Universität des Saarlandes
Fachrichtung 6.2 – Informatik
J. Siekmann, S. Autexier
C. Benzmüller, C. Brown, C. Hahn

5th Theoretical Assignment in Artificial Intelligence (SS 2005)

Issued: May 30, 2005 Due: June 6, 2005

Exercise 5.1: **10 P**

In the following, we specify a set of first order logic formulas:

(A) $\forall x.\neg(x < x)$

(B) $\forall x, y, z.(x < y \wedge y < z) \Rightarrow x < z$

(C) $\forall x.x < S(x)$

(D) $N(0)$

(E) $\forall x.N(x) \Rightarrow N(S(x))$

A model for this first-order language is given by a nonempty set $\mathcal{D}$ of objects, a binary relation $| < |$ on $\mathcal{D}$ (to interpret $<$), a function $|S|$ from $\mathcal{D}$ to $\mathcal{D}$ (to interpret S), a subset $|N|$ of $\mathcal{D}$ (to interpret N) and an element $|0|$ of $\mathcal{D}$ (to interpret 0).

1. Give a model which satisfies the set of formulas $\{$**(A)**,**(C)**,**(D)**,**(E)**$\}$ and such that $\mathcal{D}$ contains exactly two objects. (5 P)
2. Give a model that satisfies the set of formulas $\{$**(A)**,**(B)**,**(C)**,**(D)**,**(E)**$\}$. (Hint: $\mathcal{D}$ can be infinite.) (5 P)

For each part, justify your answer!

Exercise 5.2: **13 P**

1. Represent the following statement in first order logic.
   - Every student attends some class.

   (5 P)
2. Convert your first-order statement above into clause normal form as described in class. In this example, you should perform the following steps:
   (a) Eliminate implications in favor of disjunction and negation.
   (b) Skolemize to remove existential quantifiers.
   (c) Remove universal quantifiers.
   (d) Distribute $\wedge$ over $\vee$.

Show the result of each step! (8 P)

Exercise 5.3: **10 P**

Identify the most general unifier (if one exists) for the following pairs of formulas. Constants are illustrated as capitals, variables as lower case letters.

1. $Older(Father(y), y)$, $Older(Father(x), John)$ (5 P)
2. $Q(y, G(A, B))$, $Q(G(x, x), y)$ (5 P)

Exercise 5.4: **20 P**

Let S be the set of the following four clauses:

(A) $P(A, A)$

(B) $\neg P(B, B)$

(C) $\neg P(x, x) \vee P(x, f(x, y))$

(D) $\neg P(x, f(x, y)) \vee P(y, y)$

The set S is inconsistent.

1. In general, the Herbrand Universe of a set of clauses is the set of all ground terms (terms with no variables) that can be constructed from constants and function symbols which occur in the set of clauses. Describe the Herbrand Universe H_S of S. (5 P)
2. Give all ten (10) ground instances of the clauses **(A)**, **(B)**, **(C)** and **(D)** the variables to range over the two terms A and B. (5 P)
3. Give a set of four of the ten (10) ground clauses above which is inconsistent. Also, give a refutation (i.e., derive the empty clause) using resolution and factoring. Let Ψ be the set of all ground clauses which appear in this refutation. (5 P)
4. Find a refutation of the clauses in S (with variables) using resolution and factoring with the following property: Every clause involved in this refutation has a ground instance in Ψ. (This is the process of "lifting" the ground refutation to a refutation with variables.) (5 P)

Exercise 5.5: **6 P**

The *unit preference strategy* only allows resolution steps if one of the clauses is a unit clause (that is, a clause with one literal). Let S be the set of the following four propositional clauses:

(A) $P \vee Q$

(B) $\neg P \vee Q$

(C) $P \vee \neg Q$

(D) $\neg P \vee \neg Q$

Explain why the empty clause cannot be derived from S using the unit preference strategy. (Your explanation should not be more than three sentences.)

Exercise 5.6: **15 P**

The *set-of-support (sos) strategy* only allows inference rules (resolution or factoring) to be applied when one of the given clauses is in the set-of-support (sos). Whenever a new clause is inferred using resolution or factoring, the new clause is included in the set-of-support. Let Φ be the set of four clauses

(A) $P(0)$

(B) $\neg P(S(S(S(0))))$

(C) $\neg P(x) \vee P(S(x))$

(D) $\neg P(S(x)) \vee P(x)$

1. Assume the set-of-support is initially $\{(A)\}$. Derive the empty clause from Φ using the set-of-support strategy. (5 P)
2. Assume the set-of-support is initially $\{(B)\}$. Derive the empty clause from Φ using the set-of-support strategy. (5 P)
3. Assume the set-of-support is initially $\{(D)\}$. Give four clauses that can be derived from Φ using the set-of-support strategy. (5 P)

Exercise 5.7: **20 P**

Given the following clause forms, prove whether there exists an $x \in \{John, Mike, Tom\}$ that satisfies both $Climber(x)$ and $\neg Skier(x)$? If it exists, who is x:

1. $Alp(Tom)$
2. $Alp(Mike)$
3. $Alp(John)$
4. $\neg Alp(x) \vee Skier(x) \vee Climber(x)$
5. $\neg Skier(x) \vee Like(x, Snow)$
6. $\neg Climber(x) \vee \neg Like(x, Rain)$
7. $\neg Like(Tom, y) \vee \neg Like(Mike, y)$
8. $Like(Tom, y) \vee Like(Mike, y)$
9. $Like(Tom, Rain)$
10. $Like(Tom, Snow)$

Exercise 5.8: **6 P**

Suppose we are given the clause

(GM) $GoodMan(John) \vee GoodMan(Mike) \vee GoodMan(Tom)$

1. Using clause **(GM)**, prove $\exists x GoodMan(x)$ by refuting $\neg\exists x GoodMan(x)$ using resolution and factoring. (3 P)

2. Is it possible to find a particular $x \in \{John, Mike, Tom\}$ such that $GoodMan(x)$ follows from clause **(GM)**? (3 P)

Exercise Bonus: **12 P (BONUS EXERCISE)**

Let Φ be the set with the two clauses $(0 = 1)$ and $\neg(1 = 0)$ (where 0 and 1 are constant symbols). Either $(0 = 1)$ or $\neg(1 = 0)$ must be false in any first-order model with equality. (In this sense, the set Φ is *unsatisfiable*).

1. Explain why it is *not possible* to derive the empty clause from Φ using the resolution and factoring rules. (Your explanation should not be more than four sentences.) (3 P)

2. Explain why the unsatisfiable clause set Φ does not provide a counterexample to the completeness theorem for resolution given in class. (Your explanation should not be more than two sentences.) (3 P)

3. Derive the empty clause from $\Phi \cup \{x = x\}$ using the resolution, factoring and paramodulation rules. (3 P)

4. Is it possible to derive the empty clause from Φ using the resolution, factoring and paramodulation rules? (Answering "Yes" or "No" is sufficient.) (3 P)