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Fachrichtung 6.2 – Informatik
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8th Theoretical Assignment in
Artificial Intelligence (WS 2006/2007)
Solutions

Exercise 8.1

Give one analogue and one symbolic (e.g. Fregean) representation for a maze. How are these representations related to the different algorithms for finding a path through the maze?


Solution:

An example for an analogue representation of a maze is a bitmap, where the walls are represented by collections of dark points, and the empty spaces by white points. For example:

A feasible algorithm for a maze without cycles is the following: Assume your algorithm places a cursor at the starting position in the maze. As long as there is still a neighboring white point in the upward direction, move to that point. If this moves the cursor outside the maze, the algorithm is done. Otherwise, the cursor will stop at a white dot adjacent to a black dot. Then your algorithm tries to move alongside the wall represented by the black dots pixel by pixel, and "turn right" whenever possible (if there is a black dot ahead and a black dot to the right, the cursor turns left and continues in that direction).

Notice that such an algorithm cannot make use of abstraction - all it sees is a number of pixels. On the other hand, many properties of the original maze are still present in the analogue drawing, for example, the distance between points in the map correspond directly to distances in reality, as well as the angles of the walls, etc.

As an example, a more symbolic representation is to encode all the paths in the maze as a graph. For the above example, this gives:
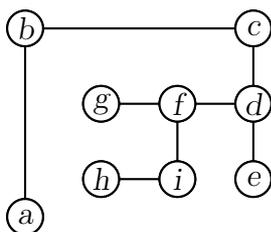
The graph can be symbolically represented as a tuple (Nodes,Edges), where:
Nodes = {a,b,c,d,e,f,g,h,i}, Edges={(a,b),(b,c),(c,d),(d,e),(d,f),(f,g),(f,i),(i,h)}.
This way, the search techniques from the lecture can be applied.

A further step is to encode the problem in first-order-logic. For example, the graph can be encoded as a conjunction of facts, namely $edge(a,b) \land edge(b,c) \land ....$ Then we need axioms like the transitivity of paths: $\forall a, b, c : path(a,b) \land path(b,c) \Rightarrow path(a,c)$, and that where there is a single edge there is also a path: $\forall a, b : edge(a,b) \Rightarrow path(a,b)$. To solve the maze problem, we can try to prove the theorem that there exists a path from the start position to the exit of the maze, and use the proof to reconstruct the path.

*Observe that there is no strict dividing line between analogue and symbolic representations.*

Exercise 8.2

Let *Man, Woman, Male, Female, Human* be concept names, and let *has-child, is-brother-of, is-sister-of, is-married-to* be role names.

1. Construct a TBox in which the concepts *Mother, Father, Grandmother, Grandfather, Aunt, Uncle, Niece, Nephew, Mother of at least three sons, Father of at most two daughters* are defined.
2. Expand the definition of *Grandmother*!
3. Expand the definition of *Niece*!
4. Draw the subsumption hierarchy!

Solution:

1.
   - *Mother $\equiv$ Woman $\sqcap \exists has\text{-}child.\top$*
   - *Father $\equiv$ Man $\sqcap \exists has\text{-}child.\top$*
   - *Grandmother $\equiv$ Woman $\sqcap \exists has\text{-}child.(Mother \sqcup Father)$*
   - *Grandfather $\equiv$ Man $\sqcap \exists has\text{-}child.(Mother \sqcup Father)$*
   - *Aunt $\equiv$ Female $\sqcap Human \sqcap \exists is\text{-}sister\text{-}of.(Mother \sqcup Father)$*
   - *Uncle $\equiv$ Male $\sqcap Human \sqcap \exists is\text{-}sister\text{-}of.(Mother \sqcup Father)$*
   - *Niece $\equiv$ Female $\sqcap Human \sqcap \exists has\text{-}child^{-1}.((\exists is\text{-}sister\text{-}of.\top) \sqcup (\exists is\text{-}brother\text{-}of.\top))$*
   - *Nephew $\equiv$ Male $\sqcap Human \sqcap \exists has\text{-}child^{-1}.((\exists is\text{-}sister\text{-}of.\top) \sqcup (\exists is\text{-}brother\text{-}of.\top))$*
   - *Mother of at least three sons $\equiv$ Woman $\sqcap (\geq 3has\text{-}child.Male \sqcap Human)$*
   - *Father of at most two daughters $\equiv$ Man $\sqcap (\leq 2has\text{-}child.Female \sqcap Human)$*

2.
   - *Woman $\sqcap \exists has\text{-}child.((Woman \sqcap \exists has\text{-}child.\top \sqcup Man \sqcap \exists has\text{-}child.\top))$*
   - *Female $\sqcap Human \sqcap \exists has\text{-}child^{-1}.((\exists is\text{-}sister\text{-}of.\top) \sqcup (\exists is\text{-}brother\text{-}of.\top))$ (this is identical to the definition in (1.))*

3. *Subsumption hierarchy*
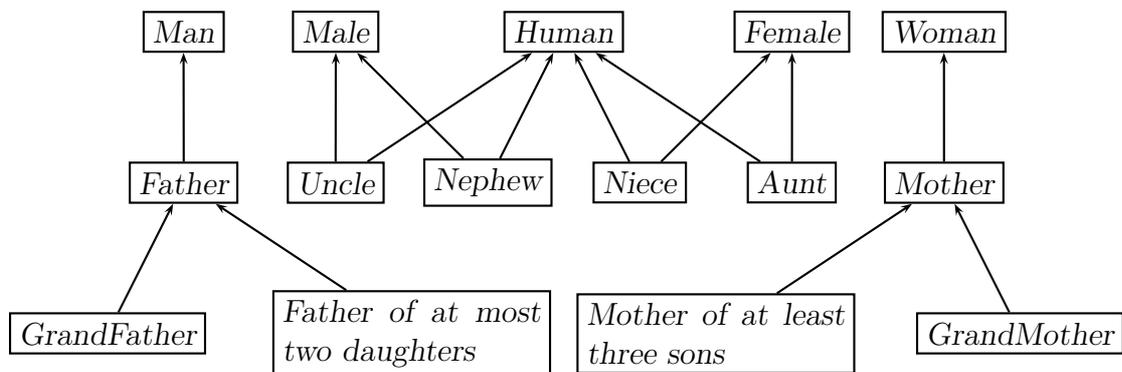
In this subsumption hierarchy, an arrow from a concept A to a concept B represents concept inclusion $A \sqsubseteq B$.

Exercise 8.3

Let $\mathcal{A}$ be the ABox that contains the following assertions:

- likes(Marc, Lydia)
- is-flatmate-of(Lydia, Phoebe)
- single(Lydia)
- likes(Marc, Phoebe)

- $\neg$ single(Phoebe)
- is-flatmate-of(Phoebe, Kate)
- $\neg$ single(Kate)

1. Indicate $(\exists\text{is-flatmate-of}.\neg\text{single})^{\mathcal{I}}$ where $\mathcal{I}$ is the interpretation function over the domain {Marc, Lydia, Phoebe, Kate}!
2. Indicate $(\forall\text{likes}.\exists\text{is-flatmate-of}.\neg\text{single})^{\mathcal{I}}$ where $\mathcal{I}$ is the interpretation function over the domain {Marc, Lydia, Phoebe, Kate}!
3. Indicate $(\forall\text{is-flatmate-of}.\neg\text{single})^{\mathcal{I}}$ where $\mathcal{I}$ is the interpretation function over the domain {Marc, Lydia, Phoebe, Kate}!
4. Is Marc an instance of the concept $\exists\text{likes}.(\text{single} \sqcap \exists\text{is-flatmate-of}.\neg\text{single})$ with respect to $\mathcal{A}$?
5. Is Marc an instance of the concept $\exists\text{likes}.(\exists\text{is-flatmate-of}.(\forall\text{is-flatmate-of}.\neg\text{single}))$ with respect to $\mathcal{A}$?

Solution:

1. *{Lydia, Phoebe}*
2. *{Marc,Lydia,Phoebe,Kate}*
3. *again, {Marc,Lydia,Phoebe,Kate}*
4. *According to (1.), $(\exists\text{is-flatmate-of}.\neg\text{single})^{I} = \{Lydia, Phoebe\}$. Therefore, $(\text{single} \sqcap \exists\text{is-flatmate-of}.\neg\text{single})^{I} = \{Lydia\}$. Because likes(Marc,Lydia) holds, Marc is indeed an instance of the given concept.*
5. *yes*

Exercise 8.4

Consider the following (politically incorrect) TBox $\mathcal{T}$ with concepts *Human, Man, Woman* and *Male*, and the role name *married-to*:

- Man $\equiv$ Human $\sqcap$ Male
- Woman $\equiv$ $\forall$ married-to . Man

Also consider the ABox $\mathcal{A}$:

- Human(Claus)
- Male(Claus)
- Woman(Grete)
- Woman(Susan)
- married-to(Grete,Claus)

Is the ABox $\mathcal{A}$ consistent w.r.t. $\mathcal{T}$? In any case, justify your findings!

Solution:

The ABox $\mathcal{A}$ is consistent w.r.t. $\mathcal{T}$ if $\mathcal{A}$ has a model that is also a model for $\mathcal{T}$.

We now try to find a model for $\mathcal{A}$ that fulfills this requirement. A model for an ABox $\mathcal{A}$ is an interpretation that satisfies all the assertions in $\mathcal{A}$. Consider an interpretation $\mathcal{I} = (\Delta^I, \cdot^I)$ where

- $\Delta^I = \{CLAUS, GRETE, SUSAN\}$ (we assume that for all three individual names Claus, Grete and Susan, there exist three corresponding objects CLAUS,GRETE,SUSAN in our domain)
- and $Claus^I = CLAUS$, $Grete^I = GRETE$, $Susan^I = SUSAN$,
- $Human^I = \{CLAUS,GRETE,SUSAN\}$,
- $Male^I = \{CLAUS\}$,
- $Man^I = \{CLAUS\}$,
- $Woman^I = \{GRETE,SUSAN\}$,
- $married\text{-}to^I = \{(GRETE,CLAUS),(CLAUS,GRETE)\}$

If we choose $\mathcal{I}$ this way, then the assertions in $\mathcal{A}$ are fulfilled. Namely:

- $Claus^I = CLAUS \in Human^I$
- $Claus^I \in Male^I$
- $Grete^I = GRETE \in Woman^I$
- $Susan^I = SUSAN \in Woman^I$
- $(GRETE,CLAUS) \in married\text{-}to^I$

Now we have to check that this model $\mathcal{I}$ is also a model for the TBox $\mathcal{T}$.

We have:

- $Man^I = \{CLAUS\} = (Human \sqcap Male)^I$,
- $Woman^I = \{GRETE,SUSAN\} = (\forall \text{ married-to . } Man)^I$

We conclude that the ABox $\mathcal{A}$ is consistent w.r.t. $\mathcal{T}$.