

Exercise 6.1 The models of BA are known as Boolean algebras. Stone's representation theorem says that every finite Boolean algebra is isomorphic to a powerset algebra. Explain why this implies that there is no Boolean algebra where B has 7 elements.

Exercise 6.2 Find a model of BA that has exactly 8 elements.

Exercise 6.3 Find a substitution θ such that $S\theta(BA) = BA$ and $Ker\theta = \{0, 1, +, \cdot\}$.

Exercise 6.4 State both versions (duality) of the following laws:

- a) Absorption
- b) Complements
- c) Distributivity
- d) Dominance
- e) Idempotence
- f) Identities
- g) Resolution

Exercise 6.5 Show that the following equations are deducible from BA. Use commutativity and associativity tacitly and mention the use of the other axioms.

- a) $xx = x$ (idempotence)
Hint: Start with $xx = xx + 0$, then use complements.
- b) $x0 = 0$ (dominance)
Hint: Start with $x0 = x0 + 0$, then use complements.
- c) $x = x(x + y)$ (absorption)
Hint: Start with $x = x + 0$, then use (b).
- d) $xy + \bar{x}z = xy + \bar{x}z + yz$ (resolution)
Hint: Start from left and use (c) in the form of $x = x(x + y)$ and $\bar{x} = \bar{x}(\bar{x} + y)$, then use (a) and complements in the form $x + \bar{x} = 1$.

Exercise 6.6 Let x, y be variables of type B . Prove $BA, 0=1 \vdash x=y$. Use the dominance law.

Exercise 6.7 Let a and b be constants of type B . Prove the following deductive equivalence, which is known as **Uniqueness of Complements (UoC)**.

$$ab = 0, a + b = 1 \stackrel{\text{BA}}{\vdash\vdash} b = \bar{a}$$

Hint: For \vdash , start with $b = b1$ and use complements.

Exercise 6.8 Show that the following equations are deducible from BA. Use instances of UoC. Make sure you understand where the stability law is used.

- a) $x = \bar{\bar{x}}$
- b) $\bar{0} = 1$
- c) $\overline{xy} = \bar{x} + \bar{y}$
- d) $\overline{x + y} = \bar{x}\bar{y}$

Exercise 6.9 Let a and b be constants of type B . Prove the following deductive equivalences. You may use all Boolean laws.

- a) $a = 1, b = 1 \stackrel{\text{BA}}{\vdash\vdash} ab = 1$
- b) $a = 0, b = 0 \stackrel{\text{BA}}{\vdash\vdash} a + b = 0$
- c) $a = ab \stackrel{\text{BA}}{\vdash\vdash} b = b + a$
- d) $\bar{a} + b = 1 \stackrel{\text{BA}}{\vdash\vdash} a = ab$
- e) $(\bar{a} + b)(a + \bar{b}) = 1 \stackrel{\text{BA}}{\vdash\vdash} a = b$

Exercise 6.10 Let a, b, c, d be constants of Type B . Find a term s such that $a = b, c = d \stackrel{\text{BA}}{\vdash\vdash} s = 1$. Hint: Use the deductive equivalences of Exercise 6.9.

Exercise 6.11 Show that the following equations are deducible from BA. You may use all Boolean laws.

- a) $(\lambda f. fx\bar{x})(+) = 1$
- b) $(\lambda xy. \bar{x} + y)0 = \lambda x.x + \bar{x}$
- c) $(\lambda f. f(fxy)y)(\lambda xy. \bar{x} + y) = x + y$