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Computer Graphics I
Assignment 5

Submission deadline for the exercises: Thursday, 7th December 2006

5.1 Fourier Transformation (30 Points)

Show that the Fourier transformation of the box function $B(x)$ is a *sinc* type function. The sinc function is defined as $sinc(x) = \frac{sin(\pi x)}{\pi x}$ and a definition of the Fourier transform can be found in Exercise 5.5.

$$B(x) = \begin{cases} 0 & \texttt{for } x \leq -1 \\ 1 & \texttt{for } -1 < x < 1 \\ 0 & \texttt{for } 1 \leq x \end{cases}$$

5.2 Sampling Theory (10 + 10 Points)

Let $f(x)$ be an infinite signal that fulfills the Nyquist property, thus the highest frequency of the signal is smaller than $\frac{1}{2T}$ if T is the sampling distance. Consider a regular sampling $f_S(x)$ of $f(x)$ with sample distance T.

a) Is an exact signal reconstruction of $f(x)$ possible? If so, why?

b) How has the reconstruction to be performed in image and Fourier space?

5.3 Antialiasing (10 + 10 Points)

a) Explain what sampling of a continuous signal means in signal and Fourier space. Further explain what aliasing of a sampled signal means in signal and Fourier space.

b) Consider an infinite signal $f(x)$ and a regular sampling $f_S(x)$ with sampling distance d that shows no aliasing artefacts. The sampling distance d is now increased step by step until the first aliasing artefacts occur.

How can we best get *an aliasing-free* sampling from these samples f_S again? The sampling distance should stay d and as much useful information as possible should be recovered.

Describe the filter procedure in Fourier and signal space. You do not have to derive the exact filter kernels (but you can of course).

5.4 Triangle Filter (30 Points)

Show that reconstructing a signal that is sampled at sampling distance 1 with the triangle filter $T(x)$ is equivalent of performing linear interpolation.

$$T(x) = \begin{cases} 0 & \texttt{for } x \leq -1 \\ x+1 & \texttt{for } -1 < x < 0 \\ -x+1 & \texttt{for } 0 \leq x < 1 \\ 0 & \texttt{for } 1 \leq x \end{cases}$$

5.5 Duality of Multiplication and Convolution* (20 Points)

The convolution of a function $f(t)$ with a second function $g(t)$ is defined as:

$$(f \otimes g)(t) = \int_{-\infty}^{+\infty} f(\tau) \cdot g(t - \tau) d\tau$$

The multiplication of two function is defined as the point-wise multiplication:

$$(f \cdot g)(t) = f(t) \cdot g(t)$$

The transformation of a signal $f(x)$ to Fourier space is given by:

$$F(k) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-2\pi ikx} dx$$

We call $\mathcal{F}$ the operator mapping f to Fourier space: $\mathcal{F}f = F$. Show that convolving in signal space is the same as multiplication in Fourier space:

$$\mathcal{F}[f \otimes g] = \mathcal{F}[f] \cdot \mathcal{F}[g]$$