CS 578 – Cryptography

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PRPs, PRFs, and Security Definitions of Ciphers (cont'd)

May 9, 2006

Administrative Announcements

- Handouts today:
  - Lecture notes, next exercise sheet
- Office hours:
  - Moved Thursday's TA office hour to Monday 14:00-16:00
  - Additional office hour by myself on Monday 12:00-13:00
- Result of quizzes sent by weekly email
- Remarks on the mid-term exam:
  - Takes place on Tuesday, May 30, 1 hour, instead of lecture 11:00 - 13:00 (likely to happen in 2 groups)
  - Preceding Friday, May 26, Q&A lecture!

Recall: Stream- and block ciphers

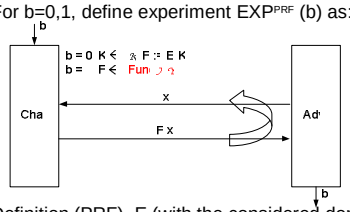
- Stream Ciphers (PRG):
  - Encrypts long messages, but one-time key use
- Block Ciphers:
  - basic "primitive" for encrypting short blocks
- Attacks on block ciphers:
  - Exhaustive Search: For const # of PT/CT pairs
  - Linear Cryptanalysis: Large (2^{42}) # of PT/CT pairs
- Defining security via games:
  - PRPs, PRFs
  - Semantic security against CT-only attacks for ciphers

Recall: PRFs and PRPs

- PRF: F: $\mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ is a PRF over $(\mathcal{K},\mathcal{X},\mathcal{Y})$ if
  - F is "efficiently" computable
  - F indistinguishable from a random function $\mathcal{X} \rightarrow \mathcal{Y}$ for a randomly drawn key
- PRP: E: $\mathcal{K} \times \mathcal{X} \rightarrow \mathcal{X}$ is a PRP over $(\mathcal{K},\mathcal{X})$ if
  - E can be efficiently computed
  - For all $K \in \mathcal{K}$ E(K,·): $\mathcal{X} \rightarrow \mathcal{X}$ is bijective.
  - For all $K \in \mathcal{K}$ D(K,·) = E^{-1}(K,·) is efficiently computable.
  - F indistinguishable from a random permutation $\mathcal{X} \rightarrow \mathcal{X}$ for a randomly drawn key
- Examples: AES, 3DES
- Any PRP on $(\mathcal{K},\mathcal{X})$ is also a PRF on $(\mathcal{K},\mathcal{X},\mathcal{X})$

Recall: PRF Attack Game

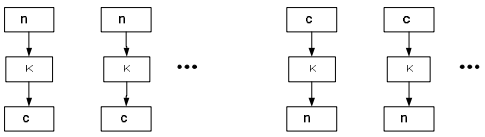
- For b=0,1, define experiment EXP^{PRF} (b) as:



- Definition (PRF). E (with the considered domains, ranges, etc.) is a PRF if for all efficient adversaries A, we have that $Adv^{PRF}[A,E] = |Pr[EXP^{PRF}(0)=1] – Pr[EXP^{PRF}(1)=1]|$ is negligible.

Recall: Electronic Codebook

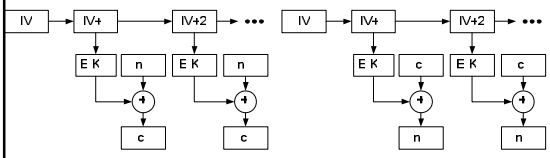
- Electronic Codebook (ECB)



- "Not secure" because the adversary can tell if two blocks encrypt the same message

Recall: Countermode

- Countermode (CTR)



- Should be “secure” if E is “secure”…

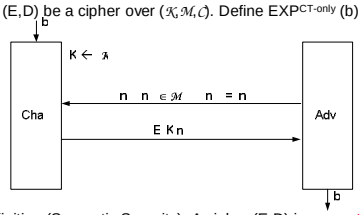
Recall: Definitions of Security

- Security always defined in two parameters:
  1. What “power” does the adversary have?
     - Adv. sees only one ciphertext (i.e. CT-only attack)
     - Adv. sees many PT/CT pair (CPA)
     - (Adv. gets chosen CTs decrypted (CCA))
  2. What “goal” is the adversary trying to achieve?
     - Semantic security: learn info about (new) PT from CT

Goal \ Power	One-time key (CT-only attack)	Many-time key (CPA)	CCA
Semantic Security	Stream ciphers (det) ctr mode	(rand) CBC (rand) ctr mode	Later

Recall: Semantic Security (CT-only attack)

- Let (E,D) be a cipher over $(\mathcal{K},\mathcal{M},\mathcal{C})$. Define $EXP^{CT\text{-}only}$ (b) as:



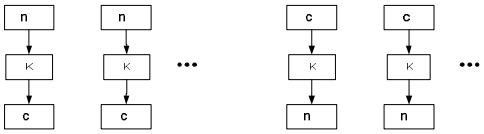
- Definition (Semantic Security). A cipher (E,D) is semantically secure under CT-only attack if for all efficient adversaries A, we have that $Adv^{CT\text{-}only}[A,E] = |Pr[EXP^{CT\text{-}only}(0)=1] - Pr[EXP^{CT\text{-}only}(1)=1]|$ is negligible.

First Implications of Semantic Security

- Semantic Security states that a CT leaks nothing about a PT to efficient adversaries
- E.g., assume an adversary A learns “the x-th bit of m” given E(K,m) with probability ½ + p
- Claim: If A is efficiently computable and p is not negligible, then E cannot be semantically secure!
  [proofsketch on the board again]
- Works not only for “x-th bit” but for any samplable predicate on messages (!!)

Semantic Security of ECB?

- Recall: Electronic Codebook (ECB)



- Semantically Secure under CT-only attack?

Semantic Security and ECB

- ECB is not semantically secure under CT-only attack
  - Consider the following messages
  - m_0 = “hello””hello”, m_1 = “hello””world”
  - Leads to outputs of the form
    c_0 = “c” ”c”, c_1 = “c” ”c*”
  - Easily distinguishable!

Constructions for Semantic Security

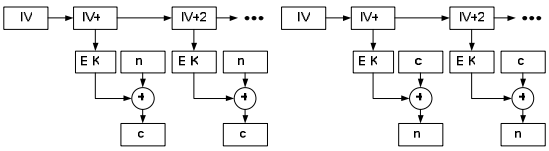
- Examples:
  - One-time Pad: $Adv^{CT\text{-}only}[A, OTP] = 0$ for all A.
  - Deterministic counter mode from a PRF E: (essentially stream ciphers built from PRF, e.g., AES, 3DES)

$E^{DETCTR}(K,m) =$

	m[1]	m[2]	…	m[L]
⊕	E[K,1]	E[K,2]	…	E[K,L]
	c[1]	c[2]	…	c[L]

Deterministic Counter mode

- Deterministic Countermode (DETCTR): IV = 0

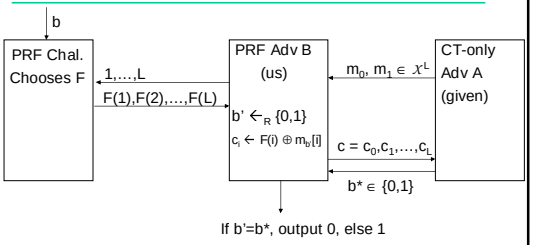


Deterministic ctr-mode Security

- Theorem: For any L>0:
  If E is a PRF over $(\mathcal{K}, \mathcal{X})$ then E^{DETCTR} (using E) is a semantically secure cipher over $(\mathcal{K}, \mathcal{X}^L, \mathcal{X}^L)$ under CT-only attack.

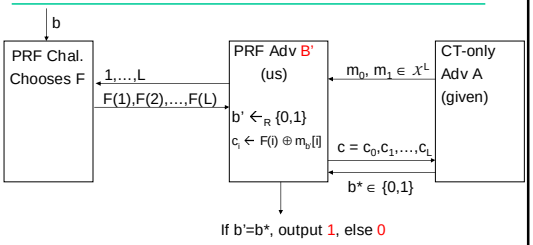
  More precisely, for any adversary A attacking E^{DETCTR} there exists a PRF adversary B such that
  $Adv^{CT\text{-}only}[A, E^{DETCTR}] = 2 \cdot Adv^{PRF}[B,E]$

Proof Overview



If b'=b*, output 0, else 1

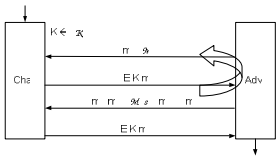
Proof Overview



If b'=b*, output 1, else 0

Semantic Security under CPA attack

- Let (E,D) be a cipher defined over $(\mathcal{K},\mathcal{M},\mathcal{C})$. For b = 0,1 define $EXP^{CPA}(b)$ as:



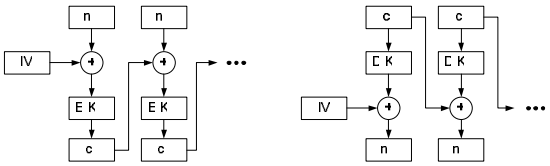
- Definition. A cipher (E,D) is semantically secure under CPA if for all efficient adversaries A, we have that
$Adv^{CPA}[A,E] = |Pr[EXP^{CPA}(0)=1] – Pr[EXP^{CPA}(1)=1]|$ is negligible

Randomized Encryption

- Fact: Stream ciphers are insecure under CPA.
- Fact: No deterministic function can be secure under CPA!!
- Consequence: Good encryption algorithms must be randomized

Security of CBC

- Cipherblock Chaining (CBC)
  - Very often used, but main problem: Sequential
  - Initial value random chosen and output as well



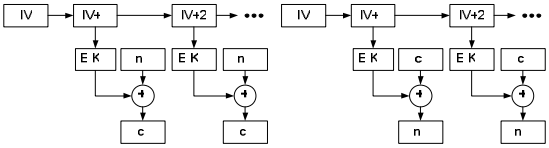
Security of CBC (cont'd)

- Cipher Block Chaining (CBC) with a random IV
- CBC theorem: For any L > 0: If E is a PRP over $(\mathcal{K}, \mathcal{X})$ then E^{CBC} is semantically secure under CPA over $(\mathcal{K}, \mathcal{X}^L, \mathcal{X}^{L+1})$.

  In particular: For any q-query adversary A attacking E^{CBC} there exists a PRF adversary B such that $Adv^{CPA}[A, E^{CBC}] \le 2 \cdot Adv^{PRF}[B,E] + 2q^2L^2 / |\mathcal{X}|$
- Note: CBC is only secure as long as $qL << |\mathcal{X}|^{1/2}$

Security of Random ctr-mode

- Random Countermode (RNDCTR)
  - New IV for every encryption (will be send as IV=c_0)
  - Note: No need for decryption here → PRF instead of PRP suffices
  - We will see: Better security than CBC



Security of Random ctr-mode (cont'd)

- Randomized counter mode: random IV for every new message to be encrypted
- Counter-mode Theorem: For any L > 0: If E is a PRF over $(\mathcal{K}, \mathcal{X}, \mathcal{X})$ then E^{RNDCTR} is semantically secure under CPA over $(\mathcal{K}, \mathcal{X}^L, \mathcal{X}^{L+1})$.

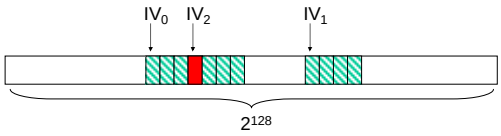
  In particular: For any q-query adversary A attacking E^{RNDCTR} there exists a PRF adversary B such that $Adv^{CPA}[A, E^{RNDCTR}] \leq 2 \cdot Adv^{PRF}[B,E] + 2q^2L/|\mathcal{X}|$

- Note: ctr-mode only secure as long as $q^2L << |\mathcal{X}|$

  Better than CBC!

Security of Random ctr-mode (cont'd)

- Sketch of PRF attack
- Attacker queries $x_0, x_1, \ldots$
- Attacker gets $E(K,x_0)$, $E(K,x_1)$, …

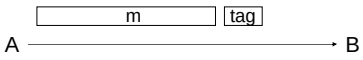


Summary of Symmetric Encryption

- Perfect secrecy and the OTP: Security against infinitely smart adversaries, but unpractical
- Stream ciphers: Fast, but keys only used once. Security treated via security of PRGs
- Block ciphers: Basic building blocks for larger encryption systems (modes of operation). Idealized into PRPs/PRFs in security proofs
- Modes of Operation: Semantic security: *The* definition of secure encryption. Captures security against all efficient adversaries. So far CT-only and CPA variants.
- Asymmetric encryption still to come
- But now forget about encryption for a while…

Message Integrity

- Goal of message integrity:



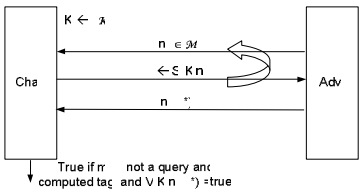
- Alice generates tag, Bob verifies tag
- Goal: Attacker cannot change message, i.e., attacker cannot generate any valid pair (m,tag)

Definition of MAC

- Definition (MAC): A message authentication code (MAC) defined over $(\mathcal{K},\mathcal{M},\mathcal{T})$ is a pair (S,V) of efficient algorithms (S,V) where
  S: $\mathcal{K}\times\mathcal{M}\rightarrow\mathcal{T}$ and V: $\mathcal{K}\times\mathcal{M}\times\mathcal{T}\rightarrow\mathcal{Bool}$
  s.t. for all $m\in\mathcal{M}$, $K\in\mathcal{K}$: $(t\leftarrow S(K,m)) \rightarrow (V(K,m,t) = true)$
- Definition (Secure MACs, intuitively):
  - Attacker's power: chosen-message attack
    Attacker outputs $m_1 \ldots m_q$, and gets $t_i \leftarrow S(K,m_i)$
  - Attacker's goal: existential forgery
    Produce valid pair (m^*,t^*), i.e., $V(K,m^*,t^*) = true$, where $(m^*,t^*) \notin \{(m_1,t_1), \ldots, (m_q,t_q)\}$
- Attacker cannot even forge MACs on non-sensitive messages

MACS

- For MAC I=(S,V), define the following MAC game:



Definition of Secure MACs

- The advantage of adversary A attacking I is $Adv^{MAC}[A,I]$ = Pr[Challenger outputs true]
- Definition (Secure MACs): A MAC I=(S,V) is a secure against existential forgery under chosen-message attacks (CMA) if for all efficient algorithms A: $Adv^{MAC}[A,I]$ is negligible.