Universität des Saarlandes
Fachrichtung 6.2 – Informatik
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9th Theoretical Assignment in Artificial Intelligence (SS 2005)
Solutions

Exercise 9.1 **(60 P)**

Consider a shopping problem, where the shopping agent can use the following operators:

$Op($ACTION:$Start,$
  EFFECT:$At(Home) \wedge Sells(HWS, Drill) \wedge Sells(SM, Milk) \wedge Sells(SM, Banana))$

$Op($ACTION:$Finish,$
  PRECOND:$Have(Drill) \wedge Have(Milk) \wedge Have(Banana))$

$Op($ACTION:$Go(?here, ?there),$
  PRECOND:$At(?here)$
  EFFECT:$At(?there) \wedge \neg At(?here))$

$Op($ACTION:$Buy(?store, ?x),$
  PRECOND:$At(?store) \wedge Sells(?store, ?x),$
  EFFECT:$Have(?x))$

Here, SM stands for *Supermarket* and HWS for *Hardware store*. We want to include money, at least in a simple way.

(a) Let CC denote a credit card that the agent can use to buy any object. Modify the description of *Buy* so that the agent has to have its credit card in order to buy anything. (5 P)

(b) Write a *PickUp*-Operator that enables the agent to $Have$ an object if it is portable and at the same location as the agent and owned by the agent. (Hint: you need to change the vocabulary of the domain.) (10 P)

(c) Whenever the agent changes his location the credit card has to change its location as well if the agent has it with him.

  i. Give a plausible specification of *Go*, such that it has $Have$ as precondition and appropriate changes of *At* as effects. (5 P)

  ii. Why is that not a correct way to update the location of objects that the agent has with him? (5 P)

  iii. How would you handle this problem in this context? Explain. (5 P)

  iv. What should be done to enable a complete correct specification of *Go*? (5 P)

(d) Assume that the credit card is at home, but $Have(CC)$ is initially false. Specify a planning problem (initial conditions and goals) for the problem to buy milk somewhere. Describe in detail how a *partial order planner (POP)* would generate a plan for that problem. To do so, do the following:

i. Give a description for every step of the planning process with new *casual links* (including the preconditions for which the causal link was added). Moreover, describe any new operator you want to add as well as the resulting *threats* (that is, steps that might delete a precondition protected by a causal link) and the ordering constraints included to resolve the threats (by ordering the threats to come before or after the causal link). You need not give a complete diagram of the plan for every step. (15 P)

ii. Give a diagram of the final plan, inclusive causal link, as well as their preconditions and ordering constraints. (15 P)

(e) Explain in detail what happens during the planning process when the agent explores a partial plan in which it leaves home without the card. (10 P)

Solution:

(a)
```
Op(Action: Buy(?store,?x)
    Precond: At(?store),Sells(?store,?x),Have(CC)
    Effect: Have(?x))
```

(b) Man benötigt folgende neue Prädikate:

- `Portable(?x)` ist wahr, wenn `x` tragbar ist.
- `At(?x,?y)` ist wahr, wenn `?x` sich an Ort `?y` befindet.
- `Owns(?x,?y)` ist wahr, wenn `?y` sich im Besitz von `?x` befindet.

```
Op(Action: PickUp(?x)
   Precond: Portable(?x), Owns(Agent,?x), At(Agent,?y), At(?x, ?y)
   Effect: Have(?x))
```

(c) i.
```
Op(Action: Go(?x,?y)
    Precond: At(Agent,?x), Have(?obj)
    Effect: At(Agent,?y), not At(Agent,?x), At(?obj,?y), not At(?obj,?x)
```

ii. Diese Methode ist nicht korrekt, weil der Agent nur ein einziges Objekt mit sich führen kann und sich nicht bewegen darf, falls er kein Objekt bei sich trägt.

iii. Man muß sich klarmachen, daß der `Go`-Operator keine korrekte Verwaltung der Orte von Objekten wahrnimmt, falls der Agent mehrere Objekte trägt. Z.B. könnte der Agent Milch im Supermarkt kaufen, und nach Hause gehen, ohne daß `At(milk, home)` gilt. Man könnte dieses Problem solange ignorieren, solange keine Lösung von diesem Fehler beeinflußt wird.

iv. Wenn sich der Agent bewegt, bewegen sich alle Objekte, die er mit sich führt. Wir benötigen also universelle Quantifikation im Effekt-Teil von `Go`, z.B. in der Form

```
forall(?obj)(if Have(?obj) then not At(?obj,?x)
                   and At(?obj,?y))
```

(d) i. Wir definieren zunächst eine Start- und eine Zielaktion:

```
Op(ACTION: Start
   PRECONDITION:
   EFFECT: At(Agent,Home), At(CC,Home), Portable(CC),
           Owns(Agent,CC), Sells(SM,Milk))
```

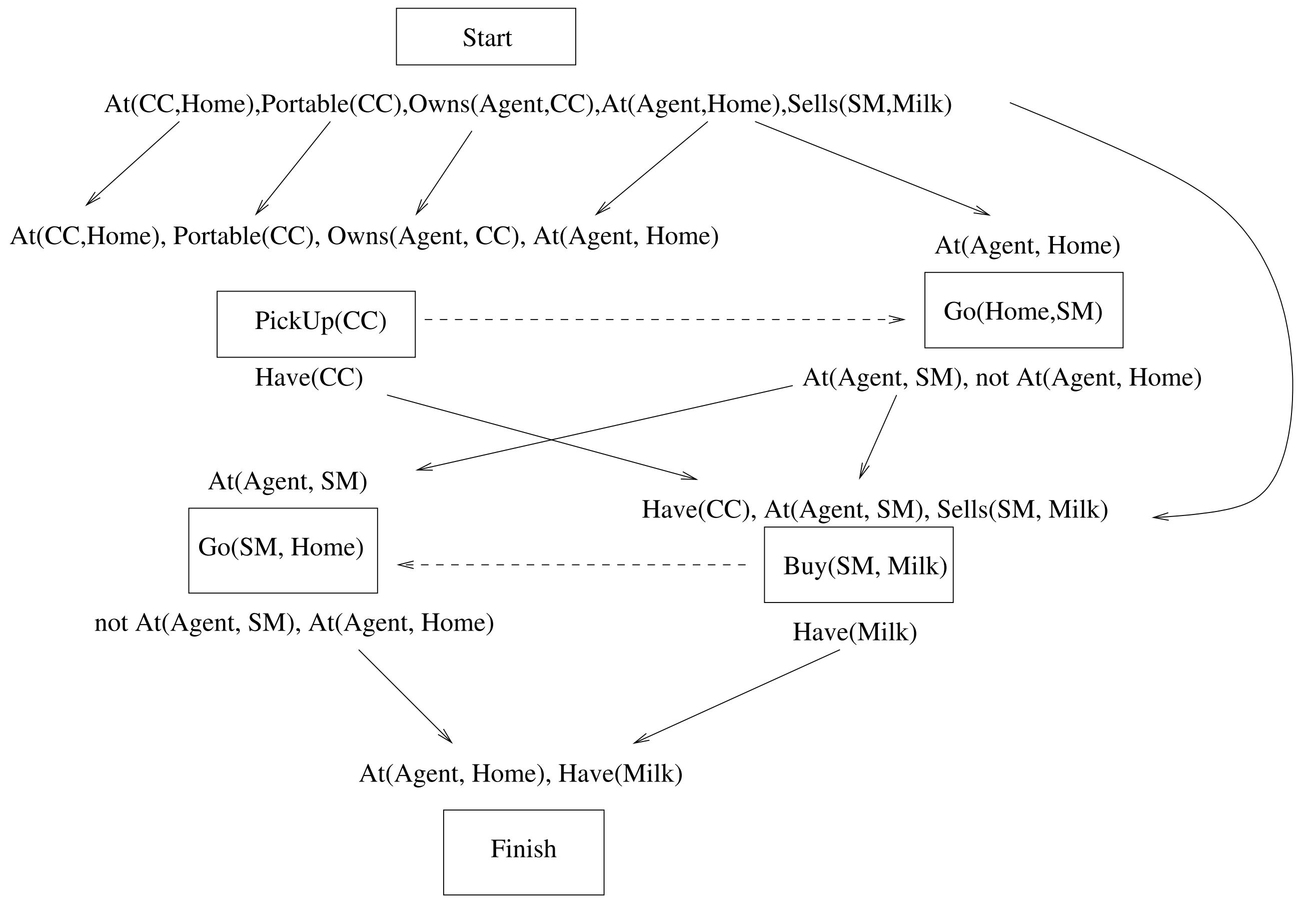

Figure 1: Der Milchkaufplan.

```
Op(ACTION: Finish
   PRECONDITION: Have(Milk), At(Agent, Home)
   EFFECT:)
```

Nun der Plan:

S_{need}	c	S_{add}	Bindings
Finish	Have(Milk)	Buy(t1,x1)	$x1/Milk$
Buy(t1,Milk)	Sells(t1,Milk)	Start	$t1/SM$
Buy(SM, Milk)	At(Agent,SM)	Go(l1, l2)	$l2/SM$
Go(l1,SM)	At(Agent,l1)	Start	$l1/Home$
Finish	At(Agent,Home)	Go(l3, l4)	$l4/Home$
Go(l3,Home)	At(Agent, l3)	Go(Home,SM)	$l3/SM$
Buy(SM,Milk)	Have(CC)	Pickup(x2)	$x2/CC$
PickUp(CC)	Portable(CC)	Start	
PickUp(CC)	Owns(Agent, CC)	Start	
PickUp(CC)	At(CC, Home)	Start	
Pickup(CC)	At(Agent, Home)	Start	

Threats: Effekt `not At(Agent, SM)` von `Go(SM,Home)` bedroht den causal link `Go(Home,SM) --At(Agent,SM)--> Buy(SM, Milk)`. Auflösung durch Promotion (dashed line) von `Go(SM,Home)` hinter `Buy(SM,Milk)`.

Der `not At(Agent, Home)` Effekt von `Go(Home,SM)` bedroht den causal link `Start --At(Agent, Home)--> PickUp(CC)`. Auflösung durch Demotion (dashed line) von `PickUp(CC)` vor `Go(Home,SM)`.

ii. Siehe Abb.1

(e) Falls der Planer das tut, wird der korrekte Plan am Ende trotzdem erzeugt. Wenn die Vorbedingung `At(Agent, x)` von `PickUp(CC)` untersucht wird, wird der Planer feststellen, daß `Go(Home,SM)` diese Bedingung bedroht. Durch das Promoten von `Go(Home,SM)` hinter `PickUp(CC)` wird diese Bedrohung aber entfernt.

Exercise 9.2 **(20 P)**

(a) Use the definition of the conditional probability to prove the following conditional version of the general product rule (asuming all probabilities are non-zero) (10 P)

$$\mathbf{P}(A, B \mid E) = \mathbf{P}(A \mid B, E)\mathbf{P}(B \mid E)$$

Note that the notation $\mathbf{P}(A \mid B, C)$ is a short-cut for $\mathbf{P}(A \mid B \wedge C)$.

Solution:

(a) $P(B \wedge E) = 0$: *In dem Fall gilt trivialerweise auch* $P(A, B \mid E) = 0$ *(da* $P(A, B \mid E) = \frac{P(A \wedge B \wedge E)}{P(E)}$*). Weiterhin gilt:*

$$P(A \mid B, E)P(B \mid E) = P(A \mid B, E)\frac{P(B \wedge E)}{P(E)} = 0$$

Somit wurde gezeigt, daß die allgemeine Produktregel für $P(B \mid E) = 0$ *gilt.*

(b) $P(B \wedge E) > 0$:

$$\begin{aligned} P(A, B \mid E) &\overset{Def.}{=} \frac{P(A \wedge B \wedge E)}{P(E)} \\ &= \frac{P(A \wedge B \wedge E)}{P(B \wedge E)} \frac{P(B \wedge E)}{P(E)} \\ &= P(A \mid B \wedge E)P(B \mid E) \end{aligned}$$

q.e.d.

(b) Prove the following conditional version of the general Bayes' rule (10 P)

$$\mathbf{P}(A \mid B, C) = \frac{\mathbf{P}(B \mid A, C)\mathbf{P}(A \mid C)}{\mathbf{P}(B \mid C)}$$

Solution:

(a) $P(A \wedge C) = 0$: *In diesem Fall gilt* $P(A \mid B, C) = 0$*. Zudem gilt* $P(A \mid C) = 0$*, und somit auch* $\frac{\mathbf{P}(B|A,C)\mathbf{P}(A|C)}{\mathbf{P}(B|C)} = 0$.

(b) $P(C) = 0$: *Diesen Fall kann es nicht geben, da in diesem Fall* $P(A \mid C)$ *bzw* $P(B \mid C)$ *nicht definiert sind.*

(c) $P(A \wedge C) \neq 0$ *und* $P(C) \neq 0$

$$\begin{aligned} P(A \mid B, C) &\overset{Def.}{=} \frac{P(A \wedge B \wedge C)}{P(B \wedge C)} \\ &= \frac{P(B \wedge A \wedge C)}{P(A \wedge C)} \frac{P(A \wedge C)}{P(B \wedge C)} \\ &= P(B \mid A, C)\frac{P(A \wedge C)P(C)}{P(C)P(B \wedge C)} \\ &= P(B \mid A, C)\frac{P(A \mid C)}{P(B \mid C)} \\ &= \frac{P(B \mid A, C)P(A \mid C)}{P(B \mid C)} \end{aligned}$$

Exercise 9.3 **(20 P)**

Three prisoners A, B and C are locked up in their cells. Everybody knows that one of them will be put to death the next morning and the two others will be pardoned. The decision who will be put to death has already been made by the governor. The prisoner A asks the guard a favour: "Please ask the governor who will be put to death tomorrow and tell one of the other two prisoners that he will be pardoned." The guard accepts and returns to tell A, that he passed the pardon message to prisoner B.

To represent that situation, we use to variables $Execute$ and $GuardSays$. The possible values of $Executed$ are $\{A, B, C\}$, the possible values for $GuardSays$ are $\{B, C\}$. The joint probability distribution for the situation is

	$Execute = A$	$Execute = B$	$Execute = C$
$GuardSays = B$	$\frac{1}{6}$	0	$\frac{1}{3}$
$GuardSays = C$	$\frac{1}{6}$	$\frac{1}{3}$	0

(a) Why are there two values $\frac{1}{6}$ in each row of the column for $Execute = A$, while for the other two columns there is one row with $\frac{1}{3}$ and the other is 0? (Hint: Compute $P(GuardSays = B|Execute = A)$ and $P(GuardSays = C|Execute = A)$). (6 P)

Solution:

In case A is executed, the guard has a choice between telling B is pardoned and telling C is pardoned. Hence, the probability of $\frac{1}{3}$ of A being executed is split equally between these.

Computing the two values resulted in $\frac{1}{2}$, which was supposed to give you the indication that the probabilities are split equally.

$$
\begin{aligned}
P(GuardSays = B|Execute = A) &= \frac{P(GuardSays = B \wedge Execute = A)}{P(Execute = A)} \\
&= \frac{\frac{1}{6}}{\frac{1}{3}} \\
&= \frac{1}{2}
\end{aligned}
$$

(b) What is the conditional probability of A being executed the next day given B is not executed the next day? (justify your answer mathematically!) (7 P)

Solution:

$$
\begin{aligned}
P(Execute = A|\neg(Execute = B)) &= \frac{P(Execute = A \wedge \neg(Execute = B))}{P(\neg(Execute = B))} \\
&= \frac{\overbrace{\frac{1}{6} + \frac{1}{6}}^{Firstcolumn}}{\underbrace{\frac{1}{6} + \frac{1}{6}}_{FirstColumn} + \underbrace{\frac{1}{3}}_{Thirdcolumn}}
\end{aligned}
$$

$$= \frac{\frac{1}{3}}{\frac{2}{3}}$$
$$= \frac{1}{2}$$

(c) What is the conditional probability of A being executed the next day if the guard says B is not executed the next day? (justify your answer mathematically!) (7 P)

Solution:

$$\begin{aligned}
P(Execute = A|GuardSays = B)) &= \frac{P(Execute = A \wedge GuardSays = B)}{P(GuardSays = B)} \\
&= \frac{\overbrace{\frac{1}{6}}^{First\ column, First\ row}}{\underbrace{\frac{1}{6} + \frac{1}{3}}_{FirstRow}} \\
&= \frac{\frac{1}{6}}{\frac{3}{6}} \\
&= \frac{1}{3}
\end{aligned}$$
