Euler-Lagrange Formalism

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Seminar *Optimisation for Visual Computing*

Motivation

- Euler-Lagrange equations appear in

  - mechanics,

  - diffusion processes,

  - ...

- ... but also in applications, e.g. for landmark matching

Basic observations

- Standard calculus:
    - considers real-valued functions
    - if f has a minimum in ξ, then it holds $f'(\xi) = 0$.
    - if f is strictly convex and $f'(\xi) = 0$, then ξ is the unique minimum of f.

Basic observations

- Calculus of variations:

  - considers real-valued functionals

  - if E is minimised by a function u, then u has to satisfy a so-called *Euler-Lagrange equation*. This is a PDE in u.

  - if E is strictly convex and satisfies the Euler-Lagrange equation, then u is the unique minimizer of E.

Euler-Lagrange equation

- PDE = Partial Differential Equation

- Solutions are functions for which a given functional is stationary

- Good for solving optimization problems

- Analogy: Fermat's theorem in calculus
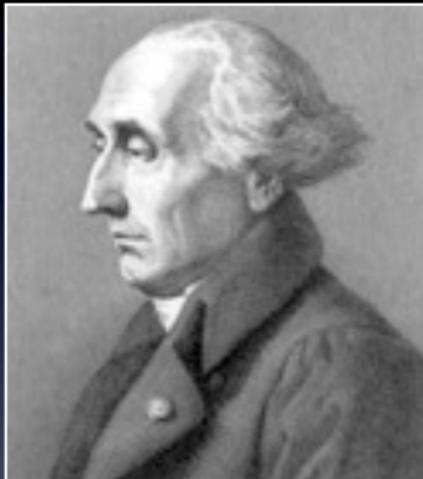
Left: Leonhard Euler. *Right:* Joseph Louis Lagrange.
Source: Wikipedia.

References

Thank you for your attention!