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Fachrichtung 6.2 – Informatik
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5th Theoretical Assignment in Artificial Intelligence (WS 2006/2007)

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Note: You need not hand these exercises in, and they are not graded. But bring along your solutions to the tutorial. Impress your tutors by presenting your favourite exercise to the class.

Exercise 5.1

1. Represent the following statement in first order logic.
   - Every student attends some class.
2. Convert your first-order statement above into clause normal form as described in class. In this example, you should perform the following steps:
   (a) Eliminate implications in favor of disjunction and negation.
   (b) Skolemize to remove existential quantifiers.
   (c) Remove universal quantifiers.
   (d) Distribute $\wedge$ over $\vee$.
   Show the result of each step!

Exercise 5.2

Identify the most general unifier (if one exists) for the following pairs of formulas. Constants are illustrated as capitals, variables as lower case letters. Use the unification algorithm presented in the lecture step by step.

1. $\mathit{Older}(\mathit{Father}(y), y)$, $\mathit{Older}(\mathit{Father}(x), \mathit{John})$
2. $Q(y, G(A, B))$, $Q(G(x, x), y)$

Exercise 5.3

In the following, we specify a set of first order logic formulas:

(A) $\forall x.\neg(x < x)$
(B) $\forall x, y, z.(x < y \wedge y < z) \Rightarrow x < z$
(C) $\forall x.x < S(x)$
(D) $N(0)$
(E) $\forall x.N(x) \Rightarrow N(S(x))$

A model for this first-order language is given by a nonempty set $\mathcal{D}$ of objects, a binary relation $|<|$ on $\mathcal{D}$ (to interpret $<$), a function $|S|$ from $\mathcal{D}$ to $\mathcal{D}$ (to interpret S), a subset $|N|$ of $\mathcal{D}$ (to interpret N) and an element $|0|$ of $\mathcal{D}$ (to interpret 0).

1. Give a model which satisfies the set of formulas {**(A)**,**(C)**,**(D)**,**(E)**} and such that $\mathcal{D}$ contains exactly two objects.
2. Give a model that satisfies the set of formulas {**(A)**,**(B)**,**(C)**,**(D)**,**(E)**}. (Hint: $\mathcal{D}$ can be infinite.)

For each part, justify your answer!

Exercise 5.4

Let S be the set of the following four clauses:

(A) $P(A, A)$
(B) $\neg P(B, B)$
(C) $\neg P(x, x) \vee P(x, f(x, y))$
(D) $\neg P(x, f(x, y)) \vee P(y, y)$

The set S is inconsistent.

1. In general, the Herbrand Universe of a set of clauses is the set of all ground terms (terms with no variables) that can be constructed from constants and function symbols which occur in the set of clauses. Describe the Herbrand Universe H_S of S.
2. Give all ten (10) ground instances of the clauses **(A)**, **(B)**, **(C)** and **(D)** where you let the variables range over the two terms A and B.
3. Give a set of four of the ten (10) ground clauses above which is inconsistent. Also, give a refutation (i.e., derive the empty clause) using resolution and factoring. Let Ψ be the set of all ground clauses which appear in this refutation.
4. Find a refutation of the clauses in S (with variables) using resolution and factoring with the following property: Every clause involved in this refutation has a ground instance in Ψ. (This is the process of "lifting" the ground refutation to a refutation with variables.)

Exercise 5.5

Given the following clause forms, prove whether there exists an $x \in \{John, Mike, Tom\}$ that satisfies both $Climber(x)$ and $\neg Skier(x)$? If it exists, who is x:

1. $Alp(Tom)$
2. $Alp(Mike)$
3. $Alp(John)$
4. $\neg Alp(x) \vee Skier(x) \vee Climber(x)$
5. $\neg Skier(x) \vee Like(x, Snow)$
6. $\neg Climber(x) \vee \neg Like(x, Rain)$
7. $\neg Like(Tom, y) \vee \neg Like(Mike, y)$
8. $Like(Tom, y) \vee Like(Mike, y)$
9. $Like(Tom, Rain)$
10. $Like(Tom, Snow)$