Assignment 4
Introduction to Computational Logic, SS 2006

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`http://www.ps.uni-sb.de/courses/cl-ss06/`

Exercise 4.1 (Implicit Conditions) The formulation of the axioms leaves implicit the universal quantification of meta-variables and the domain constraints for partial functions. For instance, the axioms

TA $\tau t = \tau s \to \tau(ts)$

TL $\tau(\lambda x.t) = \tau x \to \tau t$

look in explicit formulation as follows:

TA $\forall s,t \in \mathit{Ter}\colon\ t \in \mathit{Dom}\,A \wedge s \in \mathit{Dom}\,(At) \Longrightarrow \tau t = F(\tau s)(\tau(Ats))$

TL $\forall x \in \mathit{Var}\quad \forall t \in \mathit{Ter}\colon\ \tau(Lxt) = F(\tau x)(\tau t)$

Give explicit formulations of the axioms IA, IL and SA.

Exercise 4.2 (Construction) The lecture notes outline the construction of a model of the axiomatization of terms. Define the functions $\mathcal{N}$ and **S**. Use the following notations:

$$\begin{aligned} st &\rightsquigarrow (5,(s,t)) \\ \lambda T.t &\rightsquigarrow (6,(T,t)) \\ i &\rightsquigarrow (7,i) \qquad \text{where } i \in \mathbb{N} \end{aligned}$$

Exercise 4.3 (Trivial Substitution) Prove the following:

$$\forall t \in \mathit{Ter}\colon\ \mathbf{S}(\lambda u \in \mathit{Nam}.u)t = t$$

Exercise 4.4 (Coincidence) Prove the following:

$$\forall t \in \mathit{Ter}\quad \forall f,g \in \mathit{Dom}\,\mathbf{S}\colon\ (\forall u \in \mathcal{N}t\colon\ fu = gu) \Longrightarrow \mathbf{S}ft = \mathbf{S}gt$$

Exercise 4.5 (Counterexamples) Find counterexamples for the following claims

a) $t = t'[x' := x] \wedge x' \notin \mathcal{N}(\lambda x.t) \Longrightarrow \lambda x.t = \lambda x'.t'$

b) $t' = t[x := x'] \Longrightarrow \lambda x.t = \lambda x'.t'$

Exercise 4.6 Make sure that you can name all the sets and all the functions of the axiomatization of terms. Moreover, given the name of an axiom, you should be able to reproduce the axiom.