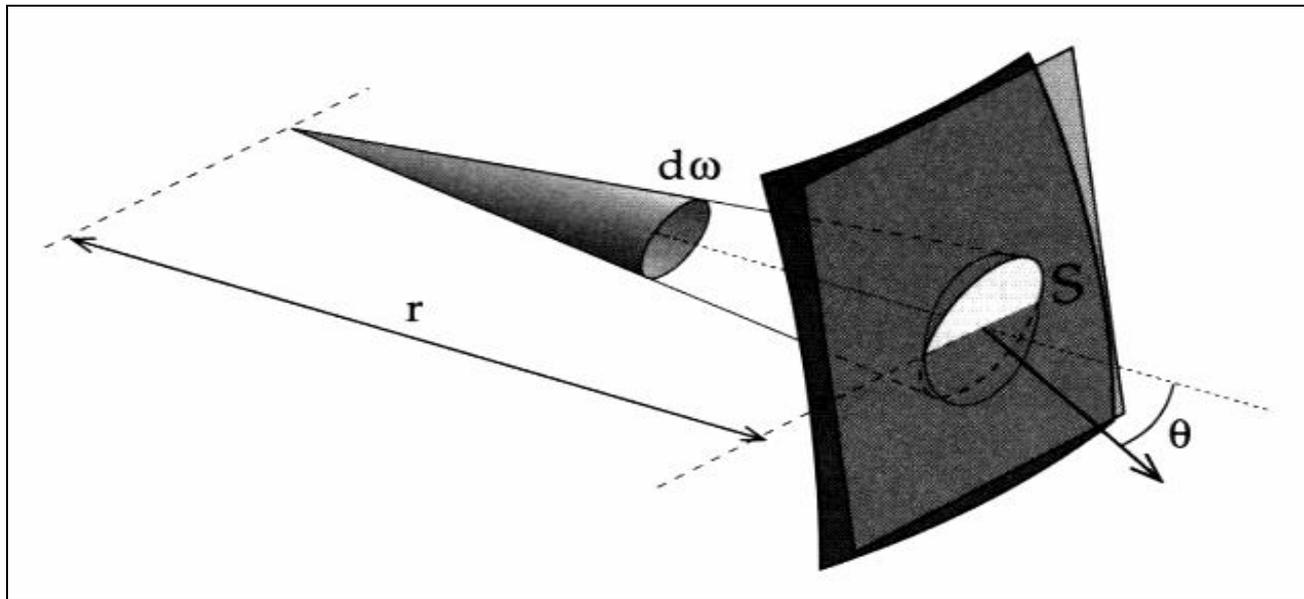
Solid Angle for a Small Area

The solid angle subtended by a small surface patch S with area ΔA is obtained (i) by projecting it orthogonal to the vector r to the origin

$$\Delta A \cos \theta$$

and (ii) dividing by the square of the distance to the origin:

$$\Delta \Omega \approx \frac{\Delta A \cos \theta}{r^2}$$



Radiometry

- **Definition:**

- Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

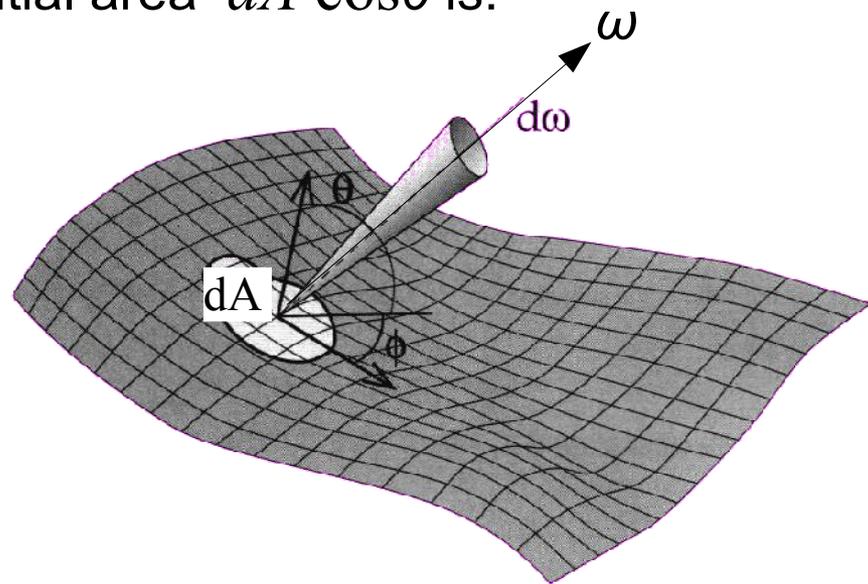
- **Radiometric Quantities**

- energy [watt second] $n \cdot h\nu$ (Photon Energy)
- radiant power (total flux) [watt] Φ
- radiance [watt/(m² sr)] L
- irradiance [watt/m²] E
- radiosity [watt/m²] B
- intensity [watt/sr] I

Radiometric Quantities: Radiance

- Radiance is used to describe radiant energy transfer.
- Radiance L is defined as
 - the power (flux) traveling at some point \underline{x}
 - in a specified direction $\underline{\omega} = (\theta, \varphi)$,
 - per unit area **perpendicular** to the direction of travel,
 - per unit solid angle.
- Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA \cos\theta$ is:

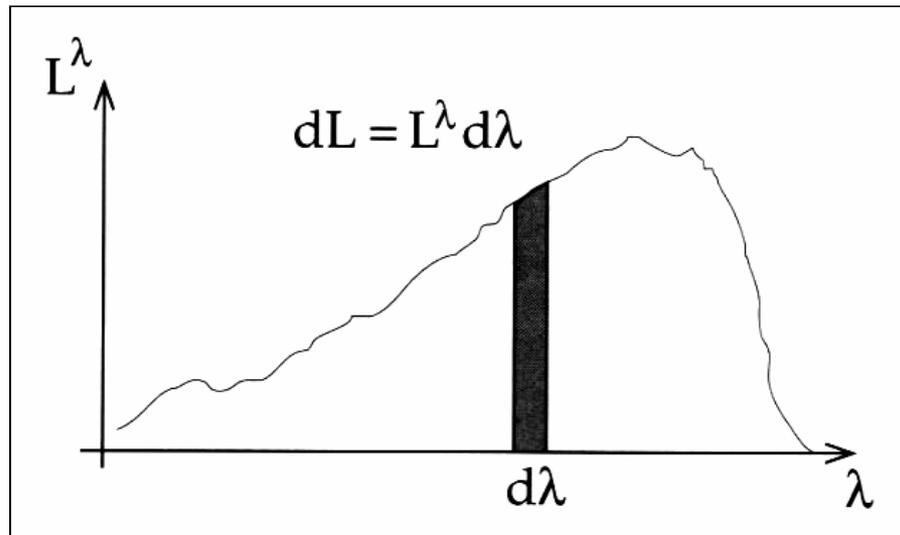
$$d^2\Phi = L(\underline{x}, \underline{\omega}) dA \cos\theta d\omega$$



Spectral Properties

- **Wavelength**

- Since light is composed of electromagnetic waves of different frequencies and wavelengths, most of the energy transfer quantities are continuous functions of wavelength.
- In graphics each measurement $L(\underline{x}, \underline{\omega})$ is for a discrete band of wavelength only (often some abstract R, B, G)



Radiometric Quantities: Irradiance

Irradiance E is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to dA , the **incoming** radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$E = \int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_i(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

Radiometric Quantities: Radiosity

Radiosity B is defined as the **total power per unit area** (flux density) leaving a surface. To obtain the total flux radiated from dA , the **outgoing** radiance L_o is integrated over the upper hemisphere Ω_+ above the surface.

$$B \equiv \frac{d\Phi}{dA}$$

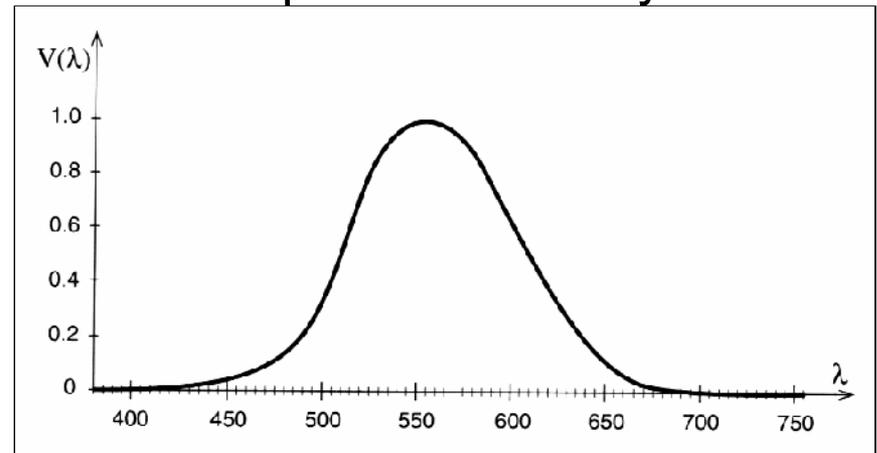
$$d\Phi = \left[\int_{\Omega} L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$B = \int_{\Omega} L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_o(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

Photometry

- **Photometry:**

- The human eye is sensitive to a limited range of radiation wavelengths (roughly from 380nm to 770nm).
- The response of our visual system is not the same for all wavelengths, and can be characterized by the luminous efficiency function $V(\lambda)$, which represents the average human spectral response.
- A set of photometric quantities can be derived from radiometric quantities by integrating them against the luminous efficiency function $V(\lambda)$.
- Separate curves exist for light and dark adaptation of the eye.



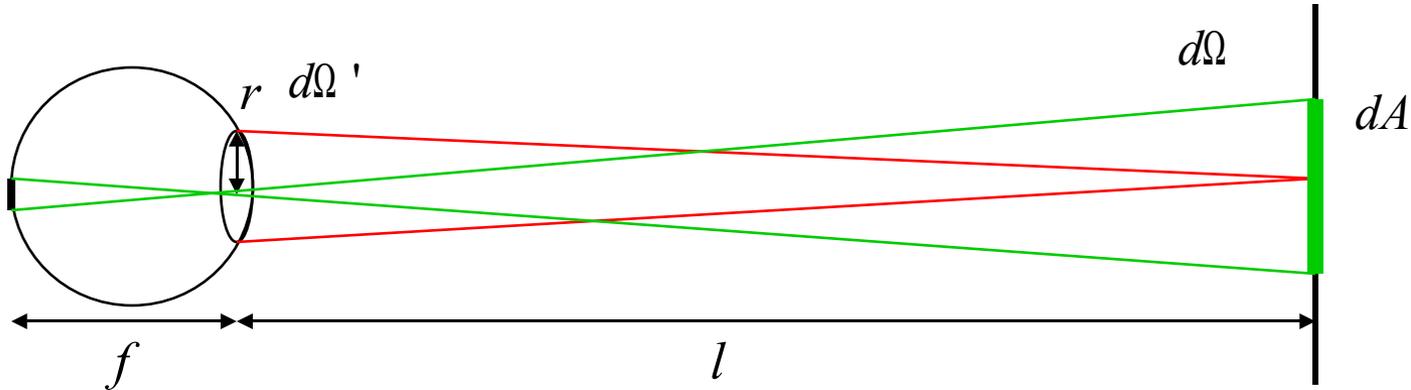
Radiometry vs. Photometry

Physics-based quantities

Perception-based quantities

Radiometry		→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m ²	Radiosity	→	Luminosity	Lux (lm/m ²)
	Irradiance		Illuminance	
W/m ² /sr	Radiance	→	Luminance	cd/m ² (lm/m ² /sr)

Perception of Light



photons / second = **flux** = energy / time = power Φ

rod sensitive to flux

angular extend of rod = **resolution** (≈ 1 arc minute²)

$d\Omega$

projected rod size = **area**

$$dA \approx l^2 \cdot d\Omega$$

Angular extend of pupil aperture ($r \leq 4$ mm) = **solid angle**

$$d\Omega' \approx \pi \cdot r^2 / l^2$$

flux proportional to area and solid angle

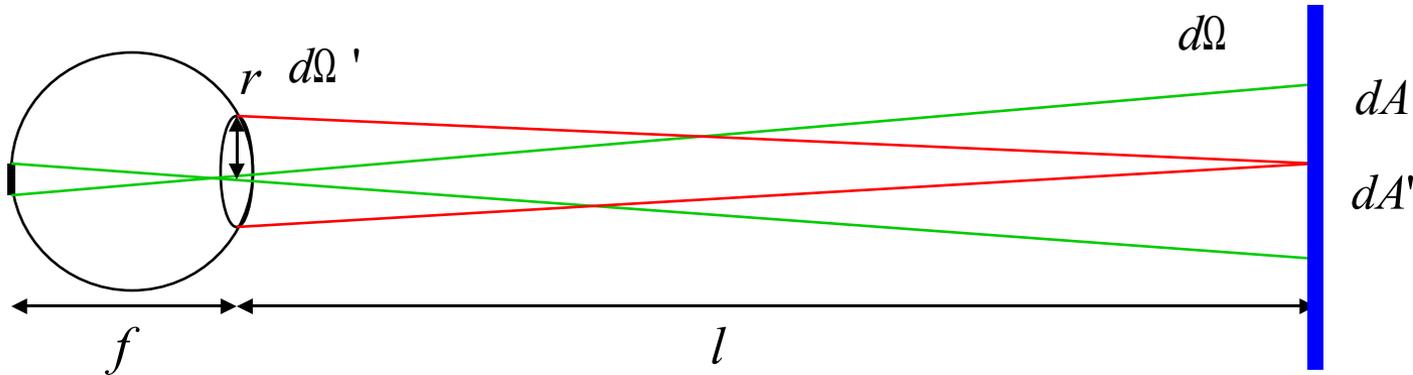
$$\Phi \propto d\Omega' \cdot dA$$

radiance = flux per unit area per unit solid angle

$$L = \frac{\Phi}{d\Omega' \cdot dA}$$

The eye detects radiance

Brightness Perception



As l increases: $\Phi_0 \propto dA \cdot d\Omega' = l^2 d\Omega \cdot \pi \frac{r^2}{l^2} = \text{const}$

- $dA' > dA$: photon flux per rod stays constant
- $dA' < dA$: photon flux per rod decreases

Where does the Sun turn into a star ?

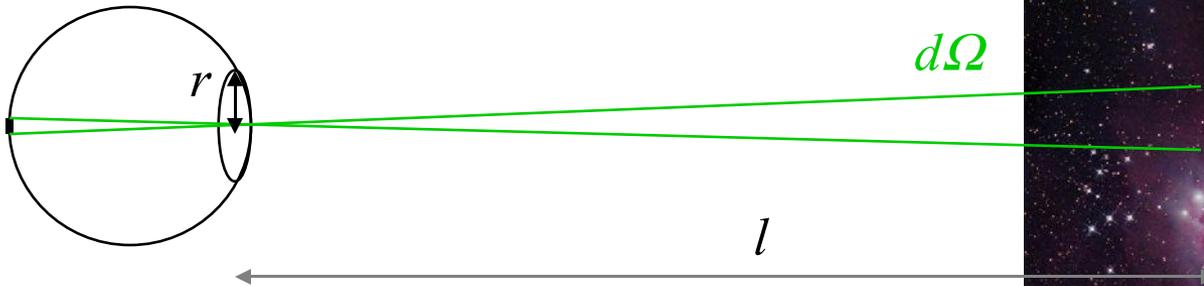
– Depends on apparent Sun disc size on retina

\Rightarrow Photon flux per rod stays the same on Mercury, Earth or Neptune

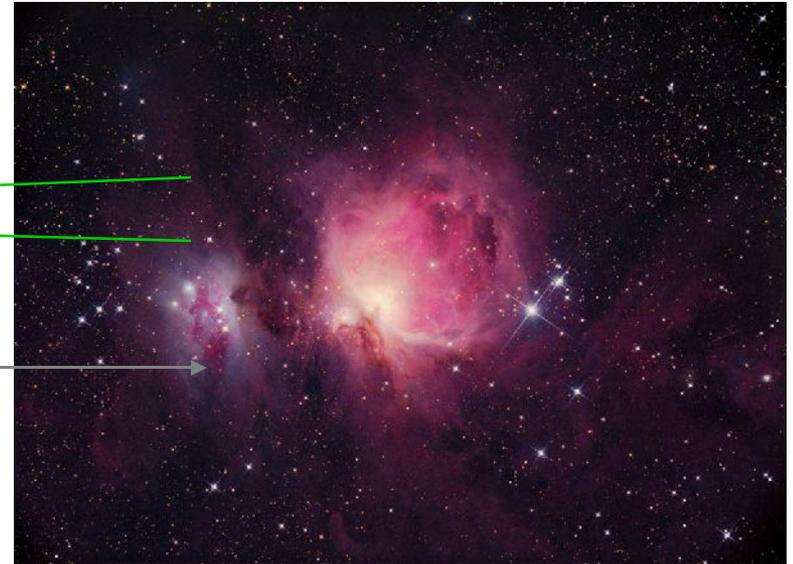
\Rightarrow Photon flux per rod decreases when $d\Omega' < 1$ arc minute (beyond Neptune)

Brightness Perception II

Extended light source

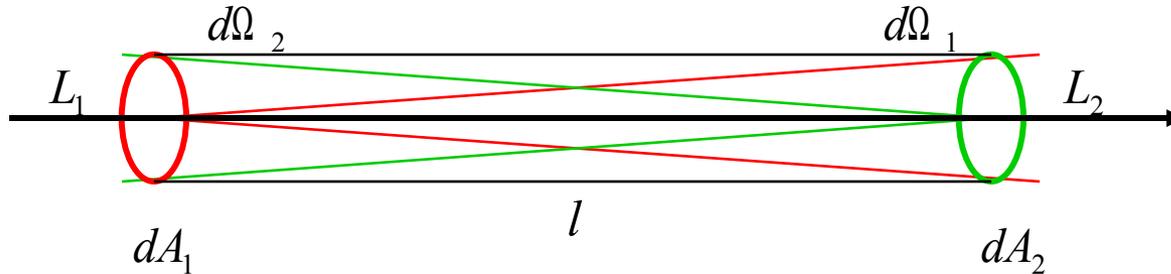


$$\Phi_0 = L_0 \cdot \pi \cdot r^2 \cdot d\Omega$$



- ⇒ Flux does not depend on distance l
- ⇒ Nebulae always appear b/w

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 \cdot d\Omega_1 \cdot dA_1 = L_2 \cdot d\Omega_2 \cdot dA_2$$

From geometry follows

$$d\Omega_1 = \frac{dA_2}{l^2} \quad d\Omega_2 = \frac{dA_1}{l^2}$$

Ray throughput

$$T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$$

$$L_1 = L_2$$

The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

Point Light Source

- **Point light with isotropic radiance**

- Power (total flux) of a point light source

- $\forall \Phi_g = \text{Power of the light source [watt]}$

- Intensity of a light source

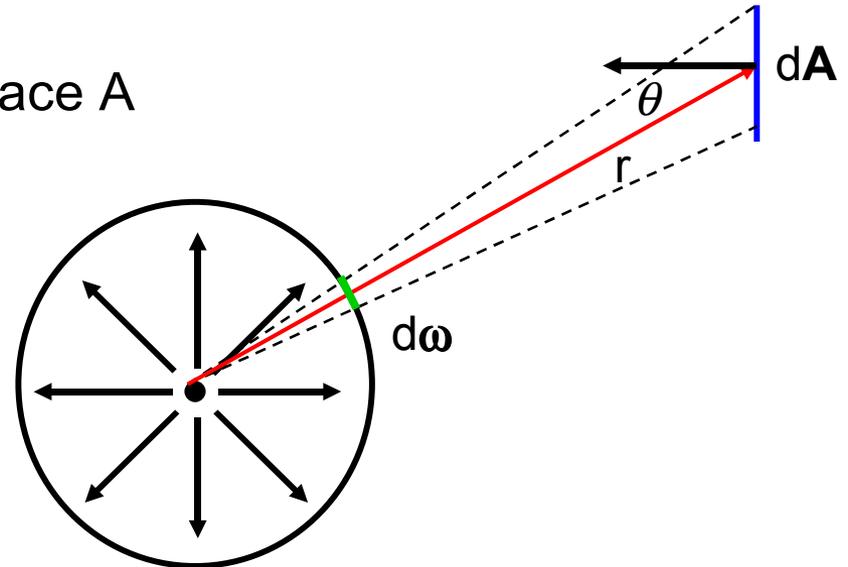
- $I = \Phi_g / (4\pi \text{ sr})$ [watt/sr]

- Irradiance on a sphere with radius r around light source:

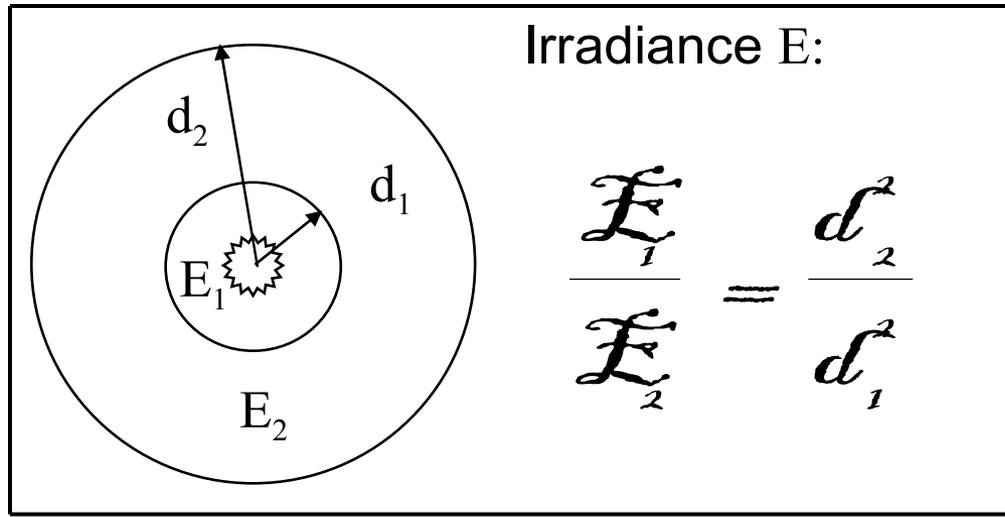
- $E_r = \Phi_g / (4\pi r^2)$ [watt/m²]

- Irradiance on some other surface A

$$\begin{aligned} E(x) &= \frac{d\Phi_g}{dA} = I \frac{d\omega}{dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{dA \cos\theta}{r^2 dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{\cos\theta}{r^2} \end{aligned}$$



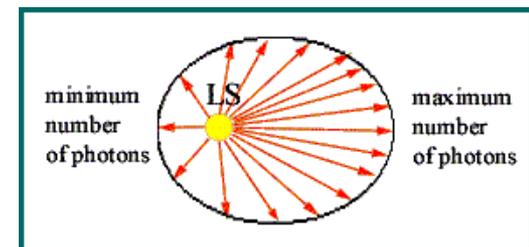
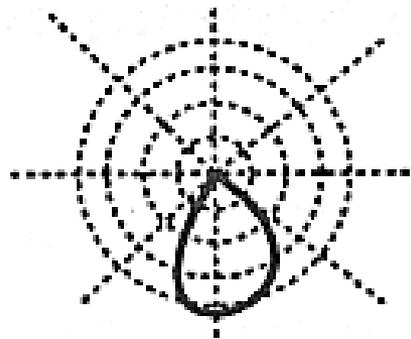
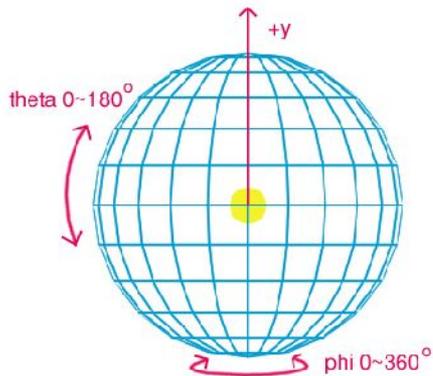
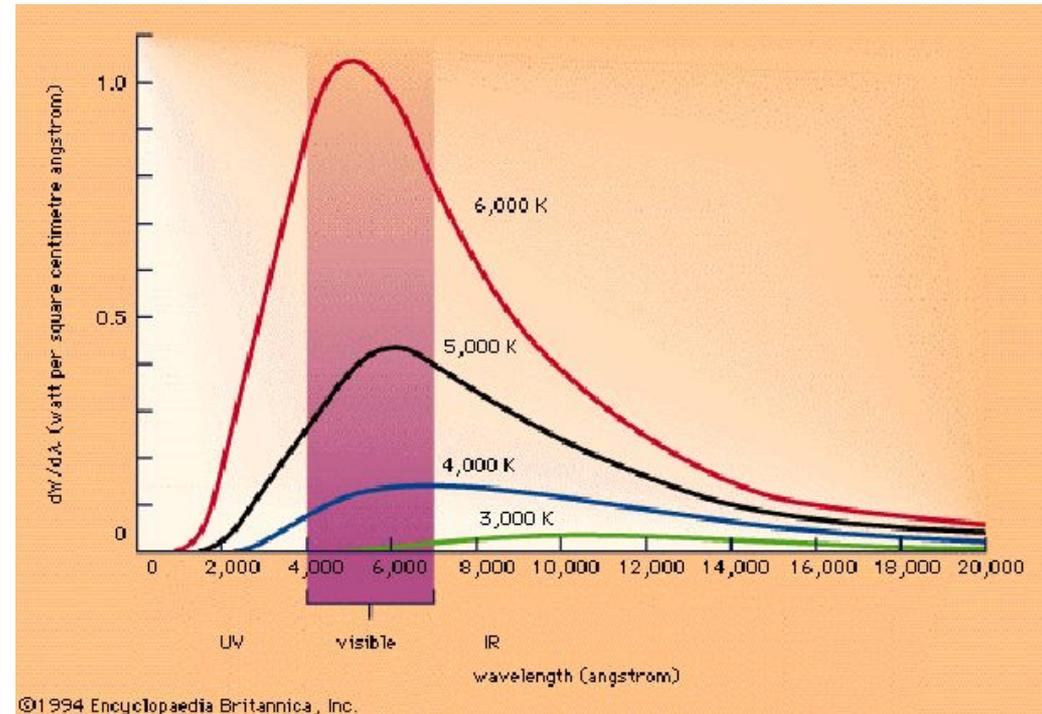
Inverse Square Law



- **Irradiance E : power per m^2**
 - Illuminating quantity
- **Distance-dependent**
 - Double distance from emitter: sphere area four times bigger
- **Irradiance falls off with inverse of squared distance**
 - For point light sources

Light Source Specifications

- **Power (total flux)**
 - Emitted energy / time
- **Active emission size**
 - Point, area, volume
- **Spectral distribution**
 - Thermal, line spectrum
- **Directional distribution**
 - Goniometric diagram



Sky Light

- **Sun**
 - Point source (approx.)
 - White light (by def.)
- **Sky**
 - Area source
 - Scattering: blue
- **Horizon**
 - Brighter
 - Haze: whitish
- **Overcast sky**
 - Multiple scattering in clouds
 - Uniform grey



Courtesy Lynch & Livingston

Light Source Classification

Radiation characteristics

- **Directional light**
 - Spot-lights
 - Beamers
 - Distant sources
- **Diffuse emitters**
 - Torchieres
 - Frosted glass lamps
- **Ambient light**
 - “Photons everywhere”

Emitting area

- **Volume**
 - neon advertisements
 - sodium vapor lamps
- **Area**
 - CRT, LCD display
 - (Overcast) sky
- **Line**
 - Clear light bulb, filament
- **Point**
 - Xenon lamp
 - Arc lamp
 - Laser diode

Surface Radiance

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L_i(\underline{x}, \underline{\omega}_i) \cos \theta_i d\underline{\omega}_i$$

- **Visible surface radiance**
 - Surface position
 - Outgoing direction
 - Incoming illumination direction
- **Self-emission**
- **Reflected light**
 - Incoming radiance from all directions
 - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function)

$L(\underline{x}, \underline{\omega}_o)$

\underline{x}

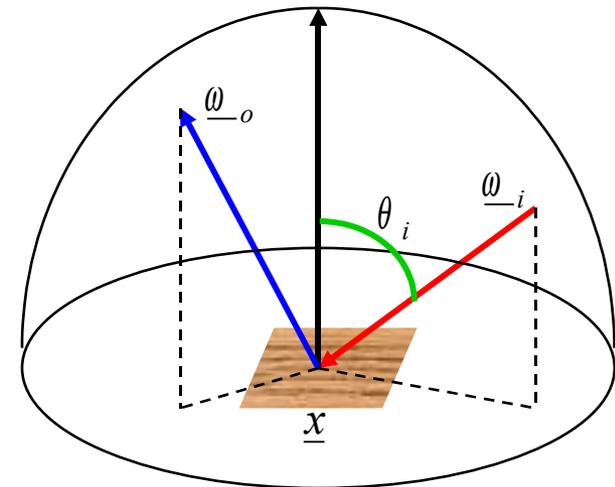
$\underline{\omega}_o$

$\underline{\omega}_i$

$L_e(\underline{x}, \underline{\omega}_o)$

$L_i(\underline{x}, \underline{\omega}_i)$

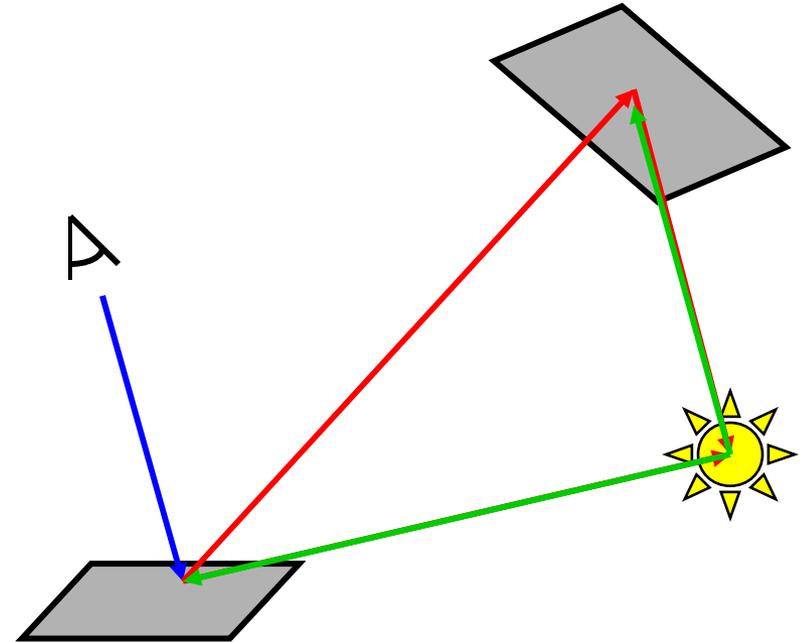
$f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o)$



Ray Tracing

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i d\underline{\omega}_i$$

- **Simple ray tracing**
 - Illumination from light sources only - **local illumination** (integral \rightarrow sum)
 - Evaluates angle-dependent reflectance function - **shading**
- **Advanced ray tracing techniques**
 - Recursive ray tracing
 - Multiple reflections/refractions (for specular surfaces)
 - Forward ray tracing
 - Stochastic sampling (Monte Carlo methods)
 - Photon mapping
 - Combination of both



Light Transport in a Scene

- **Scene**
 - Lights (emitters)
 - Object surfaces (partially absorbing)
 - **Illuminated object surfaces become emitters, too !**
 - Radiosity = Irradiance – absorbed photons flux density
 - Radiosity: photons per second per m^2 leaving surface
 - Irradiance: photons per second per m^2 incident on surface
 - **Light bounces between all mutually visible surfaces**
 - **Invariance of radiance in free space**
 - No absorption in-between objects
 - **Dynamic Energy Equilibrium**
 - emitted photons = absorbed photons (+ escaping photons)
- **Global Illumination**

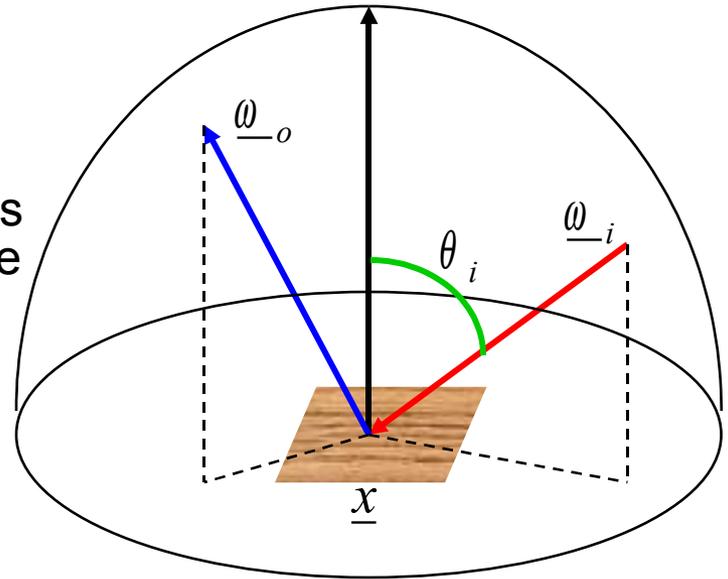
(Surface) Rendering Equation

- In Physics: *Radiative Transport Equation*
- Expresses energy equilibrium in scene

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L_i(\underline{x}, \underline{\omega}_i) \cos \theta_i d\omega_i$$

total radiance = emitted radiance + reflected radiance

- **First term: emissivity of the surface**
 - non-zero only for light sources
- **Second term: reflected radiance**
 - integral over all possible incoming directions of irradiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
 - unknown radiance appears on lhs and inside the integral
 - Numerical methods necessary to compute approximate solution



Rendering Equation II

- **Outgoing illumination at a point**

$$\begin{aligned} L(\underline{x}, \underline{\omega}_o) &= L_e(\underline{x}, \underline{\omega}_o) + L_r(\underline{x}, \underline{\omega}_o) \\ &= L_e(\underline{x}, \underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L_i(\underline{x}, \underline{\omega}_i) \cos \theta_i \, d\underline{\omega}_i \end{aligned}$$

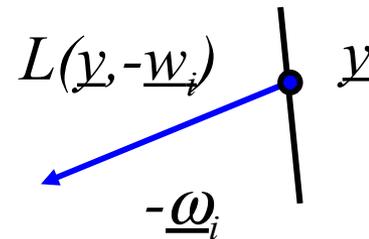
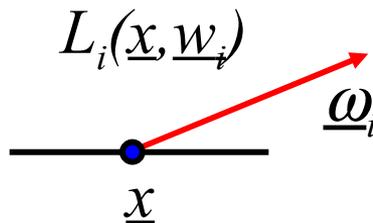
- **Linking with other surface points**

- Incoming radiance at \underline{x} is outgoing radiance at \underline{y}

$$L_i(\underline{x}, \underline{\omega}_i) = L(\underline{y}, -\underline{\omega}_i) = L(RT(\underline{x}, \underline{\omega}_i), -\underline{\omega}_i)$$

- Ray-Tracing operator

$$\underline{y} = RT(\underline{x}, \underline{\omega}_i)$$



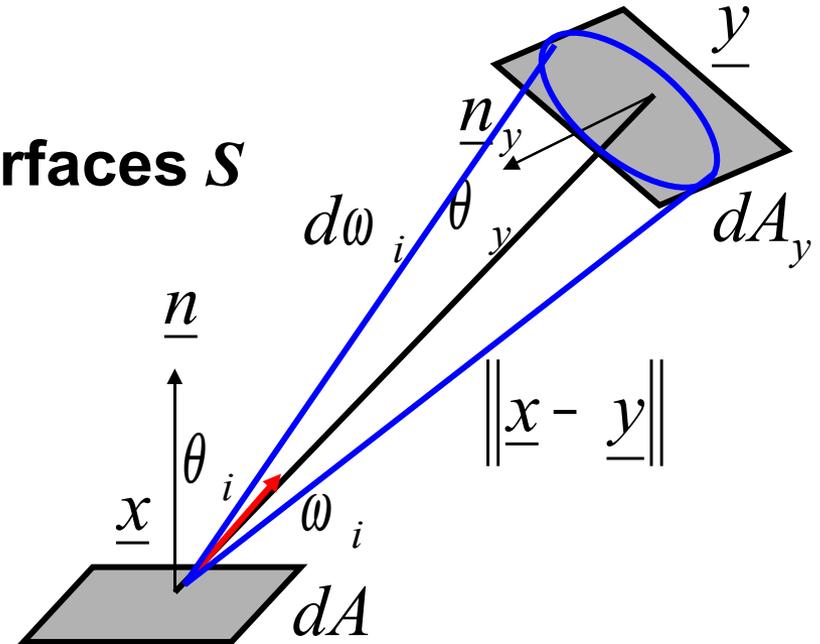
Rendering Equation III

- **Directional parameterization**

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L(\underline{y}(\underline{x}, \underline{\omega}_i), -\underline{\omega}_i) \cos \theta_i d\omega_i$$

- **Re-parameterization over surfaces S**

$$d\omega_i = \frac{\cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$$



$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\underline{y} \in S} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L(\underline{y}, \underline{\omega}_i(\underline{x}, \underline{y})) V(\underline{x}, \underline{y}) \frac{\cos \theta_i \cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$$

Rendering Equation IV

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\underline{y} \in S} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L(\underline{y}, \underline{\omega}_i(\underline{x}, \underline{y})) V(\underline{x}, \underline{y}) \frac{\cos \theta_i \cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$$

- **Geometry term** $G(\underline{x}, \underline{y}) = V(\underline{x}, \underline{y}) \frac{\cos \theta_i \cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$
- **Visibility term** $V(\underline{x}, \underline{y}) = \begin{cases} 1 & \text{if visible} \\ 0 & \text{if not visible} \end{cases}$
- **Integration over all surfaces**

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\underline{y} \in S} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L(\underline{y}, \underline{\omega}_i(\underline{x}, \underline{y})) G(\underline{x}, \underline{y}) dA_y$$

Rendering Equation: Approximations

- **Using RGB instead of full spectrum**
 - follows roughly the eye's sensitivity
- **Dividing scene surfaces into small patches**
 - Assumes locally constant reflection, visibility, geometry terms
- **Sampling hemisphere along finite, discrete directions**
 - simplifies integration to summation
- **Reflection function model**
 - Parameterized function
 - ambient: constant, non-directional, background light
 - diffuse: light reflected uniformly in all directions
 - specular: light of higher intensity in mirror-reflection direction
 - Lambertian surface (only diffuse reflection) - **Radiosity**
- *Approximations based on empirical foundations*
 - *An example: polygon rendering in OpenGL*

Radiosity Equation

$$L(\underline{x}, \underline{\omega}_o) = L_e(\underline{x}, \underline{\omega}_o) + \int_{\underline{y} \in S} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) L(\underline{y}, \underline{\omega}_i(x, y)) G(\underline{x}, \underline{y}) dA_y$$

- **Diffuse reflection only** $\rho(\underline{x}) = \int_0^{\pi/2} \int_0^{2\pi} f_r(\underline{x}) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} f_r(\underline{x}) \cos\theta \sin\theta \, d\theta \, d\phi$

$$f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o) = f_r(\underline{x}) \Rightarrow \rho(\underline{x}) = \pi f_r(\underline{x})$$

diffuse
reflectance

- **Radiance \Rightarrow Radiosity** $L(\underline{x}, \underline{\omega}_o) = B(\underline{x})/\pi$

$$\begin{aligned} B(\underline{x}) &= B_e(\underline{x}) + \rho(\underline{x}) E(\underline{x}) \\ &= B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y}) B(\underline{y}) dA_y \end{aligned}$$

- **Form factor** $F(\underline{x}, \underline{y}) = \frac{G(\underline{x}, \underline{y})}{\pi}$ percentage of light leaving dA_y that arrives at dA

Linear Operators

- **Properties**

- Fredholm integral of 2nd kind
- Global linking
 - Potentially each point with each other
 - Often sparse systems (occlusions)
- No consideration of volume effects!!

- **Linear operator**

- acts on functions like matrices act on vectors
- Superposition principle
- Scaling and addition

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y}) B(\underline{y}) dA_y$$

$$f(x) = g(x) + (K \circ f)(x)$$

$$(K \circ f)(x) \equiv \int k(x, y) f(y) dy$$

$$K \circ (af + bg) = a(K \circ f) + b(K \circ g)$$

Formal Solution of Integral Equations

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y}) B(\underline{y}) dA_y$$

- **Integral equation**

$$B = B_e + K \circ B$$

$$\Rightarrow (I - K) \circ B = B_e$$

- **Formal solution**

$$B = (I - K)^{-1} \circ B_e$$

- **Neumann series**

$$\frac{1}{1-x} = 1 + x + x^2 + \dots$$

$$\frac{1}{I-K} = I + K + K^2 + \dots$$

$$\begin{aligned} (I-K) \frac{1}{I-K} &= (I-K)(I + K + K^2 + \dots) \\ &= (I + K + K^2 + \dots) - (K + K^2 + \dots) = I \end{aligned}$$

Formal Solutions II

- **Successive approximation**

$$\begin{aligned}\frac{1}{I - K} B_e &= B_e + K \circ B_e + K^2 \circ B_e + \dots \\ &= (B_e + K \circ (B_e + K \circ (B_e + \dots)))\end{aligned}$$

- Direct light from the light source
- Light which is reflected and transported one time
- Light which is reflected and transported n-times

$$B_1 = B_e$$

$$B_2 = B_e + K \circ B_1$$

...

$$B_n = B_e + K \circ B_{n-1}$$

Lighting Simulation



Courtesy Karol Myszkowski, MPII

Wrap-up

- **Physical Quantities in Rendering**
 - Radiance
 - Radiosity
 - Irradiance
 - Intensity
- **Light Perception**
- **Light Sources**
- **Rendering Equation**
 - Integral equation
 - Balance of radiance
- **Radiosity**
 - Diffuse reflectance function
 - Radiative equilibrium between emission and absorption, escape
 - System of linear equations
 - Iterative solution