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Computer Graphics I
Assignment 9

Submission deadline for the exercises: Thursday, 25th January 2007

9.1 Homogenous Coordinates (5 + 10 + 15 Points)

a) Show that multiplying the homogenous point (x, y, z, w) with an arbitrary scalar $\alpha \neq 0$ yields an equivalent homogenous point again.

b) Show that the component wise addition of three homogenous points $(a_0, b_0, c_0, 1)$, $(a_1, b_1, c_1, 1)$, and $(a_2, b_2, c_2, 1)$ yields the center between that points.

c) Find an addition rule in homogenous space that is equivalent to standard addition after performing the projection.

9.2 Perspective Projection (30 Points)

In the lecture a perspective projection P_{persp} is given that maps the viewing frustum to the regular box $[-1, 1]^3$. In this exercise you have to derive a formula for a similar projection P' from scratch, that maps the near-plane to the $z = 0$ plane and the far-plane to the $z = 1$ plane. The viewing direction is the positive z direction, such that the viewing frustum has an x extension from $-\frac{w}{2}$ to $\frac{w}{2}$ and y extension from $-\frac{h}{2}$ to $\frac{h}{2}$ in the near plane. Find the corresponding transformation P':

$$P' = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

9.3 Perspective Projection (15 + 15 + 10 Points)

a) Compute the point where two arbitrary parallel lines seem to intersect after being projected by the perspective projection P. For which parallel lines does no such intersection point exist?

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1 \end{pmatrix}$$

b) Compute the center of projection of the following perspective projection Q.

$$Q = \begin{pmatrix} \frac{3}{4} & 0 & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & 1 & \frac{1}{4} & \frac{1}{4} \\ -1 & 0 & 0 & -1 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{5}{4} \end{pmatrix}$$

c) Compute the projection plane of projection Q'

$$Q' = \begin{pmatrix} 1 & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{5}{4} & 0 \\ 0 & \frac{1}{4} & -\frac{1}{4} & 1 \end{pmatrix}$$

9.4 Homogenous Lines in 2D* (20 Points)

Prove that the cross product between two homogenous points $p = (p_x, p_y, p_w)$ and $q = (q_x, q_y, q_w)$ yields the homogenous coordinates of the connecting line.