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# Computer Graphics I
## Assignment 6

**Submission deadline for the exercises**: Thursday, 21th December 2006

## 6.1 Gamma correction (25 Points)

Graphics card gets 8 bit color components as input from an application. These 8 bit values represent the fractional part of the color value, thus go from $0.00000000b = 0$ to $0.11111111b$ which is approximately 1. Assume that the graphics card has the capabilities to perform gamma correction by a function $f(x) = x^\gamma$ with $\gamma$ in the range of $[1, 2.5]$. Which accuracy (in number of bits) is required after the gamma correction to accurately represent the corrected signal?

## 6.2 Transforms in colorspace (50 points)

Many of today's monitors allow to change the color temperature of the image. A change in the color temperature corresponds to a movement of the white point in the color diagram and does not change the primary colors itself.
A monitor corresponding to the standard "ITU-R BT.709 (which is identical to sRGB) uses the following color coordinates $(x_r, y_r) = (0.640, 0.330), (x_g, y_g) = (0.300, 0.600), (x_b, y_b) = (0.150, 0.060)$, and a D65 white point $(x_w, y_w) = (0, 3127, 0.3290)$.

a) Can you think of an easy way to realize a movement of the white point on a monitor?

b) Calculate the necessary transformation matrix to convert a color from the color space given above into a new one with the same primaries but with the white point at $(w_x, w_y) = (0.400, 0.330)$. Explain your solution. Since this transformation is not used to change the brightness, assume a normalized white point.

## 6.3 Color Space (25 Points)

Compute the position of the sRGB color (1,1,0) in the CIE-XYZ and CIE-xy color space.

# Solutions

## 6.1 Gamma correction (35 Points)

The worst case is obviously if setting $\gamma = 2.5$. The highest accuracy is required near zero input, there we get:

$$f(0.00000000b) = 0^{2.5} = 0$$

$$f(0.00000001b) = (1/128)^{2.5} = 5.39E - 6$$

To represent the step from 0 to 5.39E-6 we need at least 18 bits for the fractional part ($log_2(5.39 \cdot 10^{-6}) = -17.5$).

## 6.2 Transforms in colorspace (50 points)

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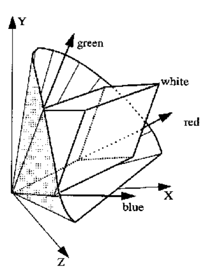
a)

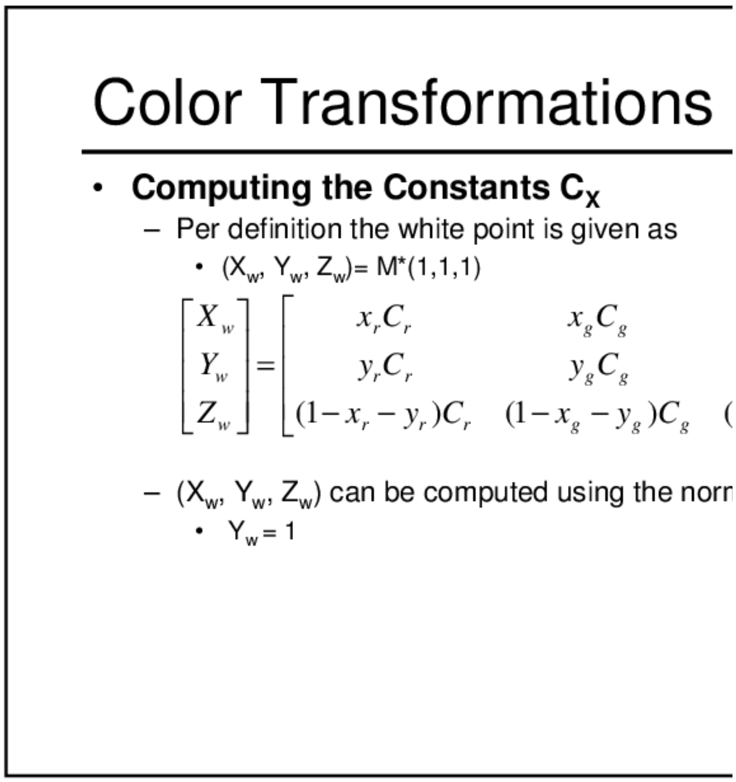



We can't change direction of vectors red, green and blue since they depend on the monitor's hardware characteristics. So the only way remaining to change the position of the white point is by changing the length of the vectors. Simply speaking: We dimm the maximum brightness of one or more primary colors.

b) Overview: let $C_{rgb} = (r, g, b)$ be the color on the first monitor, then we want to know which color $C'_{rgb}$ we need to use on the second monitor so that both monitors show the same color. With formulars:

$$C_{cie} = M \cdot C_{rgb} \text{ and } C_{cie} = M' \cdot C'_{rgb}$$
$$\Rightarrow M \cdot C_{rgb} = M' \cdot C'_{rgb} \Leftrightarrow C'_{rgb} = M'^{-1} \cdot M \cdot C_{rgb}$$

Given: $x_r, y_r, x_g, y_g, x_b, y_b, x_w, y_w$ and due to the normalization of the white point: $Y_w = 1$ (see slide of computer graphics lecture)




Then: $X_w = x_w \cdot \frac{Y_w}{y_w} \approx 0.9505$ and $Z_w = z_w \cdot \frac{Y_w}{y_w} = (1 - x_w - y_w) \cdot \frac{Y_w}{y_w} \approx 1.0891$

and for the second monitor we get: $X_{w2} \approx 1.2121$ and $Z_{w2} = 0.8182$

We need to compute $C_r, C_g, C_b$:
In equation above, set $z_r = (1 - x_r - y_r), z_g$ and $z_b$ analog for convenience issues.

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} x_r C_r + x_g C_g + x_b C_b \\ y_r C_r + y_g C_g + y_b C_b \\ z_r C_r + z_g C_g + z_b C_b \end{bmatrix} \Leftrightarrow \begin{array}{c} X_w = x_r C_r + x_g C_g + x_b C_b \\ Y_w = y_r C_r + y_g C_g + y_b C_b \\ Z_w = z_r C_r + z_g C_g + z_b C_b \end{array}$$

Solving these equations yields:

$$\text{for the first white point: } \Rightarrow C_r = 1.2778,\ C_g = 0.8763,\ C_b = 0.8763$$
$$\text{for the second white point: } \Rightarrow C_r = 0.6444,\ C_g = 1.1919,\ C_b = 1.2033$$

Thus calculating $M, M'$ and $M'^{-1}$ results in

$$M = \begin{bmatrix} 0.8178 & 0.2629 & 0.1314 \\ 0.4217 & 0.5258 & 0.0526 \\ 0.0383 & 0.0876 & 0.6922 \end{bmatrix} \qquad M' = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2127 & 0.7151 & 0.0722 \\ 0.0193 & 0.1192 & 0.9506 \end{bmatrix}$$

$$M'^{-1} = \begin{bmatrix} 3.2413 & -1.5377 & -0.4987 \\ -0.9697 & 1.8764 & 0.0416 \\ 0.0558 & -0.2041 & 1.0569 \end{bmatrix}$$

## 6.3 Color Space (25 Points)

For the CIE-XYZ color space one simply computes $(X, Y, Z) = M \cdot (1, 1, 0) = (1.08, 0.95, 0.13)$. By normalizing we get the coordinates in the CIE-xy color space: $X/(X+Y+Z) = 0.5$, $Y/(X+Y+Z) = 0.43$.