Assignment 12
Introduction to Computational Logic, SS 2006

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Exercise 12.1 (Boolean Quantifiers) Find a backward proof for

$$\mathrm{QL} \vdash \forall f = f0 \wedge f1$$

Exercise 12.2 (Cantor) Let the type of f be $T \to T \to B$. Find a backward proof for

$$\mathrm{QL} \vdash \exists f \forall g \exists x \forall h.h(fx) \leftrightarrow hg = 0$$

Exercise 12.3 (Strange) The notation $\forall f.\forall(f\forall)$ may describe many terms.

a) Make the notation more explicit by giving the type indices for the 3 occurences of $\forall$. State the indices in most general form by using the meta variables S and T.

b) Prove $\mathrm{QL} \not\vdash \forall f.\forall(f\forall) = 1$. Use a deductive argument.

Exercise 12.4 (Tricky) Find a backward proof for

$$\mathrm{QL} \vdash \exists x.fxy \to fzx$$

Exercise 12.5 (Challenge) Let the type of f be $T \to T \to B$. Find a backward proof for

$$\mathrm{QL} \vdash \exists g \forall x \exists h.h(fx) \wedge \overline{hg} = 1$$

Exercise 12.6 (Boolean Choice) Consider Propositional Logic PL.

a) How many choice functions do exist for $\mathbb{B}$?

b) Find for each choice function a closed term s describing it.

c) Prove $\mathrm{PL} \vdash fx \to f(sf) = 1$ for one of the terms describing a choice function.

d) Find a closed term describing the existential quantifier for $\mathbb{B}$ that contains no other constant than 1.

Exercise 12.7 (Definition of Identities)

a) Find a backward proof for $\mathrm{QL} \vdash \forall f.fx \to fy = \forall f.fx \leftrightarrow fy$

Hint: Use branching to reduce $\leftrightarrow$ to $\to$.

b) Explain how the above equation relates to the axiomatization of identities in QL′.

Exercise 12.8 (Duality) To establish the duality of QL′, one has to prove

$$\mathrm{QL}' \vdash x \neq y = \exists f.fx \not\doteq fy$$

Find a backward proof.

Exercise 12.9 (Identity Laws) Find backward proofs for the following claims.

a) $\mathrm{QL}' \vdash x \doteq y \to y \doteq z \to x \doteq z = 1$

b) $\mathrm{QL}' \vdash x \doteq y \to fx \doteq fy = 1$

c) $\mathrm{QL}', \mathrm{Ext} \vdash (\forall x.s \doteq t) \to (\lambda x.s) \doteq (\lambda x.t) = 1$

Exercise 12.10 Find a backward proof for

$$\mathrm{QL}' \vdash x \wedge y = (\lambda f.fxy) \doteq (\lambda f.f11)$$

You may use all laws from the slides.

Hint: For "→", use BRep and Ref. For "←", use $D\doteq$.