UNIVERSITÄT DES SAARLANDES
Prof Dr. Philipp Slusallek
Lehrstuhl für ComputerGraphik
Sven Woop (woop@graphics.cs.uni-sb.de)
Stefan Popov (popov@graphics.cs.uni-sb.de)

Monday, 29nd January 2007

Computer Graphics I
Assignment 11

Submission deadline for the exercises: Thursday, 8th February 2007

11.1 Extended Bresenham (50 Points)

Consider the Bresenham algorithm for a line with a slope in the range of 0 to 1 and integer start and endpoint. Exploit symmetry in the computation of the decision variable to draw the line from both sides simultaneously using a single decision variable d only. You do not have to implement the algorithm, but to derive how to initialize x, y, and the decision variable d and how to update them in each case.

11.2 Bresenham for Parabola (50 Points)

Develop a mid-point algorithm to draw the fixed parabola $y = \frac{1}{12}x^2 + 2$ in the range of 0 to 6. Again derive the formulas to initialize and update x,y, and the decision variable d.

Solutions

11.1 Extended Bresenham

Let the integer endpoints of the line to draw be $P = (p_x, p_y)$ and $Q = (q_x, q_y)$. As abbrevation we define $\Delta x = q_x - p_x$ and $\Delta y = q_y - p_y$. The distance function for the line is given by $D(x, y) = \Delta y \cdot x - \Delta x \cdot y + (\Delta x \cdot q_y - \Delta y \cdot p_x)$.

a) Left to Right: Lets first derive the midpoint algorithm if drawing the line from left to right. The initial values for x_0 and y_0 are $x_0 = p_x$ and $y_0 = p_y$. The initial value for the decision variable is $d_0 = D(x_0 + 1, y_0 + 1/2) = \Delta y(x_0 + 1) - \Delta x(y_0 + 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) = \Delta y - 1/2\Delta x$ as the point (x_0, y_0) lies on the line.

i) Case $d_n \leq 0$: We set $(x_{n+1}, y_{n+1}) = (x_n + 1, y_n)$ and get the new decision variable by:

$$\begin{aligned}
d_{n+1} &= D(x_{n+1} + 1, y_{n+1} + 1/2) \\
&= D(x_n + 2, y_n + 1/2) \\
&= \Delta y(x_n + 2) - \Delta x(y_n + 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n + 1) + \Delta y - \Delta x(y_n + 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n + 1) - \Delta x(y_n + 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) + \Delta y \\
&= D(x_n + 1, y_n + 1/2) + \Delta y \\
&= d_n + \Delta y
\end{aligned}$$

ii) Case $d_n > 0$: We update $(x_{n+1}, y_{n+1}) = (x_n + 1, y_n + 1)$ and get the new decision variable by:

$$\begin{aligned}
d_{n+1} &= D(x_{n+1} + 1, y_{n+1} + 1/2) \\
&= D(x_n + 2, y_n + 3/2) \\
&= \Delta y(x_n + 2) - \Delta x(y_n + 3/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n + 1) + \Delta y - \Delta x(y_n + 1/2) - \Delta x + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n + 1) + \Delta x(y_n + 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) + \Delta y - \Delta x \\
&= D(x_n + 1, y_n + 1/2) + \Delta y - \Delta x \\
&= d_n + \Delta y - \Delta x
\end{aligned}$$

b) Right to Left: Lets now do the same, but from right to left. The initial values for x_0 and y_0 are $x_0 = q_x$ and $y_0 = q_y$. The initial value for the decision variable is $d_0 = D(x_0 - 1, y_0 - 1/2) = \Delta y(x_0 - 1) - \Delta x(y_0 - 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) = -\Delta y + 1/2\Delta x$ as the point (x_0, y_0) lies on the line.

i) Case $d_n < 0$: We update $(x_{n+1}, y_{n+1}) = (x_n - 1, y_n)$ and get the new decision variable by:

$$\begin{aligned}
d_{n+1} &= D(x_{n+1} - 1, y_{n+1} - 1/2) \\
&= D(x_n - 2, y_n - 1/2) \\
&= \Delta y(x_n - 2) - \Delta x(y_n - 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n - 1) - \Delta y - \Delta x(y_n - 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n - 1) - \Delta x(y_n - 1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) - \Delta y \\
&= D(x_n - 1, y_n - 1/2) - \Delta y \\
&= d_n - \Delta y
\end{aligned}$$

ii) Case $d_n \geq 0$: We update $(x_{n+1}, y_{n+1}) = (x_n - 1, y_n - 1)$ and get the new decision variable by:

$$
\begin{aligned}
d_{n+1} &= D(x_{n+1}-1, y_{n+1}-1/2) \\
&= D(x_n-2, y_n-3/2) \\
&= \Delta y(x_n-2) - \Delta x(y_n-3/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n-1) - \Delta y - \Delta x(y_n-1/2) + \Delta x + (\Delta x \cdot q_y - \Delta y \cdot p_x) \\
&= \Delta y(x_n-1) - \Delta x(y_n+1/2) + (\Delta x \cdot q_y - \Delta y \cdot p_x) - \Delta y + \Delta x \\
&= D(x_n+1, y_n+1/2) + \Delta y - \Delta x \\
&= d_n - \Delta y + \Delta x
\end{aligned}
$$

11.2 Bresenham for Parabola

Similar to the Bresenham algorithm for lines we need a distance estimation function by for the curve. For the parabola we take the following one:

$$
D(x, y) = \frac{1}{12}x^2 + 2 - y
$$

It yields $D(x, y) = 0$ if and only if the point (x, y) lies on the curve. The distance function is smaller than 0 if the point lies above the curve and greater than 0 if it is below the curve. The initial values for x_0 and y_0 are set to $x_0 = 0$ and $y_0 = 1$ which are on the curve. The initial value for the midpoint distance d_0 can be computed by $d_0 = D(x_0 + 1, y_0 + \frac{1}{2}) = D(1, \frac{3}{2}) = 1\frac{5}{12}$.
and if $d_n > 0$ n the next step.

a) Case $d_n \leq 0$: We update $(x_{n+1}, y_{n+1}) = (x_n + 1, y_n)$ and compute the new decision variable d_{n+1} by:

$$
\begin{aligned}
d_{n+1} &= D(x_{n+1}+1, y_{n+1}+\frac{1}{2}) \\
&= D(x_n+2, y_n+\frac{1}{2}) \\
&= \frac{1}{12}(x_n+2)^2 + 2 - (y_n+\frac{1}{2}) \\
&= \frac{1}{12}(x_n+1)^2 + \frac{2}{12}(x_n+1) + \frac{1}{12} + 2 - (y_n+\frac{1}{2}) \\
&= \frac{1}{12}(x_n+1)^2 + 2 - (y_n+\frac{1}{2}) + \frac{2}{12}(x_n+1) + \frac{1}{12} \\
&= d_n + \frac{2}{12}(x_n+1) + \frac{1}{12} \\
&= d_n + \frac{1}{12}(2x_n+3)
\end{aligned}
$$

b) Case $d_n > 0$: We update $(x_{n+1}, y_{n+1}) = (x_n + 1, y_n + 1)$ and compute the new decision variable d_{n+1} by:

$$
\begin{aligned}
d_{n+1} &= D(x_{n+1}+1, y_{n+1}+\frac{1}{2}) \\
&= D(x_n+2, y_n+\frac{3}{2}) \\
&= \frac{1}{12}(x_n+2)^2 + 2 - (y_n+\frac{3}{2}) \\
&= \frac{1}{12}(x_n+1)^2 + \frac{2}{12}(x_n+1) + \frac{1}{12} + 2 - (y_n+\frac{1}{2}) - 1 \\
&= \frac{1}{12}(x_n+1)^2 + 1 - (y_n+\frac{1}{2}) + \frac{2}{12}(x_n+1) + \frac{1}{12} - \frac{12}{12} \\
&= d_n + \frac{2}{12}(x_n+1) - \frac{11}{12} \\
&= d_n + \frac{1}{12}(2x_n-9)
\end{aligned}
$$