Computer Architecture 1 - Übungsblatt 4

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Aufgabe 1

1.

- `srl R10 R21 R12`
  shifts den value stored in R21 by the value stored in R12 and stores the result in register R10.

- `seqi R5 R18 109`
  tests, if the content stored in R18 equals 109. if R18=109, 1 is stored in register R5, if not, 0 is stored in register R5.

2.

- `000000 00011 11001 01111 00000 100110`

- `001000 01001 00011 0000000001111010`

Aufgabe 2

$$
\begin{array}{rcl}
rtype(c) & = & (IR[31:26](c) = 0^6) \\
jtype(c) & = & (IR[31:28](c) = 0^4 \wedge (IR[27:26](c) = 10 \vee IR[27:26](c) = 11)) \\
itype(c) & = & /rtype(c) \wedge /jtype(c) \\
alui(c) & = & (IR[31:29](c) = 001 \wedge (IR[28:26](c) \in \{000, 001, 010, 011, 100, 101, 110, 111\})) \\
testi(c) & = & (IR[31:29](c) = 011 \wedge (IR[28:26](c) \in \{000, 001, 010, 011, 100, 101, 110, 111\})) \\
shi(c) & = & (IR[31:26](c) = 0^6 \wedge IR[5:2](c) = 0^4 \wedge IR[1:0](c) \in \{00, 10, 11\}) \\
alu(c) & = & (IR[31:26](c) = 0^6 \wedge IR[5:3](c) = 100 \\
 & & \wedge IR[2:0](c) \in \{000, 001, 010, 011, 100, 101, 110, 111\}) \\
test(c) & = & (IR[31:26](c) = 0^6 \wedge IR[5:3](c) = 101 \\
 & & \wedge IR[2:0](c) \in \{000, 001, 010, 011, 100, 101, 110, 111\}) \\
sh(c) & = & (IR[31:26](c) = 0^6 \wedge IR[5:3](c) = 0^3 \wedge IR[2:0](c) \in \{1, 00, 110, 111\}) \\
j(c) & = & (IR[31:26](c) = 0^4 10) \\
jal(c) & = & (IR[31:26](c) = 0^4 11) \\
jr(c) & = & (IR[31:26](c) = 010110) \\
jalr(c) & = & (IR[31:26](c) = 010111) \\
jump(c) & = & ((IR[31:28](c) = 0^4 \vee IR[31:28] = 0101) \\
 & & \wedge (IR[27:26](c) = 10 \vee IR[27:26](c) = 11))
\end{array}
$$

Aufgabe 3

- Lemma 1
  $e \in \{0,1\}^{30}$. e is 30-bit wide and $a[31:1]$ ist 31-bit wide so $e \neq a[31:1]$ can never be true. $\rightarrow$ nothing to prove!
  Assuming that the lemma has to be

  $$\forall i . \forall a \in \{0,1\}^{32} . \exists e \in \{0,1\}^{30} . e \neq a[31:2] \rightarrow c^i.m(a) = c^{i+1}.m(a)$$

  $e \notin CR \Longleftrightarrow a[31:2] \in CR \stackrel{(SC3)}{\Longrightarrow} /store(c^i)$.
  If we have no store in step i, the memory at address a is unchanged:

  $$c^{i+1}.m(a) = c^i.m(a)$$

  $e \in CR \Longleftrightarrow a[31:2] \notin CR \Longrightarrow$ we can have $store(c^i)$.

  $$c^{i+1}.m(a) = c^i.m(a)$$

  only holds if $a \notin \{ea(c^i), ..., ea(c^i) +_{32} d(c^i)_{32}\}$.

- Lemma 2
  If $a[31:2] \in CR$ there is no store in step i and no store in step $i+1$ because of SC 3. $(*)$

  $Ind(i)$:
  $i = 0$:

  $$c^i.m(a) = c^0.m(a)$$

  $i \rightsquigarrow i+1$:

  $$\begin{aligned} c^{i+1}.m(a) &\stackrel{(*)}{=} c^i.m(a) \\ &\stackrel{(IH)}{=} c^0.m(a) \end{aligned}$$

$\square$

Aufgabe 5

$$
\begin{aligned}
C(n) &= (n-2)\cdot C_{or} + C_{nor} \\
&= (n-2)\cdot 1 + 1 \\
&= n-1
\end{aligned}
$$

$$
\begin{aligned}
D(n) &= (\lceil \log(n) \rceil - 1)\cdot D_{or} + D_{nor} \\
&= (\lceil \log(n) \rceil - 1)\cdot 1 + 1 \\
&= \lceil \log(n) \rceil
\end{aligned}
$$

Proof:

cost

$n = 1$:

$$C(1) = 1 - 1 = 0$$

$n \rightsquigarrow n+1$:

$$
\begin{aligned}
C(n+1) &= (n-1)\cdot C_{or} + C_{nor} \\
&\overset{(IH)}{=} C(n)\cdot C_{or} + C_{nor} \\
&= C(n) + 1
\end{aligned}
$$

delay

$n = 1$:

$$D(1) = \lceil \log(1) \rceil = 0$$

$n \rightsquigarrow 2n$:

$$
\begin{aligned}
D(2n) &= (\lceil \log(2n) \rceil - 1)\cdot D_{or} + D_{nor} \\
&= (\lceil \log(n) + 1 \rceil - 1)\cdot D_{or} + D_{nor} \\
&= (\lceil \log(n) \rceil + 1 - 1)\cdot D_{or} + D_{nor} \\
&= \lceil \log(n) \rceil \cdot D_{or} + D_{nor} \\
&\overset{(IH)}{=} D(n)\cdot D_{or} + D_{nor} \\
&= D(n) + 1
\end{aligned}
$$