Universität des Saarlandes
Fachrichtung 6.2 – Informatik
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6th Theoretical Assignment in Artificial Intelligence (WS 2006/2007)
Solutions

Exercise 6.1 Consider the following axioms in first-order logic (involving the natural number 0, the successor function s that assigns each number its successor, addition $+$ and the property that a number is *even*).

- $x + 0 = x$
- $x + s(y) = s(x + y)$
- $even(0)$
- $\forall x(even(x) \Rightarrow even(s(s(x)))$
- $2 = s(s(0))$

In the following, you will show that $\forall x(even(x) \Rightarrow even(x + 2))$.

1. Transform the problem to clause normal form (CNF), such that you can apply resolution to it!
2. Use resolution with paramodulation to show that $\forall x(even(x) \Rightarrow even(x + 2))$! Indicate each step.

Solution:

1. *(1)* $x + 0 = x$
(2) $x + s(y) = s(x + y)$
(3) $even(0)$
(4) $\neg even(x) \vee even(s(s(x)))$
(5) $2 = s(s(0))$
(6) The negated conclusion yields two clauses: $even(c)$ *and* $\neg even(c+2)$*, where* c *is a fresh constant introduced by skolemization (in general, we introduce Skolem functions, but in this particular case, our* c *does not depend on any argument – it is not in the scope of a universal quantification).*

2. *(a)* $\neg even(c + s(s(0)))$ *with paramodulation from (5) and (6).*
(b) $\neg even(s(c + s(0)))$ *with paramodulation from the above clause and (2)*
(c) $\neg even(s(s(c + 0)))$ *with paramodulation from the above clause and (2)*
(d) $\neg even(c + 0)$ *with resolution from the above clause and (4)*
(e) $\neg even(c)$ *with paramodulation from the above clause and (1)*
(f) $\Box$ *with resolution from the above clause and* $even(c)$ *in (6).*

Exercise 6.2

In Russel/Norvig (pages 298-300) a resolution proof for the query *Did Curiosity kill the cat?* is given. In the following, the clauses are given in a slightly different manner.

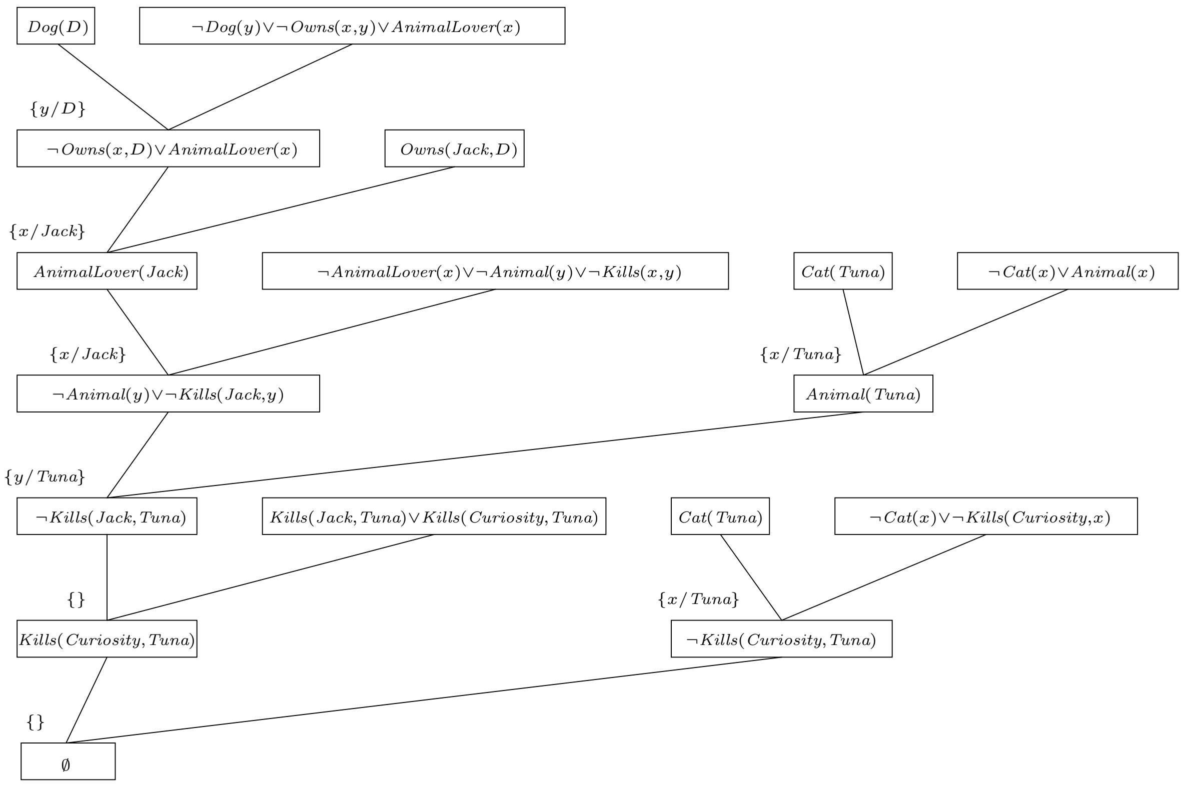
1. (a) $Dog(D)$
   (b) $Own(Jack, D)$
2. $\neg Dog(y) \vee \neg Own(x, y) \vee AnimalLover(x)$
3. $\neg AnimalLover(x) \vee \neg Animal(y) \vee \neg Kills(x, y)$
4. $Kills(Jack, Tuna) \vee Kills(Curiosity, Tuna)$
5. $Cat(Tuna)$
6. $\neg Cat(x) \vee Animal(x)$
7. $\neg Cat(x) \vee \neg Kills(Curiosity, x)$

For both resolution strategies below, give a resolution proof.

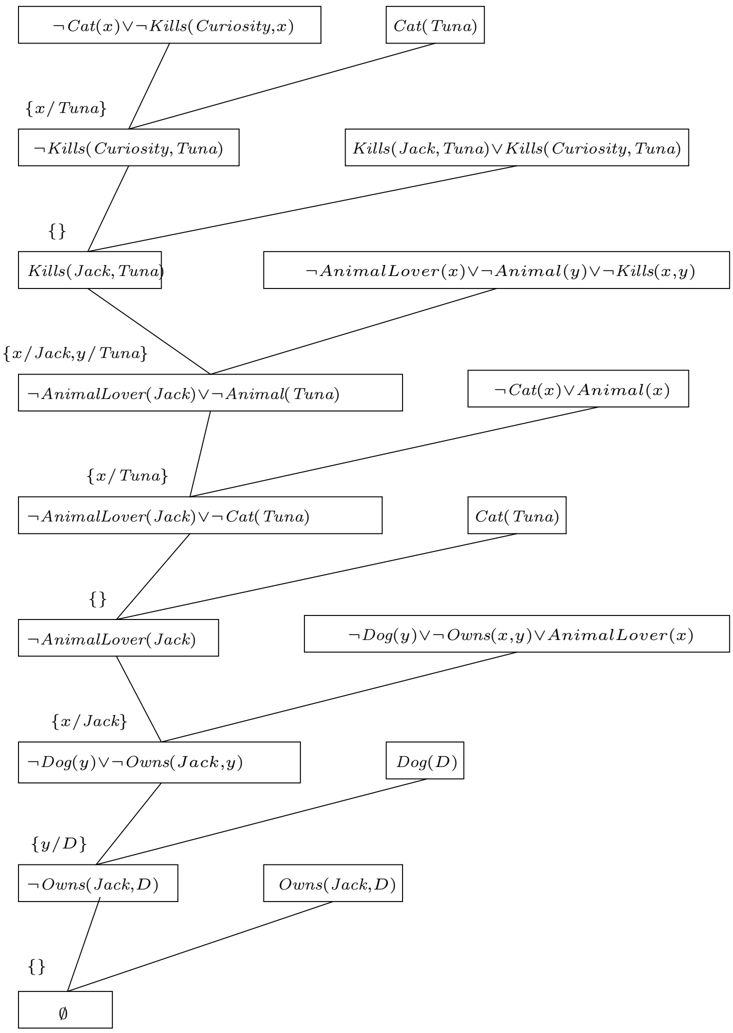
- Unit preference strategy
- Set of support strategy (Choose the clause 7 as Set of Support)

Solution:

1. *Unit Preference*



2. *Set of Support*



Exercise 6.3 In this exercise, we are concerned with the encoding of rules in a production system. The domain we chose are horses and variants. For these we can define the following, although very simplified, circumstances:

> *If a car is faster than a Porsche then it is a sportscar. It is also a sportscar if it is faster than another sportscar. If X is faster than Y and Y is faster than Z, then X is faster than Z. Furthermore, a car that is faster than a Porsche and a Ferrari is a Formula-1-car.*

(a) List predicates that are required to encode the above facts (e.g., $Faster(X, Y)$)

(b) Encode the above facts as production rules.

Solution:

(a) The following predicates are used:

$$
\begin{aligned}
Car(X) &: \text{ } X \text{ is a car} \\
Porsche(X) &: \text{ } X \text{ is a Porsche} \\
Sportscar(X) &: \text{ } X \text{ is a Sportscar} \\
Ferrari(X) &: \text{ object } X \text{ is a Ferrari} \\
Formula1car(X) &: \text{ object } X \text{ is a Ferrari} \\
Faster(X,Y) &: \text{ } X \text{ is faster than } Y
\end{aligned}
$$

(b) *Using predicates from (a), we can identify the following production rules:*

$$\begin{aligned}
Car(X) \wedge Faster(X,Y) \wedge Porsche(Y) &\Rightarrow Add(Sportscar(X)) \quad (1)\\
Car(X) \wedge Faster(X,Y) \wedge Sportscar(Y) &\Rightarrow Add(Sportscar(X)) \quad (2)\\
Faster(X,Y) \wedge Faster(Y,Z) &\Rightarrow Add(Faster(X,Z)) \quad (3)\\
Car(X) \wedge Faster(X,Y) \wedge Porsche(Y) & \\
\wedge Faster(X,Z) \wedge Ferrari(Z) &\Rightarrow Add(Formula1car(X)) \quad (4)
\end{aligned}$$

Exercise 6.4

In this exercise we prepare the construction of a RETE-network. To this end we must identify subformulas that occur frequently in the preconditions of the production rules. Then we introduce new predicates to abbreviate these subformulas. For instance, if $H(X) \wedge P(X,Y)$ is a subformula that occurs in different production rules, then we define a predicate $B(X,Y)$ by setting $B(X,Y) := H(X) \wedge P(X,Y)$.

(a) There is a multiply occurring subformula of the production rules from the previous exercise. Identify this subformula.

(b) Introduce a new predicate for the identified subformula and reformulate the production rules using this new predicate.

In the next exercise we will see that using this predicate simplifies the construction of a RETE network.

Solution:

(a) *A frequently occurring formula is described by:*

$$Car(X) \wedge Faster(X,Y)$$

(b) *For the most frequently formula*

$$Car(X) \wedge Faster(X,Y)$$

we introduce a new predicate $B1(X,Y)$ that is defined as follows:

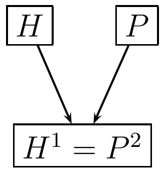
$$B1(X,Y) := Car(X) \wedge Faster(X,Y)$$

We reformulate the production rules as follows:

$$\begin{aligned}
B1(X,Y) \wedge Porsche(Y) &\Rightarrow Add(Sportscar(X))\\
B1(X,Y) \wedge Sportscar(Y) &\Rightarrow Add(Sportscar(X))\\
Faster(X,Y) \wedge Faster(Y,Z) &\Rightarrow Add(Faster(X,Z))\\
B1(X,Y) \wedge Porsche(Y) \wedge Faster(X,Z) \wedge Ferrari(Z) &\Rightarrow Add(Formula1car(X))
\end{aligned}$$

Exercise 6.5 Construct a RETE-network using the simplified production rules you obtained in the previous exercise. Note that in a RETE-network only predicates occur, but no literals. To represent a subformula $H(X) \wedge P(Y, X)$, where the variable X occurs in two different literals, we use a notation which expresses that the respective arguments of H and P must be equal. For instance we represent the above formula by $H^1 = P^2$. Assume, the literals $H(X)$ and $P(Y, Z)$ are already represented as nodes in the network:

Then the node for the formula $H(X) \wedge P(Y, X)$ is represented by



Construct a RETE-network using that notation for the production rules you obtained in the previous exercise.

Solution:

In addition to the predicate $B1(X, Y)$ defined in the previous exercise, we define the following:

$$\begin{aligned} B2(X,Y) &:= B1(X,Y) \wedge Porsche(Y) \\ B3(X,Y) &:= B1(X,Y) \wedge Sportscar(Y) \\ B4(X,Y,Z) &:= B2(X,Y) \wedge Faster(X,Z) \\ B5(X,Y,Z) &:= B4(X,Y,Z) \wedge Ferrari(Z) \\ B6(X,Y,Z) &:= Faster(X,Y) \wedge Faster(Y,Z) \end{aligned}$$

Finally here is a picture which gives the RETE-Net for the 4 production rules from the previous exercise.