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Fachrichtung 6.2 – Informatik
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4th Theoretical Assignment in Artificial Intelligence (SS 2005)
Solutions

Exercise 4.1 **(30 P)**

1. Define an admissible heuristic function h for the *Missionaries and Cannibals*-problem (Exercise 2.4 in the 2nd theoretical assignment). Explain why you have choosen this function. (10 P)

Solution:

A representation for this problem has been given in the sample solution of the 2nd theoretical assignment (exercise 2.4). A state can be defined by a triple (m, k, b).

The heuristic function h should estimate the cost (i.e, number of river-crosses). At first, we define two auxiliary functions h_1 and h_2:

$$h_1(m,k,b) = (m+k)\ div\ 2 + (m+k)\ mod\ 2$$
$$h_2(m,k,b) = \begin{cases} 1 & \text{, if } b = g \text{ and } h_1(m,k,b) > 0 \\ 0 & \text{, otherwise} \end{cases}$$

We set $h(m,k,b) = h_1(m,k,b) + h_2(m,k,b)$ and show that the heuristic function is admissible.

In one step, at most two persons can change the waterside. Therefore, at least $h_1(m,k,b)$ steps must be executed to assure that all persons could cross the river. In the case that some persons are still on the s and the boat is situated on the g, the boat must cross the river. This is expressed by h_2. Indeed, the heuristic function can be further improved.

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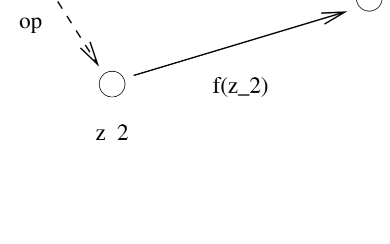
2. Show that your function h fulfils the triangle inequality (Russell/Norvig, p. 99). (10 P)

Solution:

The following figure illustrates the triangle inequality:



For two randomly chosen states z_1 and z_2, if z_1 can be transferred into z_2 by using an operator op, then the triangle inequality is fulfilled:

$$f(z_1) \leq f(z_2) + cost(op) \qquad \textit{(triangle inequality)}$$

As usual we set $f(z) = g(z) + h(z)$, where $g(z)$ are the costs we had to reach state z and $h(z)$ are the costs to reach the goal state from state z. The costs of each operation is $cost(op) = 1$. Adapting the inequality equation above, we obtain the following inequality:

$$g(z_1) + h(z_1) - g(z_2) - h(z_2) - 1 \leq 0.$$

We reformulate that formula and estimate the result:

$g(z_1) + h(z_1) - g(z_2) - h(z_2) - 1$	
$= g(z_1) - g(z_2) + h(z_1) - h(z_2) - 1$	
$= -1 + h(z_1) - h(z_2) - 1$	
$= -2 + h_1(z_1) - h_1(z_2) + h_2(z_1) - h_2(z_2)$	
$\leq -2 + 1 + h_2(z_1) - h_2(z_2)$	*$h_1(z_1) - h_1(z_2)$at most equal to 1, if two persons go from s to g.*
$\leq -2 + 1 + 1$	*$h_2(z_1) - h_2(z_2)$ at most equal to 1, if the boat crosses the river from g to s and persons are still on s.*
$= 0$	*q.e.d*

The triangle inequality is valid, due to the equivalence of the inequality above to $g(z_1) + h(z_1) \leq g(z_2) + h(z_2) + cost(op)$.

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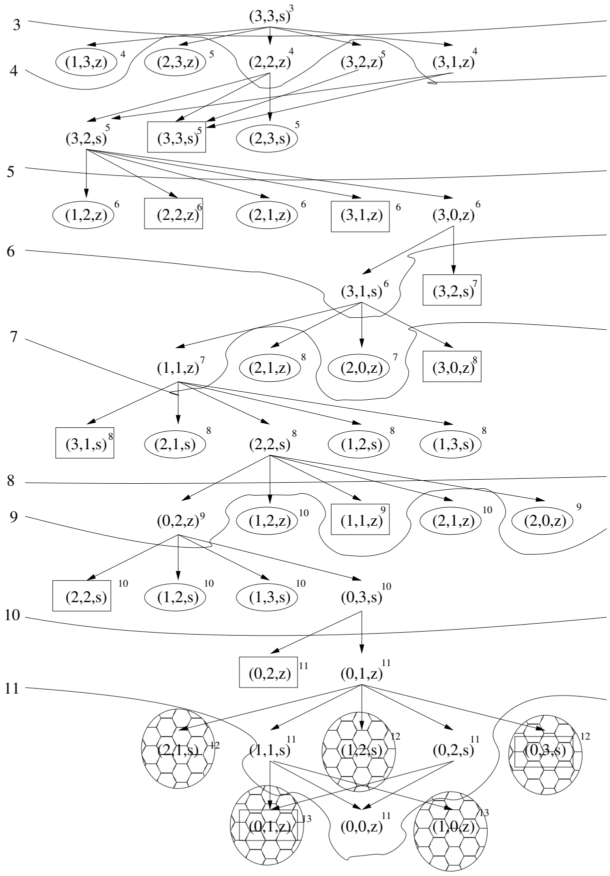
3. Explain graphically how an A*-search procedure traverses the search-space. (10 P)

Solution:

A-search filters the search space as illustrated in the following figure.*



Exercise 4.2 **(20 P)**

1. Use truth-tables to show that the following formulas of propositional logic are valid.

   - $P \wedge (Q \wedge R) \;\Leftrightarrow\; (P \wedge Q) \wedge R$ associativity of $\wedge$ (5 P)

Solution:

$F_1 = P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

P	Q	R	$P \wedge (Q \wedge R)$	$(P \wedge Q) \wedge R$	F_1
false	*false*	*false*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*
false	*true*	*true*	*false*	*false*	*true*
true	*false*	*false*	*false*	*false*	*true*
true	*false*	*true*	*false*	*false*	*true*
true	*true*	*false*	*false*	*false*	*true*
true	*true*	*true*	*true*	*true*	*true*

   - $P \vee (Q \vee R) \;\Leftrightarrow\; (P \vee Q) \vee R$ associativity of $\vee$ (5 P)

Solution:
$F_2 = P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$

P	Q	R	$P \vee (Q \vee R)$	$(P \vee Q) \vee R$	F_2
false	*false*	*false*	*false*	*false*	*true*
false	*false*	*true*	*true*	*true*	*true*
false	*true*	*false*	*true*	*true*	*true*
false	*true*	*true*	*true*	*true*	*true*
true	*false*	*false*	*true*	*true*	*true*
true	*false*	*true*	*true*	*true*	*true*
true	*true*	*false*	*true*	*true*	*true*
true	*true*	*true*	*true*	*true*	*true*

- $P \wedge (Q \vee R) \;\Leftrightarrow\; (P \wedge Q) \vee (P \wedge R)$ distributivity of $\wedge$ on $\vee$ (5 P)

Solution:
$F_3 = P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

P	Q	R	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	F_3
false	*false*	*false*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*
false	*true*	*true*	*false*	*false*	*true*
true	*false*	*false*	*false*	*false*	*true*
true	*false*	*true*	*true*	*true*	*true*
true	*true*	*false*	*true*	*true*	*true*
true	*true*	*true*	*true*	*true*	*true*

- $P \vee (Q \wedge R) \;\Leftrightarrow\; (P \vee Q) \wedge (P \vee R)$ distributivity of $\vee$ on $\wedge$ (5 P)

Solution:
$F_4 = P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

P	Q	R	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	F_4
false	*false*	*false*	*false*	*false*	*true*
false	*false*	*true*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*
false	*true*	*true*	*true*	*true*	*true*
true	*false*	*false*	*true*	*true*	*true*
true	*false*	*true*	*true*	*true*	*true*
true	*true*	*false*	*true*	*true*	*true*
true	*true*	*true*	*true*	*true*	*true*

Exercise 4.3 **(30 P)**

Assume that A, B and C are propositional constants.

1. Use truth tables to show that $\{A \vee B, \neg A \vee C\} \models B \vee C$ (10 P)

Solution:

The truth table for that problem is

A	B	C	$A \vee B$	$\neg A \vee C$	$KB(A \vee B) \wedge (\neg A \vee C)$	$B \vee C$	
true	*true*	*true*	*true*	*true*	*true*	*true*	$*$
true	*true*	*false*	*true*	*false*	*false*	*true*	$\circ$
true	*false*	*true*	*true*	*true*	*true*	*true*	$*$
true	*false*	*false*	*true*	*false*	*false*	*false*	$+$
false	*true*	*true*	*true*	*true*	*true*	*true*	$*$
false	*true*	*false*	*true*	*true*	*true*	*true*	$*$
false	*false*	*true*	*false*	*true*	*false*	*true*	$\circ$
false	*false*	*false*	*false*	*true*	*false*	*false*	$+$

Checking all lines, where the knowledge base is true, then $B \vee C$ is also true (see all the lines marked with $*$). Thus, $\{A \vee B, \neg A \vee C\} \models B \vee C$ holds.

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2. Does also $\{B \vee C\} \models (A \vee B) \wedge (\neg A \vee C)$ hold? Justify your answer. (5 P)

Solution:

Do the check by checking when $B \vee C$ holds, then the KB is true. This fails for the lines marked by $\circ$.

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3. Does $\{\neg(B \vee C)\} \models \neg((A \vee B) \wedge (\neg A \vee C))$ hold? Justify your answer. (5 P)

Solution:

Negate the values in the columns KB and $B \vee C$ above, and check whenever $\neg(B \vee C)$ is true, then so is $\neg(KB)$. This are the lines marked by $+$.

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4. Explain why the resolution rule preserves satisfiability. (5 P)

Solution:

Consider the second question above. It is a simplified form of resolution and shows for any A, B, and C, that if the knowledge base $\{A \vee B, \neg A \vee C\}$ is satisfiable, then so is the resolvent $B \vee C$. Hence, if KB is satisfiable, then so is $KB \cup \{B \vee C\}$.

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5. Let F be a propositional formula and KB be a finite set of propitional formulas. Assume we can derive the empty clause from $KB \vee \{\neg F\}$. Explain why we can conclude $KB \models F$ (5 P)

Solution:

The empty clause is unsatisfiable, and so is any set of clauses that contains the empty clause. Thus by resolution we have derived from $KB \wedge \neg F$ an unsatisfiable knowledge base. Since resolution preserves satisfiability (see before), we know conversely that $KB \wedge \neg F$ must have been unsatisfiable, i.e. $\not\models KB \wedge \neg F$.

$$\begin{aligned} \not\models KB \wedge \neg F \quad &\Leftrightarrow \quad \not\models \neg(KB \Rightarrow F) \\ &\Leftrightarrow \quad \models KB \Rightarrow F \\ &\overset{\text{Deduction Theorem}}{\Leftrightarrow} \quad KB \models F \end{aligned}$$

Exercise 4.4 **(20 P)**

The unicorn is a mammal if it is horned. If the unicorn is either immortal or a mammal, then it is horned. If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.

1. Encode these statements in propositional logic (5 P)

Solution:

(a) $Horned \Rightarrow Mammal$
(b) $(Immortal \vee Mammal) \Rightarrow Horned$
(c) $Mythical \Rightarrow Immortal$
(d) $\neg Mythical \Rightarrow (\neg Immortal \wedge Mammal)$

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2. Use resolution to prove that the unicorn is a mammal (15 P)

Solution:

(1) $\neg Horned \vee Mammal$
(2) i. $\neg Immortal \vee Horned$
    ii. $\neg Mammal \vee Horned$
(3) $\neg Mythical \vee Immortal$
(4) i. $Mythical \vee \neg Immortal$
    ii. $Mythical \vee Mammal$
(5) $\neg Mammal$
(6) $\neg Horned$ *(Resolve 1 and 5)*
(7) $Mythical$ *(Resolve 4b and 5)*
(8) $Immortal$ *(Resolve 7 and 3)*
(9) $\neg Immortal$ *(Resolve 6 and 2a)*
(10) *empty clause (Resolve 8 and 9)*
