Computer Architecture 1 - Übungsblatt 3

Dirk Heine, 2030941
Jan Hendrik Dithmar, 2031259
Steffen Heil, 2019358

Aufgabe 1:

Kosten:

$$\begin{aligned}
C_{hdec}(1) &= 0 \\
C_{hdec}(n) &= C_{hdec}(n-1) + 2^{n-1} \cdot (C_{and} + C_{or}) \\
&= C_{hdec}(n-1) + 2^{n-1} \cdot 2 \\
&= C_{hdec}(n-1) + 2^{n} \\
&= 2^n + 2^{n-1} + ... + 2^2 + C_{hdec}(1) \\
&= 2^n + 2^{n-1} + ... + 2^2 + 0 \\
&= 2^n + 2^{n-1} + ... + 2^2 + 2^1 + 2^0 - 3 \\
&= (\sum_{i=0}^{n} 2^i) - 3 \\
&= (2^{n+1} - 1) - 3 \\
&= 2^{n+1} - 4
\end{aligned}$$

Beweis:

$$\begin{aligned}
C_{hdec}(1) &= 0 = 2^{1+1} - 4 \\
C_{hdec}(n) &= 2 \cdot 2^{n-1} + C_{hdec}(n-1) \\
&\overset{\text{IV}}{=} 2^n + (2^n - 4) \\
&= 2^{n+1} - 4
\end{aligned}$$

□

Tiefe:

$$\begin{aligned}
D_{hdec}(1) &= 0 \\
D_{hdec}(n) &= D_{hdec}(n-1) + max\,\{D_{and}, D_{or}\} \\
&= D_{hdec}(n-1) + 1 \\
&= D_{hdec}(n-2) + 1 + 1 \\
&= ... \\
&= n - 1 + D_{hdec}(1) \\
&= n - 1 + 0 \\
&= n - 1
\end{aligned}$$

Beweis:

$$\begin{aligned} D_{hdec}(1) &= 0 = 1 - 1 \\ D_{hdec}(n) &= D_{hdec}(n-1) + 1 \\ &\overset{\text{IV}}{=} n - 2 + 1 \\ &= n - 1 \end{aligned}$$

□

Aufgabe 2:

(a)

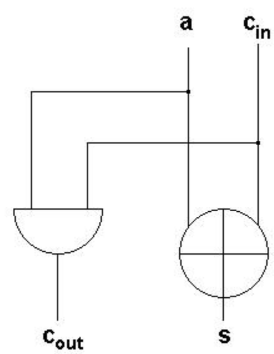

Abbildung 1: 1-bit Half-Adder

(b)

$$CCI(0) \;:=\; (c_{in} \;=\; c_{out})$$

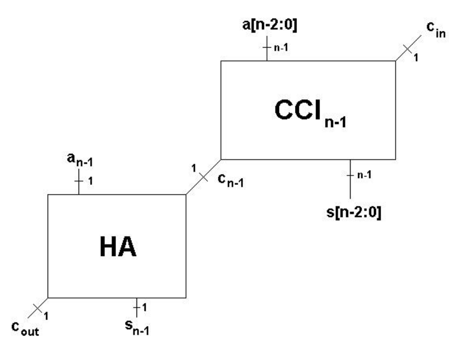

Abbildung 2: n-bit Half-Adder

(c)

Wertetabelle (*):

a	c_{in}	c_{out}	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Korrektheit des Half-Adder:

$$\langle a_0 \rangle \ + \ c_{in} \overset{(*)}{=} \langle c_{out}, s \rangle$$

Induktionsanfang: $n = 0$

$$\langle a\,[-1:0] \rangle + c_{in} = 0 + c_{in} = c_{in} \overset{\text{n.Def.}}{=} c_{out} = \langle c_{out}, s\,[-1:0] \rangle$$

Induktionsschritt: $n \hookrightarrow n + 1$

$$\begin{aligned}
\langle a[n:0] \rangle + c_{in} &= a_n \cdot 2^n + \langle a[n-1:0] \rangle + c_{in} \\
&\overset{\text{IA}}{=} a_n \cdot 2^n + \langle c_{n-1}, s[n-1:0] \rangle \\
&= a_n \cdot 2^n + c_{n-1} \cdot 2^n + \langle s[n-1:0] \rangle \\
&= (a_n + c_{n-1}) \cdot 2^n + \langle s[n-1:0] \rangle \\
&\overset{\text{HA}}{=} \langle c_n, s_n \rangle \cdot 2^n + \langle s[n-1:0] \rangle \\
&= \langle c_n, s[n:0] \rangle \\
&= \langle c_{out}, s[n:0] \rangle
\end{aligned}$$

$\square$

Aufgabe 4:

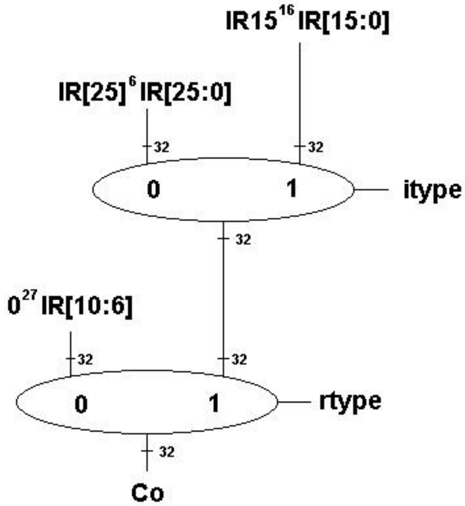

Abbildung 3: Co-Berechnung