Lecture 11:

Medical Image Registration
Mutual Information, Elastic Regularisation, Landmarks

Contents

Medical Image Registration

What is the Medical Image Registration Problem?

Given: Two medical images (e.g. CT or MRT scans).
Wanted: Realistic deformation field that allows to compare/combine information.

Example for a Typical Medical Image Registration Task

- Hand Matching with CT images
  (*Martin-Fernández/Munoz-Moreno/Martin-Fernández/Alberola-Lopez 2003*)

(a) Left: Reference Image. **(b) Middle:** Template image. **(c) Right:** Matched template image.

Medical Image Registration (2)

Fields of Application

- Radiotherapy (e.g. tumor therapy, improved diagnosis)
- Pathology/Histology (e.g. slice registration)

Challenges in Medical Image Data

- Large displacements
- Deformations constrained to elastic motion
- Intensity differences
- Images may be created by different image acquisition methods
- Heavy noise (e.g. speckle noise in ultrasound images)
- Manually set displacements (landmarks) have to be respected

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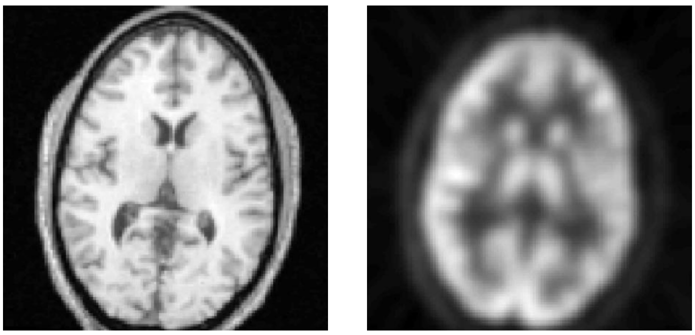
Mutual Information (1)

Mutual Information

Combining Information from Different Image Acquisition Methods

- Registration of MRT and PET Scans



Images of a human brain. **(a) Left:** MRT scan. **(b) Right:** PET scan. *Author:* W.-H. Liao (2003).

- Problem: How to register images that visualise completely different properties of the same object. Is there some common information that still allows registration?

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Mutual Information

- *Idea:* Use distance measure in the data term that specifies how independent the grey value distributions in both images are (probabilistic measure).
- *Known:* Defining $I_1 = f(x, y, t)$ and $I_2 = f(x+u, y+v, t+1)$ we usually consider data terms that comply with the following distance measure

$$d_{\mathrm{OF}}(I_1, I_2) = \Psi\Big((I_1 - I_2)^2\Big) .$$

- *New:* Mutual information is defined using **probabilistic density functions** *(Viola 1995, Viola/Wells 1997)*

$$d_{\mathrm{MI}}(I_1, I_2) = \int\limits_{[0,255]\times[0,255]} p^{[I_1,I_2]}(a,b)\ \log \frac{p^{[I_1,I_2]}(a,b)}{p^{[I_1]}(a)\ p^{[I_2]}(b)}\ da\, db .$$

  - $p^{[I_1]}(a)$, $p^{[I_2]}(b)$ are the **marginal** probability density functions of I_1 and I_2
  - $p^{[I_1,I_2]}(a,b)$ is the **joint** probability density function of I_1 and I_2

What Is Actually Measured by Mutual Information?

- *Explanation:* Mutual information measures the distance between the joint probability density function of I_1 and I_2 in the independent case given by

$$p^{[I_1,I_2]}_{\text{independent}}(a,b) = p^{[I_1]}(a)\ p^{[I_2]}(b)$$

and the actually measured joint probability density function of I_1 and I_2

$$p^{[I_1,I_2]}_{\text{measured}}(a,b) = p^{[I_1,I_2]}(a,b) .$$

Given a grey value a in I_1 and a grey value b in I_2, we can thus measure if the relation between both grey values exceeds the random probability by

$$\frac{p^{[I_1,I_2]}_{\text{measured}}(a,b)}{p^{[I_1,I_2]}_{\text{independent}}(a,b)} = \frac{p^{[I_1,I_2]}(a,b)}{p^{[I_1]}(a)\ p^{[I_2]}(b)} .$$

Here, larger ratios than 1 indicate a dependency of the grey values a and b.

Mutual Information (4)

Properties of Mutual Information

- *Positivity:* Mutual information is always nonnegative (without proof), i.e. it holds

$$d_{\mathrm{MI}}(I_1, I_2) = \int\limits_{[0,255]\times[0,255]} p^{[I_1,I_2]}(a,b)\ \log \frac{p^{[I_1,I_2]}(a,b)}{p^{[I_1]}(a)\ p^{[I_2]}(b)}\ da\, db \geq\ 0\ .$$

- *Symmetry:* Since $p^{[I_1,I_2]}(a,b) = p^{[I_2,I_1]}(a,b)$ the following holds for I_1 and I_2

$$d_{\mathrm{MI}}(I_1, I_2) = d_{\mathrm{MI}}(I_2, I_1)\ .$$

- *Minimum:* If I_1 and I_2 are independent, $p^{[I_1,I_2]}(a,b)$ can be factorised as follows

$$p^{[I_1,I_2]}(a,b) = p^{[I_1]}(a)\ p^{[I_2]}(b) \quad \rightarrow \quad \log \frac{p^{[I_1,I_2]}(a,b)}{p^{[I_1]}(a)\ p^{[I_2]}(b)} = \log\ 1 = 0$$

  Then the mutual information attains its minimum given by $d_{\mathrm{MI}}(I_1, I_2) = 0$.

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Mutual Information (5)

Using Mutual Information in Practice

- *Attention:* Since we are actually interested in maximising the mutual information (the dependency of both distributions), we have to consider the **negative version**

$$\overline{d_{\mathrm{MI}}}(I_1, I_2)\ =-\ d_{\mathrm{MI}}(I_1, I_2) = \ -\int\limits_{[0,255]\times[0,255]} p^{[I_1,I_2]}(a,b)\ \log \frac{p^{[I_1,I_2]}(a,b)}{p^{[I_1]}(a)\ p^{[I_2]}(b)}\ da\, db\ .$$

- *Computation of the Probabilistic Densities:* The probabilistic densities are computed via the **Parzen estimators**

$$\begin{aligned} p^{[I_1]}(a) \quad &= \quad K_\sigma * \frac{1}{|\Omega|} \int\limits_{\Omega} \mathbf{1}_{(I_1(x,y)=a)}\ dx\ dy\ , \\ p^{[I_1,I_2]}(a,b) \quad &= \quad K_\sigma * \frac{1}{|\Omega|} \int\limits_{\Omega} \mathbf{1}_{\big((I_1(x,y)=a)\wedge(I_2(x+u,y+v)=b)\big)}\ dx\ dy\ , \end{aligned}$$

  which can be considered as **Gaussian smoothed normalised histograms**.

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Mutual Information (6)

Mutual Information for Block Matching Techniques

- *Idea:* Use mutual information as matching cost for block matching
- *Example:* Using a 2-D search space $\mathcal{S}_d$ with displacements $u, v \in \{-d, ..., d\}$ yields the following block matching approach

$$\mathbf{u}_{i,j} = \underset{u,v \in \mathcal{S}_d}{\operatorname{argmin}} \;\; \overline{d_{\mathrm{MI}}}\Big(\mathcal{N}_m\big(f_{i,j}\big),\; \mathcal{N}_m\big(g_{i+u,j+v}\big)\Big) \,.$$

  where $\mathcal{N}_m(f, i, j)$ is a patch of size $(2m+1) \times (2m+1)$ around the pixel $f_{i,j}$

- *Attention:* The marginal and joint probability density functions are only computed for the **local patches** $\mathcal{N}_m\big(f_{i,j}\big)$ and $\mathcal{N}_m\big(g_{i+u,j+v}\big)$ (and not for the whole image).
- *Realisation:* The required cost map is derived in three steps
  - compute the normalised histograms for the separate and the joint case
  - smooth histograms with Gaussian to obtain probability density functions
  - compute cost map using the negative mutual information, i.e. $\overline{d_{\mathrm{MI}}}$

Mutual Information (7)

Mutual Information for Variational Methods

- *Idea:* Use mutual information as data term for variational methods
- *Example:* Replacing the data term of the method of Horn and Schunck with the expression for mutual information yields the functional
  (Hermosillo 2001)

$$E(\mathbf{w}) = -\int\limits_{[0,255]\times[0,255]} p^{[I_1, I_2^{\mathbf{w}}]}(a,b)\; \log \frac{p^{[I_1, I_2^{\mathbf{w}}]}(a,b)}{p^{[I_1]}(a)\; p^{[I_2^{\mathbf{w}}]}(b)} \; da\, db + \alpha \int\limits_{\Omega} \left(|\nabla u|^2 + |\nabla v|^2\right) dx\, dy$$

  with $p^{[I_1]}(a)$, $p^{[I_2^{\mathbf{w}}]}(b)$ and $p^{[I_1, I_2^{\mathbf{w}}]}(a,b)$ defined as before.

- *Remark:* In order to make explicit that we consider the motion compensated second frame $f(\mathbf{u}+\mathbf{w})$ in our data term, we replaced I_2 by $I_2^{\mathbf{w}}$.
- *Attention:* This energy functional is highly non-convex and its optimisation is non-trivial. However, the Euler-Lagrange equations can be stated explicitly.

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Mutual Information for Variational Methods

- *Minimisation:* The associated Euler-Lagrange equations are given by the PDEs

$$
\begin{aligned}
0 &= -\Big(\frac{\partial}{\partial b} L^{[I_1,I_2^{\mathbf{w}}]}\Big)\big(f(\mathbf{x}), f(\mathbf{x}+\mathbf{w})\big)\; f_x(\mathbf{x}+\mathbf{w}) - \alpha\Delta u \\
0 &= -\Big(\frac{\partial}{\partial b} L^{[I_1,I_2^{\mathbf{w}}]}\Big)\big(f(\mathbf{x}), f(\mathbf{x}+\mathbf{w})\big)\; f_y(\mathbf{x}+\mathbf{w}) - \alpha\Delta v
\end{aligned}
$$

  with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$ where the non-linear operator $L^{[I_1,I_2^{\mathbf{w}}]}$ is defined as follows

$$
L^{[I_1,I_2^{\mathbf{w}}]}(a,b) = 1 + p^{[I_1,I_2^{\mathbf{w}}]}(a,b)\log\frac{p^{[I_1,I_2^{\mathbf{w}}]}(a,b)}{p^{[I_1]}(a)\;p^{[I_2^{\mathbf{w}}]}(b)}\;.
$$

- *Attention:* Since these equations are difficult to linearise, one often performs a steepest descent/gradient descent approach. While such an approach works in most cases, it easily gets trapped in the next local minimum.

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The Steepest Desecent/Gradient Descent Approach

- *Known:* The first variation of a typical 2-D energy functional is given by

$$
\mathbf{v}(u,v) = \begin{pmatrix} v_1(u,v) \\ v_2(u,v) \end{pmatrix} = \begin{pmatrix} F_u - \frac{\partial}{\partial x}F_{u_x} - \frac{\partial}{\partial y}F_{u_y} \\ F_v - \frac{\partial}{\partial x}F_{v_x} - \frac{\partial}{\partial y}F_{v_y} \end{pmatrix}
$$

  known from the Euler-Lagrange equations, where we seek $\mathbf{v} = 0$.

- *New:* In order to minimize an anergy functional it makes sense to walk from a given solution in the direction of the steepest descent

$$
\begin{aligned}
u^{k+1} &= u^k - \gamma\, v_1(u^k, v^k) \\
v^{k+1} &= v^k - \gamma\, v_2(u^k, v^k)
\end{aligned}
$$

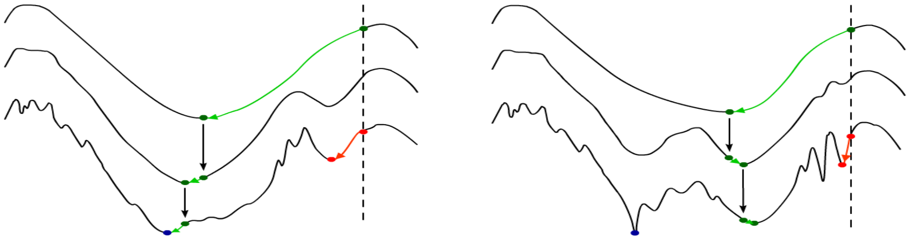
  where γ is the speed factor (step size) and $-\mathbf{v} = (-v_1, -v_2)^\top$ is the direction of the steepest descent (since $\mathbf{v}$ is the direction of the steepest ascent).

- *Remark:* The Euler–Lagrange equations represent the steady-state of the gradient descent approach, since nothing changes for $\mathbf{v} = 0$.

Mutual Information (10)

The Steepest Desecent/Gradient Descent Approach

- *Visualisation*: Since a gradient descent (red) gets trapped in the next local minima, one often uses a coarse-to-fine initialisation strategy (green)



global minimum found good local minimum found

- *Application to Our Problem:* In the case of the energy functional for mutual information the gradient descent equations are given by

$$u^{k+1} = u^k - \gamma\Big(- \Big(\frac{\partial}{\partial b} L^{\left[I_1, I_2^{\mathbf{w}^k}\right]}\Big)\big(f(\mathbf{x}), f(\mathbf{x}+\mathbf{w}^{\mathbf{k}})\big)\ f_x(\mathbf{x}+\mathbf{w}^{\mathbf{k}}) - \alpha \Delta u^k\Big) ,$$

$$v^{k+1} = v^k - \gamma\Big(- \Big(\frac{\partial}{\partial b} L^{\left[I_1, I_2^{\mathbf{w}^k}\right]}\Big)\big(f(\mathbf{x}), f(\mathbf{x}+\mathbf{w}^{\mathbf{k}})\big)\ f_y(\mathbf{x}+\mathbf{w}^{\mathbf{k}}) - \alpha \Delta v^k\Big) .$$

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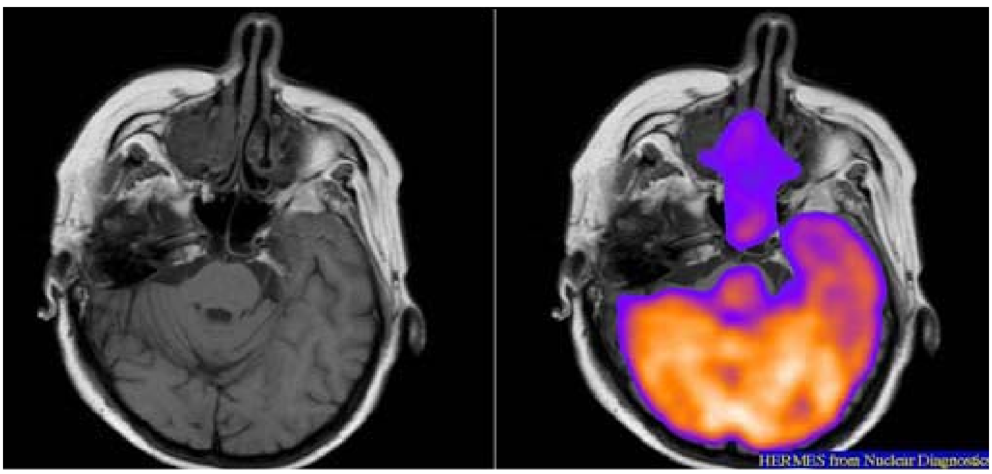
Mutual Information (11)

Results for Mutual Information

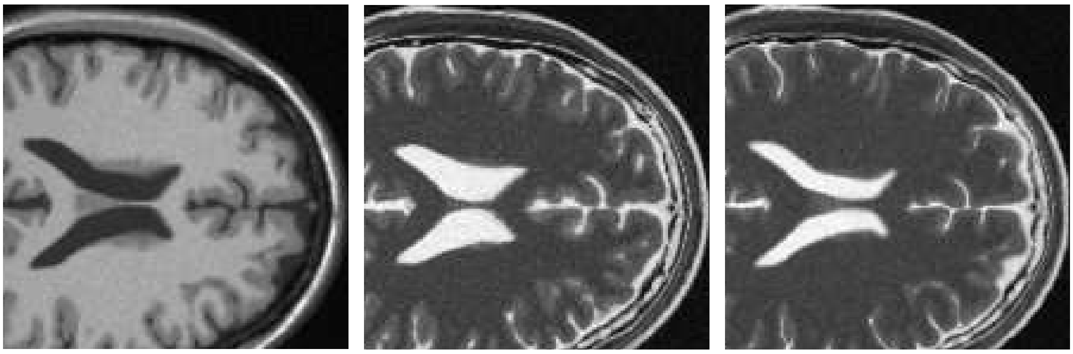
- Mapping of Magneto Resonance Tomography (MRT) and Positron Emission Tomography (PET) Images via Mutual Information
  (Willdendrup/Svarer 2004)



(a) Left: MRT image. **(b) Right:** Registered PET image blended over MRT image.

Results for Mutual Information

- Mapping of T1/T2 MRT Brain Images via Mutual Information
  (Heidmann/Mahnke/Potts/Modersitzki/Fischer 2004)



Mapping of T1/T2 brain images. **(a) Left:** T1-MRT image (reference) **(b) Center:** T2-MRT image (template).**(c) Right:** Registered T2-MRT image (matched template).

Information from different image acquisition methods can be combined!

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Normalised Gradient Fields

Can We Find A Simpler Useful Distance Measure Than Mutual Information?

- *Idea:* Use normalised gradient fields to construct data term
- *Example:* Given an image $f(\mathbf{x})$ the corresponding normalised gradient field is given by the directional vector

$$\mathbf{n}(f,\mathbf{x}) = \begin{cases} \frac{\nabla f(\mathbf{x})}{|\nabla f(\mathbf{x})|} & \text{if} \quad \nabla f(\mathbf{x}) \neq 0\,, \\ 0 & \text{else} \end{cases}$$

A suitable distance that favours small angles between the gradient field of both images is thus given by the negative squared scalar product
(Haber/Modersitzki 2006)

$$d_{\mathrm{NGF}}(I_1, I_2) = -\Big(\mathbf{n}(I_1,\mathbf{x})^\top \mathbf{n}(I_2,\mathbf{x})\Big)^2 .$$

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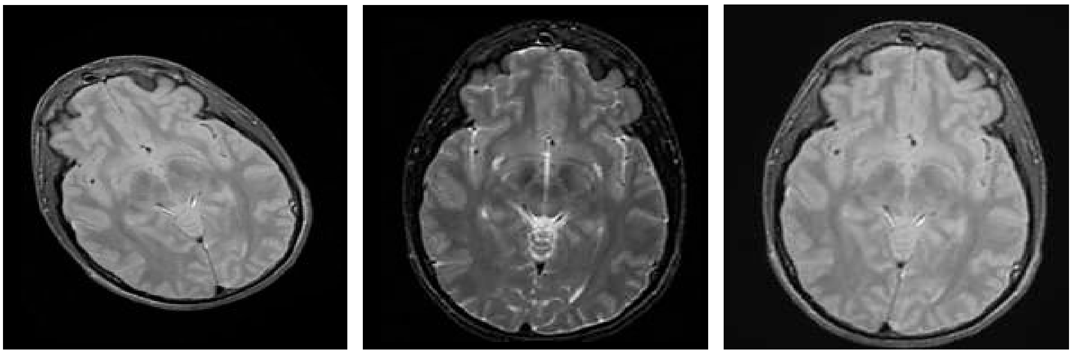
Results for Normalised Gradient Fields

- Mapping of T1/T2 MRT Brain Images via Normalised Gradient Fields
  (Haber/Modersitzki 2004)



Mapping of T1/T2 brain images. **(a) Left:** T1-MRT image (reference) **(b) Center:** T2-MRT image (template).**(c) Right:** Registered T2-MRT image (matched template).

Normalised gradient fields are a simple but good alternative

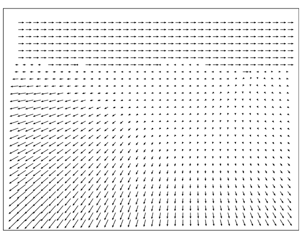
Elastic Regularisation

Can We Model the Elastic Deformations of a Tissue in a Suitable Way?

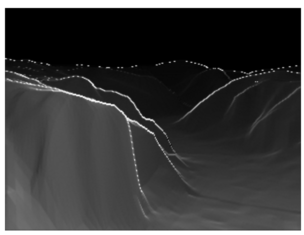
- *Idea:* Use smoothness term that can be derived from elasticity theory
- *Example:* If the tissue is elastic, no new tissue should appear or vanish during the registration. This can be encouraged via the penaliser

$$\left(\operatorname{div}\begin{pmatrix}u\\v\end{pmatrix}\right)^2 = \left(\nabla\cdot\mathbf{u}\right)^2 = (u_x+v_y)^2$$

  which models a **divergence-free flow field** (containing neither sinks nor sources).
- *Visualisation:* By the example of the Yosemite sequence we can see that large divergence expressions occur at locations where information appears or vanishes (e.g. at discontinuities).



Flow field



Magnitude of divergence

Elastic Regularisation (2)

Elastic Regularisation in Variational Approaches

- *Idea:* Assume both a smooth and elastically deformed flow field
- *Example:* The energy functional for the Horn and Schunck method with elastic deformation term is given by

$$E(u,v) = \int_\Omega \underbrace{(f_x\, u + f_y\, v + f_t)^2}_{\text{data term}} + \mu\, \underbrace{\left(|\nabla u|^2 + |\nabla u|^2\right)}_{\text{smoothness term}} + (\lambda + \mu)\, \underbrace{\left((u_x + v_y)^2\right)}_{\text{elasticity term}}\, dx\, dy\ ,$$

  where the weights μ and λ are the so-called **Lamé constants** that are related to parameters that describe the elasticity of the deformed material.

- *Minimisation:* The corresponding Euler-Lagrange equations are given by the following **linear** equations which are known as **Navier-Lamé (NLE)** equations

$$\begin{aligned} 0 &= J_{11}\, u + J_{12}\, v + J_{13} - \mu \Delta u - (\lambda+\mu)(u_{xx} + v_{xy})\ , \\ 0 &= J_{12}\, u + J_{22}\, v + J_{23} - \mu \Delta v - (\lambda+\mu)(u_{xy} + v_{yy}) \end{aligned}$$

  with associated **boundary conditions** that read

$$\mathbf{n}^\top \begin{pmatrix} (\lambda+2\mu)u_x + (\lambda+\mu)v_y \\ \mu u_y \end{pmatrix} = 0 \quad \text{and} \quad \mathbf{n}^\top \begin{pmatrix} \mu v_x \\ (\lambda+2\mu)v_y + (\lambda+\mu)u_x \end{pmatrix} = 0.$$

Elastic Regularisation (3)

Elastic Regularisation in Variational Approaches

- *Generic Form:* These Euler-Lagrange equations can be rewritten in the form

$$\begin{aligned} 0 &= J_{11}\, u + J_{12}\, v + J_{13} - \operatorname{div}\left(D_1\, \nabla u\right) - \operatorname{div}\left(D_2\, \nabla v\right)\ , \\ 0 &= J_{12}\, u + J_{22}\, v + J_{23} - \operatorname{div}\left(D_2\, \nabla u\right) - \operatorname{div}\left(D_3\, \nabla v\right)\ , \end{aligned}$$

  with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

- *New:* So far we have obtained only Euler-Lagrange equations that contain either divergence expressions for u or for v. Now expressions for **both unknowns** occur.
- *Notation:* The **anisotropic** diffusion tensors D_1, D_2 and D_3 are given by

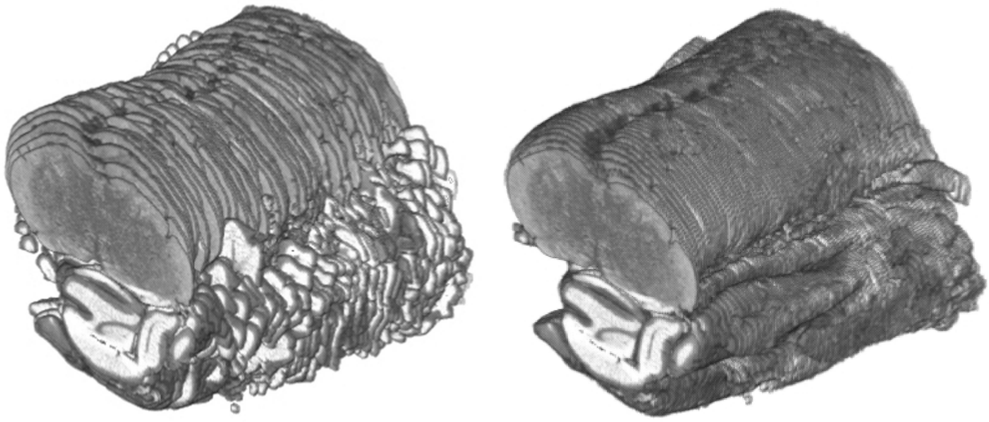
$$D_1 = \begin{pmatrix} (\lambda+2\mu) & 0 \\ 0 & \mu \end{pmatrix}\ , \quad D_2 = \begin{pmatrix} 0 & \frac{(\lambda+\mu)}{2} \\ \frac{(\lambda+\mu)}{2} & 0 \end{pmatrix}\ , \quad D_3 = \begin{pmatrix} \mu & 0 \\ 0 & (\lambda+2\mu) \end{pmatrix}\ .$$

- *Attention:* The Euler-Lagrange equations for elastic regularisation are **linear**, since the associated diffusions tensors do not depend on the unknowns u and v.

Elastic Regularisation (4)

Results for Elastic Regularisation (Variational Approach with Warping)

- Registering Histologic Slices of a Rat Brain Acquired by a Flat Bed Scanner
  (Wirtz/Fischer/Modersitzki/Schmitt 2004)



(a) Left: Original slices (unregistered). **(b) Right:** Registered slices.

Elastic Regularisation (5)

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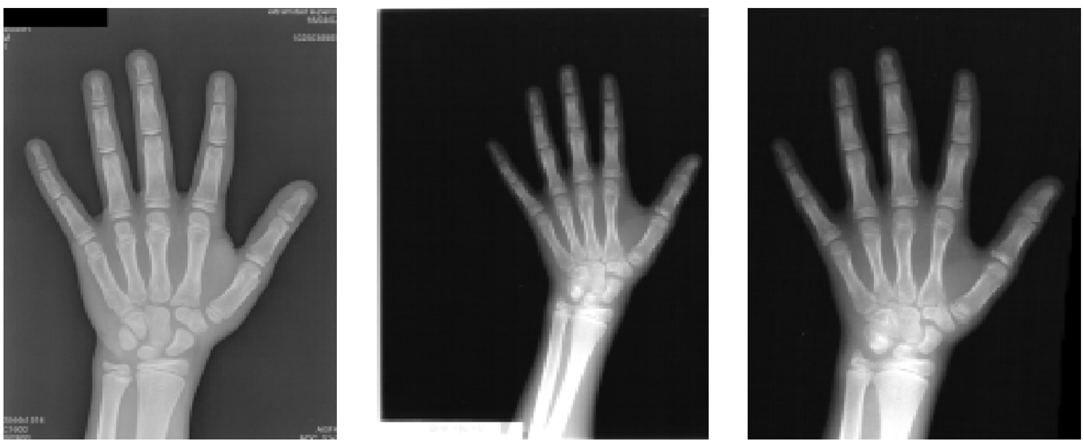
Results for Elastic Regularisation (Non-Variational Approach)

- Hand Matching with CT Images
  (*Martin-Fernández/Munoz-Moreno/Martin-Fernández/Alberola-Lopez 2003*)



(a) Left: Reference Image. **(b) Middle:** Template image. **(c) Right:** Matched template image.

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Curvature-Based Regularisation

Are There Any Alternatives for the Smoothness Term?

- *Idea:* Assume locally affine transformations (instead of locally constant ones)
- *Example:* Instead of penalising the first derivatives and thus assuming the displacement field to be locally constant, one penalises the second derivatives

$$(\Delta u)^2 + (\Delta v)^2 = u_{xx}^2 + 2u_{xx}u_{yy} + u_{yy}^2 + v_{xx}^2 + 2v_{xx}u_{yy} + v_{yy}^2 \,.$$

- *Explanation:* Affine transformations are a **subset** (for $s_1 = s_2 = 0$) of the following functions that are all mapped to zero by the curvature regulariser

$$\begin{aligned} u(x,y) &= s_1\, xy + a\, x + b\, y + c\,, \\ v(x,y) &= s_2\, xy + d\, x + e\, y + f\,. \end{aligned}$$

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Curvature-Based Regularisation in Variational Approaches

- *Example:* The energy functional for the Horn and Schunck method with curvature-based regularisation is given by

$$E(u,v) = \int_\Omega \underbrace{(f_x\, u + f_y\, v + f_t)^2}_{\text{data term}} + \alpha \underbrace{\left((\Delta u)^2 + (\Delta u)^2\right)}_{\text{smoothness term}} \; dx\, dy\,.$$

- *Minimisation:* The corresponding Euler-Lagrange equations are given by the following **linear** equations

$$\begin{aligned} 0 &= J_{11}\, u + J_{12}\, v + J_{13} + \alpha \overbrace{\left(u_{xxxx} + 2u_{xxyy} + u_{yyyy}\right)}^{\Delta^2 u}\,, \\ 0 &= J_{12}\, u + J_{22}\, v + J_{23} + \alpha \underbrace{\left(v_{xxxx} + 2v_{xxyy} + v_{yyyy}\right)}_{\Delta^2 v} \end{aligned}$$

with Neumann (reflecting) boundary conditions

$$\begin{aligned} &\mathbf{n}^\top \nabla \Delta u = 0\,, \quad \mathbf{n}^\top \begin{pmatrix} \Delta u \\ 0 \end{pmatrix} = 0\,, \quad \mathbf{n}^\top \begin{pmatrix} 0 \\ \Delta u \end{pmatrix} = 0\,, \\ &\mathbf{n}^\top \nabla \Delta v = 0\,, \quad \mathbf{n}^\top \begin{pmatrix} \Delta v \\ 0 \end{pmatrix} = 0\,, \quad \mathbf{n}^\top \begin{pmatrix} 0 \\ \Delta v \end{pmatrix} = 0\,. \end{aligned}$$

Curvature-Based Regularisation (3)

Curvature-Based Regularisation in Variational Approaches

- *New:* Since the smoothness term contains derivatives of order 2, we used an extended version of our general formulation of the Euler-Lagrange equations

$$0 \stackrel{!}{=} F_u - \frac{\partial}{\partial x} F_{u_x} - \frac{\partial}{\partial y} F_{u_y} + \frac{\partial}{\partial xx} F_{u_{xx}} + \frac{\partial}{\partial xy} F_{u_{xy}} + \frac{\partial}{\partial yx} F_{u_{yx}} + \frac{\partial}{\partial yy} F_{u_{yy}} ,$$

$$0 \stackrel{!}{=} F_v - \frac{\partial}{\partial x} F_{v_x} - \frac{\partial}{\partial y} F_{v_y} + \frac{\partial}{\partial xx} F_{v_{xx}} + \frac{\partial}{\partial xy} F_{v_{xy}} + \frac{\partial}{\partial yx} F_{v_{yx}} + \frac{\partial}{\partial yy} F_{v_{yy}} .$$

  with associated boundary conditions that read

$$\mathbf{n}^\top \begin{pmatrix} F_{u_x} - \partial_x F_{u_{xx}} - \partial_y F_{u_{xy}} \\ F_{u_y} - \partial_x F_{u_{yx}} - \partial_y F_{u_{yy}} \end{pmatrix} = 0 , \qquad \mathbf{n}^\top \begin{pmatrix} F_{u_{xx}} \\ F_{u_{xy}} \end{pmatrix} = 0 , \qquad \mathbf{n}^\top \begin{pmatrix} F_{u_{yx}} \\ F_{u_{yy}} \end{pmatrix} = 0 ,$$

$$\mathbf{n}^\top \begin{pmatrix} F_{v_x} - \partial_x F_{v_{xx}} - \partial_y F_{v_{xy}} \\ F_{v_y} - \partial_x F_{v_{yx}} - \partial_y F_{v_{yy}} \end{pmatrix} = 0 , \qquad \mathbf{n}^\top \begin{pmatrix} F_{v_{xx}} \\ F_{v_{xy}} \end{pmatrix} = 0 , \qquad \mathbf{n}^\top \begin{pmatrix} F_{v_{yx}} \\ F_{v_{yy}} \end{pmatrix} = 0 .$$

- *Attention:* The actual implementation of the boundary conditions for higher order regularisers is highly non-trivial.

Curvature-Based Regularisation (4)

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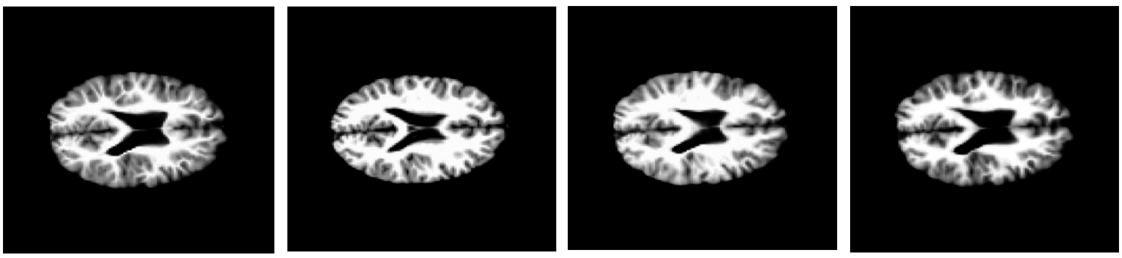
Results for Curvature-Based Regularisation (Variational Approach)

- Matching of T1 MRT Brain Images with Curvature-Based and Elastic Regulariser (*Agostino/Modersitzki/Maes/Vandermeulen/Fischer/Suetens 2003*)



(a) Outer Left: T-1 Reference image. **(b) Middle Left:** T-1 Template image. **(c) Middle Right:** Matched template image curvature-based regularisation. **(d) Outer Right:** Matched template image with elastic regularisation.

Curvature-based regularisation yields reasonable results, but is not as good elastic regularisation

Landmarks

How Can We Integrate Correspondences that are Pre-Given by the User?

- *Idea:* Set displacements at certain points fixed to pre-given values.
- *Example:* By introducing a binary confidence function $c(x, y)$ that is defined as

$$c(x,y) = \begin{cases} 1 & \text{if the displacement } u, v \text{ is known at location } (x,y) \\ 0 & \text{else} \end{cases}$$

we can modify the Euler-Lagrange equations of the method of Horn and Schunck in such a way that they become
(Fischer/Modersitzki 2003)

$$\begin{aligned} 0 &= (1-c)\left(J_{11}\,u + J_{12}\,v + J_{13} - \alpha\Delta u\right) + c\,(u - u_{\text{given}}) \\ 0 &= (1-c)\left(J_{12}\,u + J_{22}\,v + J_{23} - \alpha\Delta v\right) + c\,(v - v_{\text{given}}) \end{aligned}$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

Landmarks

- *Explanation:* For a better understanding let us consider the following two cases
- *Case 1:* If $c(x, y) = 0$, i.e. the displacement for the point is **not known** we obtain the original Euler-Lagrange equations given by

$$\begin{aligned} 0 &= J_{11}\,u + J_{12}\,v + J_{13} - \alpha\Delta u\ , \\ 0 &= J_{12}\,u + J_{22}\,v + J_{23} - \alpha\Delta v\ . \end{aligned}$$

- *Case 2:* If $c(x, y) = 1$, i.e. the displacement for the point is **known** we obtain the following equations

$$\left.\begin{aligned} 0 = u - u_{\text{given}} \\ 0 = v - v_{\text{given}} \end{aligned}\right\} \rightarrow \quad v = v_{\text{given}}\ , \quad u = u_{\text{given}}\ .$$

- *Attention:* The given information propagates via the smoothness term also to all other points of the solution (**global effect**).

Why Can't We Use a Variational Model instead of Modifying the PDEs?

- *Straightforward Idea:* Let us consider the following approach to integrate landmarks explicitly in the model

$$E(u,v) = \int_\Omega (1-c)\Big(\underbrace{(f_x\,u + f_y\,v + f_t)^2}_{\text{data term}} + \alpha\,\underbrace{(|\nabla u|^2 + |\nabla u|^2)}_{\text{smoothness term}}\Big) + c\,\underbrace{\Big((u-u_{\text{given}})^2 + (v-v_{\text{given}})^2\Big)}_{\text{landmark term}}\,dx\,dy\,.$$

The corresponding Euler-Lagrange equations are then given by

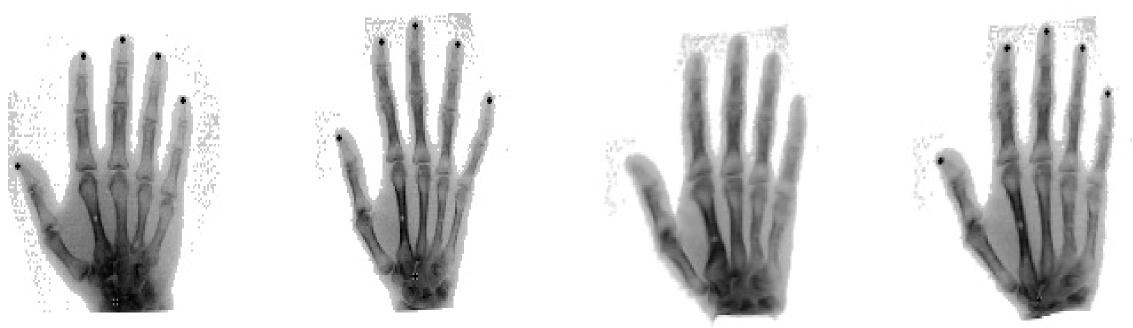
$$\begin{aligned} 0 &= (1-c)\Big(J_{11}\,u + J_{12}\,v + J_{13}\Big) - \alpha\,\text{div}\,\big((1-c)\,u\big) + c\,\big(u - u_{\text{given}}\big)\,, \\ 0 &= (1-c)\Big(J_{12}\,u + J_{22}\,v + J_{23}\Big) - \alpha\,\text{div}\,\big((1-c)\,v\big) + c\,\big(v - v_{\text{given}}\big)\,. \end{aligned}$$

- *Attention:* Unfortunately we do not obtain exactly the desired result. Since c depends on x, y it remains within the divergence expression of the smoothness term as a **binary diffusivity** (similar to image-driven methods).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Results for Landmarks (PDE-Based Approach)

- Hand Matching of CT Images with and without Landmarks (*Fischer/Modersitzki 2003*)



(a) Outer Left: Reference image. **(b) Middle Left:** Template image. **(c) Middle Right:** Matched template image without landmarks. **(d) Outer Right:** Matched template image with landmarks.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Summary

- The concept of mutual information from information theory allows the registration of images obtained by different image acquisition devices.
- A simpler alternative is given by normalised gradient fields which are much more convenient to implement.
- Elastic regularisers from elasticity theory allow to address the problem of non-rigid deformations of tissues in medical images.
- Curvature based regularisers can estimate of locally affine transformations.
- Landmarks allow interaction with user by integrating pre-given displacements. This yields semi-automatic registration approaches.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

Literature

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