



## Assignment 5 Introduction to Computational Logic, SS 2006

Prof. Dr. Gert Smolka, Dipl.-Inform. Mathias Möhl  
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**Exercise 5.1 (Renaming)** Prove the following propositions in the lecture notes:

- a) Proposition 2.6
- b) Proposition 2.7

**Exercise 5.2 (Boolean Connectives)** Find a categorical specification that extends Bool with the constants

$$\neg: B \rightarrow B$$
$$\wedge, \vee: B \rightarrow B \rightarrow B$$

and forces their canonical interpretations.

**Exercise 5.3 (Quantifiers)** Find a specification Quant that extends Bool with the constants

$$\forall, \exists: (C \rightarrow B) \rightarrow B$$

and has the following properties:

- 1) Every proper model gives the constants  $\forall$  and  $\exists$  their canonical interpretations.
- 2) For every non-empty set  $X$  there exists a model that interprets  $B$  as  $\mathbb{B}$  and  $C$  as  $X$ .

**Exercise 5.4 (Identity)** Find a specification Ident that extends Bool with the constant

$$\doteq: C \rightarrow C \rightarrow B$$

and has the following properties:

- 1) Every proper model gives the constant  $\doteq$  its canonical interpretation.
- 2) For every non-empty set  $X$  there exists a model that interprets  $B$  as  $\mathbb{B}$  and  $C$  as  $X$ .

**Exercise 5.5 (Linear)** Find a specification Linear that extends Bool with the constants

$$\forall: (V \rightarrow B) \rightarrow B$$

$$o: V \qquad \text{origin}$$

$$\sigma: V \rightarrow V \qquad \text{successor}$$

and has the following properties:

- 1) Every proper model  $\mathcal{A}$  of Linear interprets  $V$  with the recursively defined set  $X := \{\mathcal{A}o\} \cup \{\mathcal{A}\sigma v \mid v \in X\}$
- 2) Linear has a proper model that interprets  $V$  as an infinite set.
- 3) For every  $n \geq 2$  there exists a proper model of Linear that interprets  $V$  with a finite set that has  $n$  elements.

**Exercise 5.6 (Infinite Chain)** Find a categorical specification Chain that extends Linear with the constant

$$\doteq: V \rightarrow V \rightarrow B$$

and has the property that every proper model interprets  $V$  as an infinite set.

**Exercise 5.7 (Natural Numbers)** Find a categorical specification of the natural numbers that extends Bool with the following constants:

$$\forall: (N \rightarrow B) \rightarrow B$$

$$\underline{0}, \underline{1}: N$$

$$\leq: N \rightarrow N \rightarrow B$$

$$+, \cdot: N \rightarrow N \rightarrow N$$