Computer Graphics I

Assignment 8

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8.1 C^1 and G^1 continuity

a. For two curves q_1, q_2 it holds:

$$C^1 \Rightarrow G^1$$

$$\begin{aligned} C^1 &\Leftrightarrow q_1'(n) = q_2'(n) \\ &\overset{(*)}{\Rightarrow} \nabla q_1(n) = \nabla q_2(n) \Leftrightarrow G^1 \end{aligned}$$

where $(*)$ holds for $|q_1'(n)| \neq 0$.

b.

$$\begin{aligned} \gamma(t) &= (t^2, t) \\ \gamma'(t) &= (2t, 1) \\ \nu(t) &= (t^3 + 4t + 1, 2t + 1) \\ \nu'(t) &= (3t^2 + 4, 2) \end{aligned}$$

$$\gamma(1) = (1, 1), \quad \nu(0) = (1, 1) \Rightarrow C^0$$

$$\gamma'(1) = (2, 1), \quad \nu'(0) = (4, 2) \Rightarrow 2\gamma'(1) = \nu'(0) \Rightarrow G^1$$

$$\gamma'(1) \neq \nu'(0) \Rightarrow \text{ not } C^1$$

c.

$$\begin{aligned} \gamma(t) &= (t, t^2 - 2t) \\ \gamma'(t) &= (1, 2t - 2) \\ \nu(t) &= (t + 1, t^2 + 1) \\ \nu'(t) &= (1, 2t) \end{aligned}$$

$$\gamma'(1) = (1, 0) = \nu'(0) = (1, 0) \Rightarrow C^1 \Rightarrow G^1$$

8.2 Hermite Spline

a.

$$(I) \quad p(0) = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{=\begin{pmatrix} t^3 \\ t^2 \\ t^1 \\ t^0 \end{pmatrix} \text{for } t=0} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$(II) \quad p(1) = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$(III) \quad p'(0) = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}}_{=\begin{pmatrix} 3t^2 \\ 2t \\ 1t^0 \\ 0 \end{pmatrix} \text{for } t=0} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$(IV) \quad p'(1) = \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ from } (I), \quad c = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ from } (III)$$

$$\Rightarrow \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ a_2 & b_2 & 0 & 0 \\ a_3 & b_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ a_2 & b_2 & 0 & 0 \\ a_3 & b_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

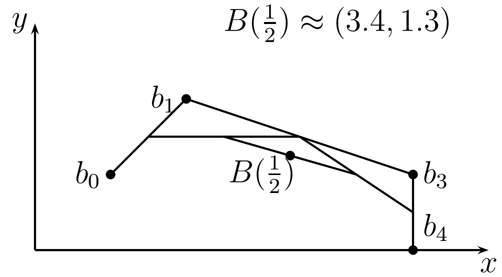
$$\Rightarrow a_1 = -b_1; a_2 = -1 - b_2; a_3 = -b_3$$
$$\Rightarrow -3b_1 + 2b_1 = -1 \Leftrightarrow b_1 = 1; -3 - 3b_2 + 2b_2 = -1 \Leftrightarrow b_2 = -2;$$
$$3b_3 + 2b_3 = -1 \Leftrightarrow b_3 = 1$$
$$\Rightarrow b = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$
$$\Rightarrow a = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

b.

$$\begin{aligned} P(t) &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (1-t)^2(1+2t) + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} t(1-t)^2 \\ &+ \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} t^2(1-t) + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} (3-2t)t^2 \\ &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (2t^3 + 3t^2 + 1) + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} (t^3 - 2t^2 + t) \\ &+ \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} (t^3 - t^2) + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} (-2t^3 - 3t^2) \\ &= \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} t^3 + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} t + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{aligned}$$

8.3 DeCasteljau

a.



b.

$$\begin{aligned}
B_2^3\left(\frac{1}{2}\right) &= \frac{1}{2}\cdot B_1^2\left(\frac{1}{2}\right)+\frac{1}{2}\cdot B_0^2\left(\frac{1}{2}\right)\\
&= \frac{1}{2}\cdot\left(\frac{1}{2}\cdot B_2^1\left(\frac{1}{2}\right)+\frac{1}{2}\cdot B_1^1\left(\frac{1}{2}\right)\right)+\frac{1}{2}\cdot\left(\frac{1}{2}\cdot B_1^1\left(\frac{1}{2}\right)+\frac{1}{2}\cdot B_0^1\left(\frac{1}{2}\right)\right)\\
&= \frac{1}{4}\cdot B_2^1\left(\frac{1}{2}\right)+\frac{1}{2}\cdot B_1^1\left(\frac{1}{2}\right)+\frac{1}{4}\cdot B_0^1\left(\frac{1}{2}\right)\\
&= \frac{1}{4}\left(\frac{1}{2}(5,0)+\frac{1}{2}(5,1)\right)+\frac{1}{2}\left(\frac{1}{2}(5,1)+\frac{1}{2}(2,2)\right)+\frac{1}{4}\left(\frac{1}{2}(2,2)+\frac{1}{2}(1,1)\right)\\
&= \left(\frac{5}{4},\frac{1}{8}\right)+\left(\frac{7}{4},\frac{3}{4}\right)+\left(\frac{3}{8},\frac{3}{8}\right)\\
&= \left(\frac{27}{8},\frac{10}{8}\right)=(3.375,1.250)
\end{aligned}$$

8.4 Bezier Splines

$$\begin{aligned}
A(t) &= a_0(1-t)^3+3a_1t(1-t)^2+3a_2t^2(1-t)+a_3t^3\\
&= a_0(-t^3+3t^2-3t-1)+3a_1(t^3-2t^2+t)+3a_2(-t^3+t^2)+a_3t^3\\
&= t^3(-a_0+3a_1-3a_2+a_3)+t^2(3a_0-6a_1+3a_2)+t(-3a_0+3a_1)-3a_0\\
A'(t) &= 3t^2(-a_0+3a_1-3a_2+a_3)+2t(3a_0-6a_1+3a_2)+(-3a_0+3a_1)
\end{aligned}$$

analogously:

$$\begin{aligned}
B'(t) &= 3t^2(-b_0+3b_1-3b_2+b_3)+2t(3b_0-6b_1+3b_2)+(-3b_0+3b_1)\\
A'(1) &= 3(-a_0+3a_1-3a_2+a_3)+2(3a_0-6a_1+3a_2)-(3a_0+3a_1)=-3a_2+3a_3\\
B'(0) &= -3b_0+3b_1\\
A'(1) &= -3a_2+3a_3=3(a_3-a_2)\overset{a_3-a_2=b_1-b_2}{=}3(b_1-b_0)\\
&= -3b_0+3b_1=B'(0)\Rightarrow C^1; a_3=b_0\Rightarrow C^0
\end{aligned}$$