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Computer Graphics I
Assignment 8

Submission deadline for the exercises: January 18, 2007

8.1 C^1 and G^1 continuity (25 Points)

a) Show that C^1 continuity implies G^1 continuity if the first derivative vectors have a length not equal to zero.

b) Consider the two spline segments $\gamma(t) = (t^2, t)$ and $\nu(t) = (t^3 + 4t + 1, 2t + 1)$, both defined on the interval $0 \leq t \leq 1$. Show that the curves join at $\gamma(1)$, $\nu(0)$ with C^0 and G^1, but not with C^1 continuity.

c) Show that the two spline segments $\gamma(t) = (t, t^2 - 2t)$ and $\nu(t) = (t + 1, t^2 + 1)$ are both C^1 and G^1 continuous where they join at $\gamma(1) = \nu(0)$.

8.2 Hermite Spline (25 Points)

The following cubic polynomial defines a spline curve in 3D:

$$p(t) = at^3 + bt^2 + ct + d \texttt{ with } a, b, c, d \in \mathcal{R}^3$$

a) Compute the coefficients $a, b, c,$ and d of the polynomials such that $p(0) = (0, 1, 0)^T$, $p(1) = (1, 0, 1)^T$, $\frac{dp}{dt}(0) = (1, 0, 1)^T$, $\frac{dp}{dt}(1) = (0, -1, 0)^T$. You have to write down the constraints and solve an system of equations.

b) Compute the same coefficients by using the Hermite basis.

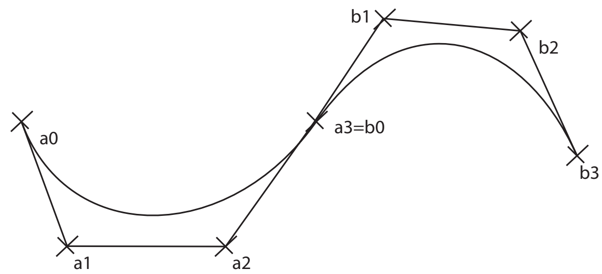
8.3 DeCasteljau (25 Points)

a) You are given the Bezier spline $B(t)$ with the control points $b_0 = (1, 1)$, $b_1 = (2, 2)$, $b_3 = (5, 1)$, $b_4 = (5, 0)$. Use the DeCasteljau algorithm graphically to find the value $B\left(\frac{1}{2}\right)$ on the spline.

b) Now apply the algorithm numerically to get the exact point $B\left(\frac{1}{2}\right)$ on the spline.

8.4 Bezier Splines (25 Points)

Prove that two Bezier splines A and B are C^1 continous at the join point if $a_3 - a_2 = b_1 - b_0$.



8.5 Invariance of Bezier Splines under Affine Transformations* (30 Points)

Prove that a Bezier spline is invariant under affine transformations, which means that transforming the control points is equivalent to transforming the complete curve.