Assignment 11
Introduction to Computational Logic, SS 2006

Prof. Dr. Gert Smolka, Dipl.-Inform. Mathias Möhl
www.ps.uni-sb.de/courses/cl-ss06/

Exercise 11.1 (Inconsistence) Let $x, y : B$ be variables. Prove the following.

a) $\text{QL} \nvdash 0 = 1$

b) $0 = 1 \overset{\text{BA}}{\vdash\dashv} x = \bar{x}$

c) $0 = 1 \overset{\text{BA}}{\vdash\dashv} x = y$

Exercise 11.2 (Duality) Find a Boolean equation e such that $e \overset{\text{BA}}{\not\vdash\!\dashv} \hat{e}$.

Exercise 11.3 (∃ Defined) Let $\text{QL}' = \text{PL} \cup \{\forall 1, \forall I, \exists f = \overline{\forall x.\overline{fx}}\}$. Find conversion proofs for the following claims.

a) $\text{QL}' \vdash \exists x.0 = 0$

b) $\text{QL}' \vdash fx \to \exists f = 1$

Moreover, prove the following.

c) $\text{QL} \vdash\dashv \text{QL}'$

Exercise 11.4 (Q→-Laws) Prove that the following equations called Q→-Laws are deducible from QL. Use conversion proofs with tautologies, $\forall\vee$, $\forall\wedge$, $\forall\wedge'$, dM and their duals.

a) $q \to \exists f = \exists x.q \to fx$

b) $q \to \forall f = \forall x.q \to fx$

c) $\exists f \to q = \forall x.fx \to q$

d) $\forall f \to q = \exists x.fx \to q$

e) $\forall f \to \exists g = \exists x.fx \to gx$

Exercise 11.5 (Backward Proofs) Find backward proofs showing that $s = 1$ is deducible from QL for the following terms s. You may use all quantifier laws from the slides.

a) $\forall f \to \exists f$

b) $\exists x.\ fx \to \forall f$

c) $\forall f \vee \forall g \to \forall x.\ fx \vee gx$

d) $\forall f \rightarrow \forall x.\ fx \vee gx$

e) $(\exists f \rightarrow \forall h) \rightarrow \forall x.\ fx \rightarrow hx$

f) $(\forall x.\ fx \vee hx) \rightarrow \forall f \vee \exists h$

g) $(\exists x \forall y. fxy) \rightarrow \forall y \exists x. fxy$

h) $\forall x.\ fx \rightarrow \exists f$

i) $(\forall x.\ fx \rightarrow gx) \rightarrow \forall f \rightarrow \forall g$

j) $(\forall x.\ fx \rightarrow gx) \rightarrow \exists f \rightarrow \exists g$

k) $(\forall x.\ fx \rightarrow \overline{gx}) \rightarrow \forall y.\ gy \rightarrow \overline{fy}$

Exercise 11.6 (Counterexample) Let $f, g \colon B \rightarrow B$ be different variables. Prove

$$\text{QL} \nvdash \forall f \vee \forall g = \forall x.\ fx \vee gx$$

by contradiction.

Exercise 11.7 (Deductivity) Find terms s, t such that

1) s and t closed

2) $s = 1 \overset{\text{BA}}{\vdash} t = 1$

3) $\text{BA} \overset{\text{BA}}{\nvdash} s \rightarrow t = 1$

Hint: Use constants $a \colon B$ and $f \colon B \rightarrow B$.

Exercise 11.8 Make sure that you understand the proofs of the following laws on the slides in detail

a) Golden Rule

b) $\forall \wedge$

c) de Morgan

d) Cantor