



Assignment 8 Introduction to Computational Logic, SS 2006

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Exercise 8.1 (Modelling) Four girls agree on some rules for a party:

- i) Whoever dances which Richard must also dance with Peter and Michael.
- ii) Whoever does not dance with Richard is not allowed to dance with Peter and must dance with Christophe.
- iii) Whoever does not dance with Peter is not allowed to dance with Christophe.

Express these rules as simple as possible.

- a) Describe each rule with a Boolean equation. Do only use the parameters c (Christophe), p (Peter), m (Michael), r (Richard).
- b) Give the prime tree for the solution set of your equation system. Use the order $c < p < m < r$. Treat parameters like variables.

Exercise 8.2 Find a conversion proof for

$$\text{BA}, a = \bar{a} \vdash 0 = 1$$

Exercise 8.3 Find a closed specification Bool' that is deductively equivalent to Bool .

Exercise 8.4 Verify the following deductive equivalence with three conversion proofs:

$$\{xy = 0, x + y = 1\} \stackrel{\text{BA}}{\vdash} \{0 = 1\}$$

Exercise 8.5 (Closed Specifications) Let A be a closed specification. Prove that the following equivalence holds:

$$A \models e \iff \forall I: I \models A \implies I \models e$$

Exercise 8.6 Let x be a variable that does not occur in $s = t$. Find a Gödel-style proof (derivation) of $\lambda x.s = \lambda x.t$ from $s = t$ that does not employ ξ .

Exercise 8.7

- a) Verify the deductive equivalence $\{s = t\} \vdash \{\lambda x.s = \lambda x.t\}$ with two conversion proofs.
- b) Prove $\forall e \exists e' : e' \text{ closed} \wedge \{e\} \vdash \{e'\}$.

Exercise 8.8 (Semantic Entailment) Peter and Maria discuss semantic entailment. Peter claims that the following equivalence holds:

$$A \models e \iff \forall \mathcal{I} : \mathcal{I} \models A \implies \mathcal{I} \models e$$

Maria thinks that Peter is wrong, since the definition of semantic entailment in the lecture notes is more complex than Peter's equivalence. After some time she produces a counterexample that shows that Peter's equivalence is wrong.

- a) Find a counterexample and prove that it falsifies Peter's equivalence.
- b) Peter acknowledges the counterexample but claims that there is some condition γ such that:
 - (i) Every specification is deductively equivalent to a specification that satisfies γ .
 - (ii) Every specification that satisfies γ satisfies his equivalence.
 Find such a condition γ .

Exercise 8.9 (Significant Variables) Determine the significant variables of the Boolean term $(\bar{x} + y)(x + y)(y + z)$.

Exercise 8.10 (Boolean Equation Systems) For each of the following Boolean equation systems E

- a) $\{x = y\}$
- b) $\{xy = \bar{x} + \bar{y}\}$
- c) $\{x = x \leftrightarrow y, x = \bar{y}\}$
- d) $\{((x \rightarrow y) \rightarrow x) \rightarrow x = 1\}$

do the following:

- i) Find a prime tree t such that $\forall \mathcal{I} : \mathcal{T} \subseteq \mathcal{I} \implies (\mathcal{I} \models E \iff \mathcal{I} \models t = 1)$ where \mathcal{T} is the two-valued Boolean algebra. Use the variable order $x < y$.
- ii) If possible, give a $\sigma \in \{x, y\} \rightarrow \mathbb{B}$ such that there is no interpretation \mathcal{I} such that $\mathcal{I} \models E$ and $\mathcal{T}, \sigma \subseteq \mathcal{I}$.
- iii) Draw a minimal graph representing the prime trees from (a)-(d)

Exercise 8.11 (Decision Graph) Let $E = \{xy = xz, yz = xz\}$ and $x < y < z$ be variables of type B.

- a) Find a prime tree t that represents the solutions of E in \mathcal{T} .
- b) Draw a minimal graph whose nodes represent the subtrees of t .
- c) Represent the graph by a table that represents an injective function φ such that:
 - i) $Dom \varphi = \{2, \dots, n\}$ where $n \geq 1$
 - ii) if $\varphi n = (u, n_0, n_1)$, then $n > n_0, n_1$Label each non-terminal node of the graph with the number representing it.