Lecture 10:

Stereo Reconstruction III

Correlation and Variational Methods, Graph Cuts

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Quality Measures

How to Measure the Quality of a Disparity Map if a Ground Truth is Given?

- *Idea:* Quantify difference between estimated disparity $\mathbf{u}^{\mathrm{e}}$ and ground truth $\mathbf{u}^{\mathrm{t}}$

- Average Absolute Disparity Error (AADE)

$$AADE = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{M} \sqrt{\left(u_{i,j}^{\mathrm{t}} - u_{i,j}^{\mathrm{e}}\right)^2 + \left(v_{i,j}^{\mathrm{t}} - v_{i,j}^{\mathrm{e}}\right)^2}$$

- Percentage of Bad Pixels (BP)

$$BP = \frac{100}{N} \sum_{i=1}^{N} (\Delta \mathbf{u}_{i,j} > 1) \quad \text{with} \quad \Delta \mathbf{u}_{i,j} = \sqrt{\left(u_{i,j}^{\mathrm{t}} - u_{i,j}^{\mathrm{e}}\right)^2 + \left(v_{i,j}^{\mathrm{t}} - v_{i,j}^{\mathrm{e}}\right)^2}$$

- *Attention:* The average angular error from optic flow computation is hardly used in stereo matching. In the ortho-parallel case, angles do not make sense anyway.

Optic Flow Goes Stereo (1)

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Optic Flow Goes Stereo

Can We Exploit Our Knowledge on Variational Optic Flow Methods?

◆ *Idea:* Integrate epipolar constraint into variational optic flow methods

◆ *Strategy:* **Reparameterise** general displacement field $\mathbf{u} = (u, v)^\top$ used in optical flow estimation w.r.t to direction of epipolar line (**requires fundamental matrix**)

◆ *Step 1:* The epipolar constraint tells us that for each point $\mathbf{m}_1 = (x_1, y_1)^\top$ in the left image the corresponding epipolar line $\mathbf{l}_2 = (a, b, c)^\top$ in the right image reads

$$\mathbf{l}_2(\mathbf{m}_1) = \begin{pmatrix} a(\mathbf{m}_1) \\ b(\mathbf{m}_1) \\ c(\mathbf{m}_1) \end{pmatrix} = \begin{pmatrix} f_{11}\,x_1 + f_{12}\,y_1 + f_{13} \\ f_{21}\,x_1 + f_{22}\,y_1 + f_{23} \\ f_{31}\,x_1 + f_{32}\,y_1 + f_{33} \end{pmatrix} = \mathcal{F}\,\widetilde{\mathbf{m}}_1\ ,$$

The corresponding equation that actually describes the epipolar line is given by

$$a(\mathbf{m}_1)\,x + b(\mathbf{m}_1)\,y + c(\mathbf{m}_1) = 0\ .$$

Here, the x and y denote **general coordinates** in the second image (not a point).

Optic Flow Goes Stereo (2)

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Integration of the Epipolar Constraint

◆ *Step 2:* Decompose the displacement vector $\mathbf{u}(\mathbf{x})$ w.r.t. to the direction of the epipolar line $\mathbf{l}_2$. This yields the new parameterisation of $\mathbf{u}(\mathbf{x})$ given by

$$\mathbf{u}(p(\mathbf{x})) = p(\mathbf{x}) \underbrace{\frac{1}{\sqrt{a^2(\mathbf{x})+b^2(\mathbf{x})}} \begin{pmatrix} b(\mathbf{x}) \\ -a(\mathbf{x}) \end{pmatrix}}_{\text{epipolar tangent } \mathbf{e}(\mathbf{x})} + q(\mathbf{x}) \underbrace{\frac{1}{\sqrt{a^2(\mathbf{x})+b^2(\mathbf{x})}} \begin{pmatrix} -a(\mathbf{x}) \\ -b(\mathbf{x}) \end{pmatrix}}_{\text{epipolar normal } \mathbf{e}^\perp(\mathbf{x})}$$

- $e(\mathbf{x})$ is the direction of the epipolar line
- $e(\mathbf{x})^\perp$ is the direction orthogonal to $e(\mathbf{x})$
- $p(\mathbf{x})$ is the projection of $u(\mathbf{x})$ onto $e(\mathbf{x})$
- $q(\mathbf{x})$ is the projection of $u(\mathbf{x})$ onto $e(\mathbf{x})^\perp$
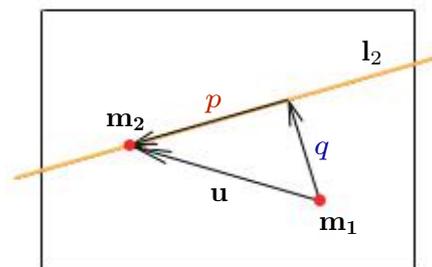
The distance from $\mathbf{m}_1$ to the epipolar line $\mathbf{l}_2$ is known: $q(\mathbf{x}) = \frac{a(\mathbf{x})x+b(\mathbf{x})y+c(\mathbf{x})}{\sqrt{a^2(\mathbf{x})+b^2(\mathbf{x})}}$.

$\rightarrow$ only the **displacement along the epipolar line**, $p(\mathbf{x})$, must be estimated.

Optic Flow Goes Stereo (3)

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Standard Reparameterisation Framework

◆ *Step 3:* As for other reparameterisation approaches, we can reformulate this new parameterisation as a **matrix vector product**. Thus we obtain

$$\mathbf{w} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{b(\mathbf{x})}{\sqrt{a^2(\mathbf{x})+b^2(\mathbf{x})}} & \frac{-a(\mathbf{x})\big(a(\mathbf{x})x+b(\mathbf{x})y+c(\mathbf{x})\big)}{a^2(\mathbf{x})+b^2(\mathbf{x})} \\ \frac{-a(\mathbf{x})}{\sqrt{a^2(\mathbf{x})+b^2(\mathbf{x})}} & \frac{-b(\mathbf{x})\big(a(\mathbf{x})x+b(\mathbf{x})y+c(\mathbf{x})\big)}{a^2(\mathbf{x})+b^2(\mathbf{x})} \\ 0 & 1 \end{pmatrix}}_{M_{\text{epi}}} \underbrace{\begin{pmatrix} p \\ 1 \end{pmatrix}}_{\mathbf{r}_{\text{epi}}}$$

This allows us to rewrite a linearised constancy assumption with 3×3 motion tensor J in terms of a new 2×2 **stereo tensor** J_{epi} that is defined as follows

$$\mathbf{w}^\top J \, \mathbf{w} = \big(M_{\text{epi}} \, \mathbf{r}_{\text{epi}}\big)^\top J \big(M_{\text{epi}} \, \mathbf{r}_{\text{epi}}\big) = \mathbf{r}_{\text{epi}}^\top \underbrace{M_{\text{epi}}^\top J \, M_{\text{epi}}}_{J_{\text{epi}}} \mathbf{r}_{\text{epi}} \; .$$

Optic Flow Goes Stereo (4)

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Optic Flow Approaches with Epipolar Constraint

◆ *Example 1:* If we embed the parameterisation that respects the epipolar line into the method of Horn and Schunck we obtain the following energy functional

$$E(\mathbf{r}_{\text{epi}}) = \int_\Omega \underbrace{\mathbf{r}_{\text{epi}}^\top J_{\text{epi}} \, \mathbf{r}_{\text{epi}}}_{\textbf{data term}} + \alpha \underbrace{(|\nabla p|^2)}_{\textbf{smoothness term}} \; dx \, dy \; .$$

◆ *Remark:* By penalising deviations from the smoothness of p and not of u and v, we assume the displacement (only) to be **smooth along the epipolar line**.

◆ *Minimisation:* The corresponding Euler-Lagrange equation that has to be fulfilled is given by the following **linear** PDE

$$J_{\text{epi}_{11}} p + J_{\text{epi}_{12}} - \alpha \Delta p = 0$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla p = 0$. The resulting linear system of equations after a discretisation can then be solved using e.g. SOR.

Optic Flow Goes Stereo (5)

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Optic Flow Approaches with Epipolar Constraint

◆ *Example 2:* Since stereo images often contain rather large displacements let us also consider a Horn and Schunck variant with warping

$$E(p) = \int_\Omega \underbrace{\left(f(x+u(p),y+v(p),t+1)-f(x,y,t)\right)^2}_{\textbf{data term}} + \alpha \underbrace{\left(|\nabla p|^2\right)}_{\textbf{smoothness term}} \; dx\,dy \, .$$

The corresponding Euler-Lagrange equation is given by the **nonlinear** PDE

$$\frac{\partial}{\partial p}u(p) \; f_x(x+u(p),y+v(p),t+1) \; \left(f(x+u(p),y+v(p),t+1) - f(x,y,t)\right)$$

$$+ \frac{\partial}{\partial p}v(p) \; f_y(x+u(p),y+v(p),t+1) \; \left(f(x+u(p),y+v(p),t+1) - f(x,y,t)\right) - \alpha\Delta p = 0$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla p = 0$.

◆ *Attention:* Since u and v depend both on p we additional inner derivatives appear compared to the case without reparameterisation.

Optic Flow Goes Stereo (6)

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Optic Flow Approaches with Epipolar Constraint

◆ *Remark:* This Euler-Lagrange equation can be further simplified. Exploiting the fact that the direction $\mathbf{e}$ of the epipolar line is given by

$$\mathbf{e} = \begin{pmatrix} \frac{\partial}{\partial p}u(p) \\ \frac{\partial}{\partial p}v(p) \end{pmatrix} = \begin{pmatrix} \frac{b}{\sqrt{a^2+b^2}} \\ \frac{-a}{\sqrt{a^2+b^2}} \end{pmatrix}$$

we obtain the following PDE

$$\left(\mathbf{e}^\top \nabla f(x+u(p),y+v(p),t+1)\right)\left(f(x+u(p),y+v(p),t+1) - f(x,y,t)\right) - \alpha\Delta p = 0 \, .$$

◆ *Minimisation:* After applying a coarse-to-fine warping strategy with linearisation, we obtain the following PDE at each warping level in motion tensor notation

$$J^k_{\text{epi}_{11}}dp^k + J^k_{\text{epi}_{12}} - \alpha\Delta dp^k - \alpha\Delta p^k \, .$$

This **linear** equation has a unique solution (related to a **convex problem!**)

Optic Flow Goes Stereo (7)

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Optic Flow Approaches with Epipolar Constraint

◆ *Remark:* If we denote by J^k the compensated 3×3 motion tensor at warping level k of an optic flow method, the corresponding 2×2 stereo tensor reads

$$J_{\mathrm{epi}}^k = {M_{\mathrm{epi}}^k}^\top \, J^k \, M_{\mathrm{epi}}^k \, .$$

Please note that the entries of M_{epi}^k **must be recomputed at each level** using a suitable coarse grid fundamental matrix (cf. theoretical assignment 8).

◆ *Example 3:* In order to obtain more accurate results one can embed the epipolar constraint into the 2-D variant of the method of Brox *et al.*. Then one obtains *(Slesareva/Bruhn/Weickert 2005)*

$$E(p) = \int_\Omega \underbrace{\Psi_{\mathrm{D}}\Big(\big(f(\mathbf{x}) - f(\mathbf{x} + \mathbf{w}(p))\big)^2 + \gamma \, \big|\nabla f(\mathbf{x}) - \nabla f(\mathbf{x} + \mathbf{w}(p))\big|^2 \Big)}_{\text{data term}} \, d\mathbf{x}$$

$$+ \, \alpha \int_\Omega \underbrace{\Psi_{\mathrm{S}}\Big(|\nabla p|^2 \Big)}_{\text{smoothness term}} \, d\mathbf{x} \, .$$

Optic Flow Goes Stereo (8)

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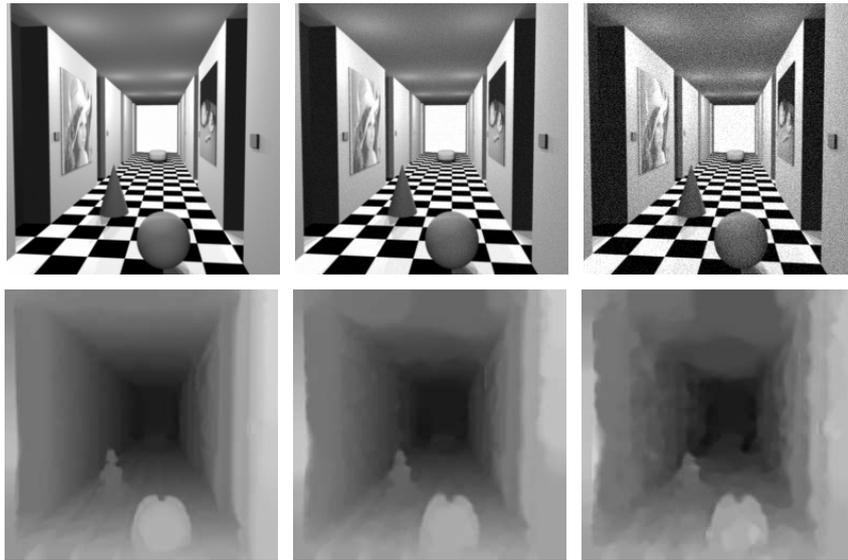29 30
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Results for the Corridor Stereo Pair (Ortho-Parallel Camera Setup)

Results for the Corridor stereo pair (University of Bonn). **(a) Upper Left:** Left frame. **(b) Upper Center:** Right Frame. **(c) Upper Right:** Ground Truth. **(d) Lower Left:** Correlation technique. **(e) Lower Center:** Variational method with Nagel/Enkelmann regularisation, warping and epipolar constraint (*Author:* L. Alvarez). **(f) Lower Right:** Method of Slesareva *et al.* (*Author:* N. Slesareva).

Optic Flow Goes Stereo (9)

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Results for the Corridor Stereo Pair under Gaussian Noise

Results for the Corridor stereo pair with Gaussian noise (University of Bonn). **Upper Row:** Left frame with noise $\sigma_\mathrm{n} = 1, \sqrt{10}, 10$. **Lower Row:** Results of the Slesareva *et al.* method for the corresponding noise levels (*Author:* N. Slesareva).

Optic Flow Goes Stereo (10)

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Results for the Corridor Stereo Pair with and without Noise

◆ Qualitative Evaluation for the Corridor Stereo Pair

- Overall performance in terms of the average absolute disparity error

Technique	AADE
Correlation method	0.4978
Alvarez *et al.*	0.2639
Slesareva *et al.*	**0.1731**

- Impact of Gaussian noise of zero mean and different standard deviation

Technique	Noise level σ_n	AADE
Slesareva *et al.*	1	0.1952
Slesareva *et al.*	$\sqrt{10}$	0.2519
Slesareva *et al.*	10	0.3297

Optic Flow Goes Stereo (11)

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Results for the Pentagon Stereo Pair (Converging Camera Setup)

◆ *Step 1:* Estimation of the fundamental matrix from Brox *et al.* correspondences with a total least squares approach (cf. Lecture 9).

◆ *Step 2:* Reestimation of the correspondences using the method of Slesareva *et al.* (→ Brox *et al.* with epipolar constraint).

Results for the Pentagon Stereo Pair (CMU Image Database).**(a) Upper Left:** Left frame. **(b) Upper Right:** Right Frame. **(c) Lower Left:** Original Brox *et al.* for estimating the fundamental matrix, then Slesereva *et al.* (Brox *et al.* with epipolar constraint) for the stereo matching (*Author:* N. Slesareva).
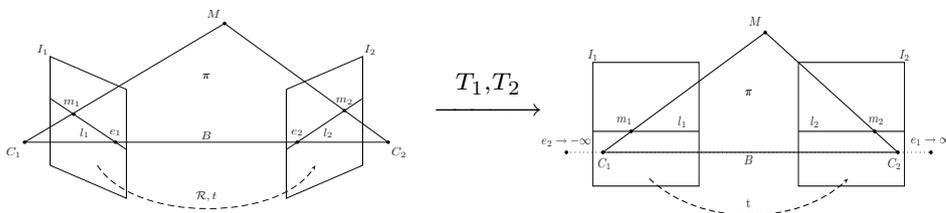
Image Rectification

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Image Rectification

Can We Simplify the Estimation if the System is Calibrated?

◆ *Idea:* Apply transformation to images from a calibrated converging camera setup such that epipolar lines become horizontal (as in an ortho-parallel camera setup)

◆ *Strategy:* Compute linear transformations T_1, T_2 for pixel positions in both views

- determine a new joint orientation $\mathcal{R}$ for the two cameras w.r.t. to the world coordinate system (orientation must be parallel to the baseline)

- compute new projection matrices for both cameras by replacing the original orientations $\mathcal{R}_1$ and $\mathcal{R}_2$ of both cameras by the new joint one $\mathcal{R}$

- determine transformations T_1, T_2 from old to new projection matrices

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Image Rectification (2)

Consequences of the Image Rectification

◆ *Horizontal Displacements:* Instead of a vector-valued displacement field only the displacement in x-direction remains to be estimated

$$\mathbf{u}(p) = \begin{pmatrix} u(p) \\ v(p) \end{pmatrix} \quad \rightarrow \quad \mathbf{u}(p) = \begin{pmatrix} p \\ 0 \end{pmatrix} .$$

◆ *Negative Displacements:* In the ortho-parallel case the displacement p along the x-direction is always negative. This yields a further restriction of the search space

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Block Matching/Correlation Techniques (1)
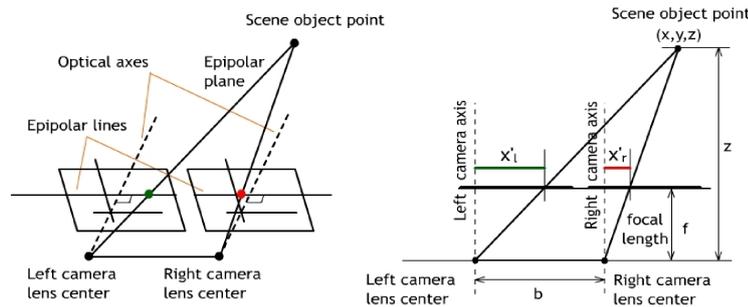
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Block Matching/Correlation Techniques

How Can We Realise Methods for the Ortho-Parallel Case?

◆ *Block Matching:* Let us restrict the search space $\mathcal{S}_d$ to negative displacements, i.e. for the displacement p holds $p \in -d, ..., 0$. This yields the formulation

$$d_{i,j} = \operatorname*{argmin}_{p \in \mathcal{S}_d} \sum_{\substack{\Delta i, \Delta j \\ \in \mathcal{N}_m}} \left(f_{i+\Delta i, j+\Delta j} - g_{(i+p)+\Delta i, v+\Delta j} \right)^2$$

where $\mathcal{N}_m$ is a neighbourhood of size m, i.e. patch of size $(2m+1) \times (2m+1)$.

◆ *Remark:* Due to the restricted search space, the computational effort for the exhaustive search decreases approximately by a factor of $4d$

$$O((2m+1)^2(2d+1)^2 NM) \quad \rightarrow \quad O((2m+1)^2(d+1)NM) .$$

Please note also that usually $d \gg m$. For usual values of $d \geq 10$ this corresponds to speedups of more than 40 compared to the unrestricted 2-D case (optic flow).

Block Matching/Correlation Techniques (2)

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Correlation Techniques

◆ *Correlation Technique:* The normalised cross correlation (NCC) with restricted search space $\mathcal{S}_d$ is given by

$$d_{i,j} = \underset{p \in \mathcal{S}_d}{\operatorname{argmax}} \frac{\underset{\substack{\Delta i,\, \Delta j \\ \in \mathcal{N}_m}}{\sum} (f_{i+\Delta i, j+\Delta j} - \bar{f}_{i,j}) \cdot (g_{(i+p)+\Delta i, j+\Delta j} - \bar{g}_{i+p,j})}{\sqrt{\underset{\substack{\Delta i,\, \Delta j \\ \in \mathcal{N}_m}}{\sum} (f_{i+\Delta i, j+\Delta j} - \bar{f}_{i,j})^2} \sqrt{\underset{\substack{\Delta i,\, \Delta j \\ \in \mathcal{N}_m}}{\sum} (g_{(i+p)+\Delta i, j+\Delta j} - \bar{g}_{i+p,j})^2}}$$

◆ *Remark:* As for the block matching method the computation for the correlation technique with restricted search space is speeded up significantly.

◆ *Observation:* Often the block matching and correlation techniques work better for stereo than for optic flow problems. This is due to the fact that the ortho-parallel stereo problem **does not require** the estimation of a **displacement direction**.

Variational Methods (1)

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Variational Methods

How Can We Design Variational Methods for the Ortho-Parallel Case?

◆ *Idea:* Use general model with the fundamental matrix for the ortho-parallel case

◆ *Example 1:* The corresponding Horn and Schunck variant for 1-D displacements in x-direction in stereo tensor notation is given by the functional

$$E(\mathbf{r}_{\text{epi}}) = \int_\Omega \underbrace{\mathbf{r}_{\text{epi}}^\top J_{\text{epi}}\, \mathbf{r}_{\text{epi}}}_{\textbf{data term}} + \alpha \underbrace{(|\nabla p|^2)}_{\textbf{smoothness term}} \; dx\, dy \, .$$

where due to the special structure of $\mathcal{F}$ (cf. theoretical assignment) the 2×2 stereo tensor J_{epi} comes down to a **direct sub-matrix** of the motion tensor J

$$J_{\text{epi}} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}^\top}_{M_{\text{epi}}^\top} \underbrace{\begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{pmatrix}^\top}_{J} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}}_{M_{\text{epi}}} = \begin{pmatrix} J_{11} & J_{13} \\ J_{13} & J_{33} \end{pmatrix} \, .$$

Variational Methods (2)

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Variational Methods

◆ *Remark:* All terms involving derivatives in y-direction (J_{12}, J_{22}, J_{23}) cancel out.

◆ *Connection 1:* This is not surprising, since we know that the y-component of the displacement field is zero. Thus we can rewrite the Horn/Schunck data term as

$$\mathbf{w}^\top J\, \mathbf{w} = \begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}^\top \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{pmatrix} \begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} p \\ 1 \end{pmatrix}^\top \begin{pmatrix} J_{11} & J_{13} \\ J_{13} & J_{33} \end{pmatrix} \begin{pmatrix} p \\ 1 \end{pmatrix}.$$

◆ *Connection 2:* The same result is obtained when linearising the following data term with quadratic penaliser that only considers displacements in x-direction

$$\Big(f(x+p, y, t+1) - f(x, y, t)\Big)^2 \quad \rightarrow \quad \Big(f_x\, p + f_t\Big)^2 = \begin{pmatrix} p \\ 1 \end{pmatrix}^\top \begin{pmatrix} J_{11} & J_{13} \\ J_{13} & J_{33} \end{pmatrix} \begin{pmatrix} p \\ 1 \end{pmatrix}.$$

Variational stereo methods for the ortho-parallel case are thus **both** special cases of optic flow methods and special instances of general variational stereo methods.

Variational Methods (3)

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Variational Methods

◆ *Minimisation:* The corresponding Euler-Lagrange equation is the same as the one for the general variational stereo method. It is given by the **linear** PDE

$$J_{\mathrm{epi}_{11}}\, p + J_{\mathrm{epi}_{12}} - \alpha \Delta p = 0$$

which can be rewritten in terms of the original motion tensor J as

$$J_{11}\, p + J_{13} - \alpha \Delta p = 0$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla p = 0$. The resulting linear system of equations after a discretisation can then be solved using e.g. SOR.

◆ *Remark:* Since we have only one variable to solve for instead of two as in the optic flow case the performance will be about twice as fast.

Variational Methods

◆ *Example 2:* Let us also consider the ortho-parallel Horn and Schunck method with warping as basis functional. Then we obtain

$$E(p) = \int_\Omega \underbrace{\left(f(x+p, y, t+1) - f(x, y, t)\right)^2}_{\textbf{data term}} + \alpha \underbrace{\left(|\nabla p|^2\right)}_{\textbf{smoothness term}} \, dx \, dy \, .$$

The corresponding Euler-Lagrange equation is given by the **nonlinear** PDE

$$f_x(x+p, y, t+1) \left(f(x+p, y, t+1) - f(x, y, t)\right) - \alpha \Delta p = 0$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla p = 0$.

◆ *Minimisation:* After applying a coarse-to-fine warping strategy with linearisation, we obtain the following PDE at each warping level in motion tensor notation

$$J_{11}^k \, dp^k + J_{13}^k - \alpha \Delta dp^k - \alpha \Delta p^k \, .$$

This **linear** equation has a unique solution (related to a **convex problem!**)

Variational Methods

◆ *Remark:* If we denote by J^k the compensated 3×3 motion tensor at warping level k of an optic flow method, the corresponding 2×2 stereo tensor reads

$$J_{\text{epi}}^k = M_{\text{epi}}^k{}^\top \, J^k \, M_{\text{epi}}^k = \begin{pmatrix} J_{11}^k & J_{13}^k \\ J_{13}^k & J_{33}^k \end{pmatrix} \, .$$

Evidently, in the ortho-prallel case the stereo tensor J_{epi}^k can be directly derived from the corresponding motion tensor J^k **without any recomputation** of M_{epi}^k.

◆ *Example 3:* Once again, one can also modify the methods of Brox *et al.* in such a way that it only computes displacements in x-direction. Then one obtains *(Slesareva/Bruhn/Weickert 2005)*

$$E(p) = \int_\Omega \underbrace{\Psi_{\text{D}}\left(\left(f(x+p, y, t+1) - f(x, y, t)\right)^2 + \gamma \, |\nabla f(x+p, y, t+1) - \nabla f(x, y, t)|^2\right)}_{\textbf{data term}} \, dx \, dy$$

$$+ \alpha \int_\Omega \underbrace{\Psi_{\text{S}}\left(|\nabla p|^2\right)}_{\textbf{smoothness term}} \, dx \, dy \, .$$

Graph Cuts (1)

M I
🏰 A
1 2
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Graph Cuts

How Can We Obtain Shaper Discontinuities that Preserve Small Details?

◆ *Idea:* Consider nonconvex penalisers in the smoothness term

◆ *Strategy:* Model stereo approach based on a **discrete energy functional** that can be minimised very well by graph cut algorithms from graph theory

- Discretise solution using n different **integer labels** l_i for $i = 1, ..., n$ (the labels represent the different disparities, e.g. $l_1 = 0, ..., l_n = -n$).

- Assign initial label l_i to each pixel, e.g. by using the NCC approach (the label l_i of a pixel p is denoted by $L_p \in (l_0, ..., l_n)$, p is a numbering)

- Minimise discrete energy functional by sequence of two different moves: **Swap moves** and **expansion moves**. These moves redistribute the labels.

- Both moves are performed by creating a **weighted graph** based on the energy functional and cutting it optimally w.r.t. a minimal cutting cost.

◆ *Remark:* For two labels the solution is **optimal**. For more labels, however, only a local minimiser can be found. Nevertheless, the local solution is often very good.

Graph Cuts (2)

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Which Energy Functionals Can Be Minimised?

◆ *Modelling:* Discrete energy functionals that can be minimised via graph cuts consist of a data and smoothness term (as in the continuous case). They read

$$E(f) = \sum_{p \in \Omega} D_p(L_p) + \sum_{\{p,q\} \in \mathcal{N}} V_{\{p,q\}}(L_p, L_q)$$

where the data term for the stereo problem is given by the well known constraint

$$D_p(L_p) = \big(f(x_p + L_p, y_p, t+1) - f(x_p, y_p, t)\big)^2$$

and the smoothness term represents a regulariser based on differences that reads

$$V_{\{p,q\}}(L_p, L_q) = \Psi_S\left(\big(L_p - L_q\big)^2\right).$$

◆ *Remark:* Remember that the solution is discrete, i.e. that values for all possible solutions can actually be computed (which of course is still too expensive)

Graph Cuts (3)

M I
A

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Which Penaliser Functions are Typically Used for The Smoothness Term?

◆ *Semi-Metric Functions*: Semi-metric functions V satisfy three properties

    • nonnegativity, i.e. $V(L_p, L_q) \geq 0$ for all labels L_p and L_q

    • identity of indiscernibles, i.e. $V(L_p, L_q) = 0$ if and only if $L_p = L_q$

    • symmetry, i.e. $V(L_p, L_q) = V(L_q, L_p)$

◆ *Metric Functions*: Metric function V satisfy one additional property

    • triangle inequality, i.e. $V(L_p, L_q) \leq V(L_p, L_a) + V(L_a, L_q)$

◆ *Remark:* Convex functions such as $(L_p - L_q)^2$ are only semi-metric functions!
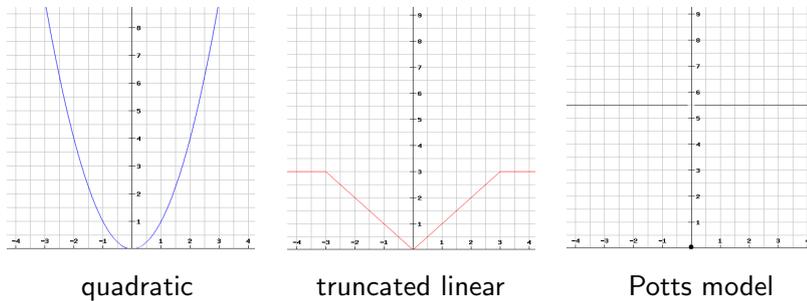
quadratic   truncated linear   Potts model

Graph Cuts (4)

M I
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The Swap Move

◆ *Swap Move:* Labels $l_i = \alpha$ and $l_j = \beta$ are **redistributed optimally** among pixels that are already labelled l_i or l_j. Pixels with other labels keep their labels.

◆ *Prerequisite:*

$V(L_p, L_q)$ is a semi-metric

◆ *Old Pixel Sets:*

P_α, P_β: pixels with label α, β

$P_{\alpha\beta}$: pixels with label α or β

The swap move. *Authors:* Boykov/Veksler/Zabih.

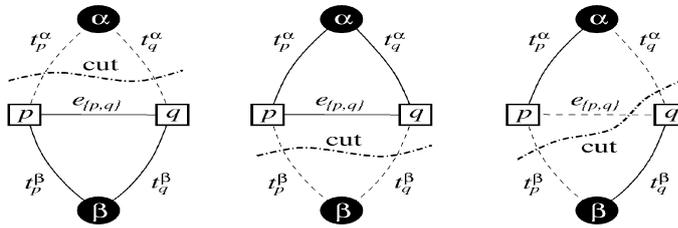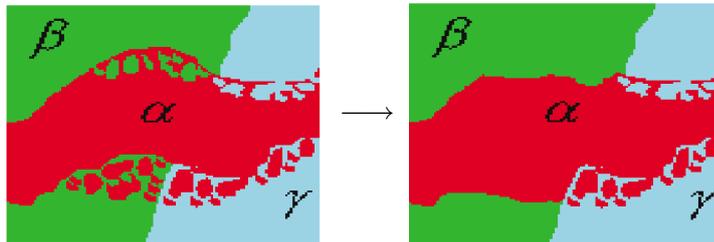| edge	weight	for
t_p^α	$D_p(\alpha) + \sum_{\substack{q \in \mathcal{N}_p \\ q \notin \mathcal{P}_{\alpha\beta}}} V(\alpha, f_q)$	$p \in \mathcal{P}_{\alpha\beta}$
t_p^β	$D_p(\beta) + \sum_{\substack{q \in \mathcal{N}_p \\ q \notin \mathcal{P}_{\alpha\beta}}} V(\beta, f_q)$	$p \in \mathcal{P}_{\alpha\beta}$
$e_{\{p,q\}}$	$V(\alpha, \beta)$	$\{p,q\} \in \mathcal{N}$ $p,q \in \mathcal{P}_{\alpha\beta}$

The Swap Move

◆ *Main Cases:* There are three main cases for cuts that appear during a swap move

Main cases of the swap move. *Authors:* Boykov/Veksler/Zabih.

◆ *Example:* The following example shows the effect of a swap move for three labels

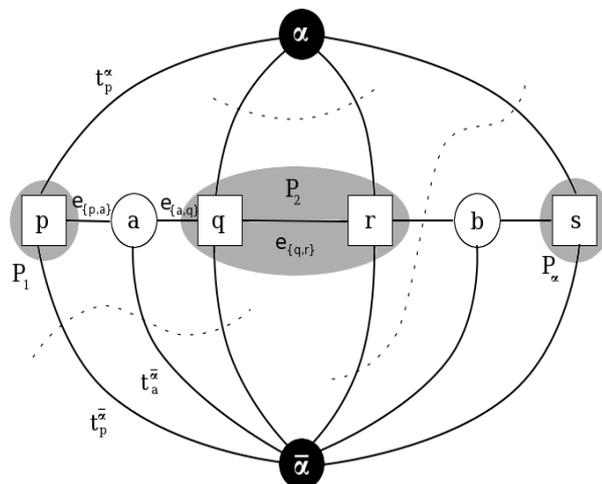$\alpha\beta$-swap move for the labels α, β, γ. *Authors:* Boykov/Veksler/Zabih.

The Expansion Move

◆ *Expansion Move:* The set of pixels with label $l_i = \alpha$ is **expanded optimally** to other pixels that had previously a different label, i.e. a label with $l_i \neq \alpha$.

◆ *Prerequisite:*

$V(L_p, L_q)$ is a metric

edge	weight	for
$t_p^{\bar{\alpha}}$	∞	$p \in \mathcal{P}_\alpha$
$t_p^{\bar{\alpha}}$	$D_p(f_p)$	$p \notin \mathcal{P}_\alpha$
t_p^{α}	$D_p(\alpha)$	$p \in \mathcal{P}$
$e_{\{p,a\}}$	$V(f_p, \alpha)$	
$e_{\{a,q\}}$	$V(\alpha, f_q)$	$\{p,q\} \in \mathcal{N},\ f_p \neq f_q$
$t_a^{\bar{\alpha}}$	$V(f_p, f_q)$	
$e_{\{p,q\}}$	$V(f_p, \alpha)$	$\{p,q\} \in \mathcal{N},\ f_p = f_q$

The expansion move. *Authors:* Boykov/Veksler/Zabih.

Graph Cuts (7)

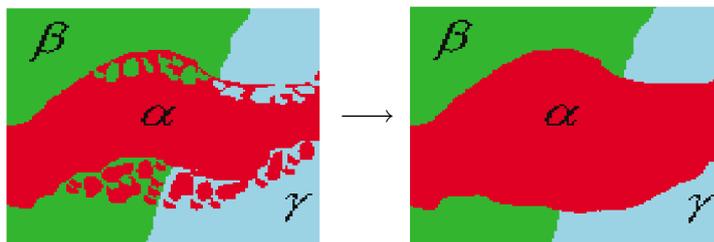| M	I
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The Expansion Move

◆ *Main Cases:* There are three main cases for cuts during an expansion move
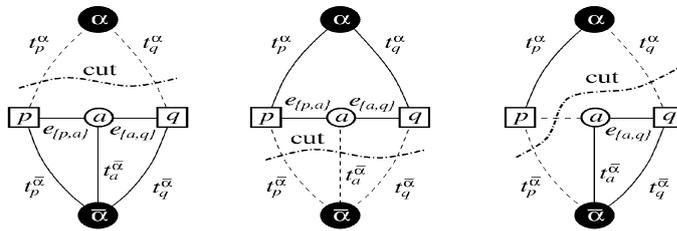
Main cases of the expansion move. *Authors:* Boykov/Veksler/Zabih.

◆ *Example:* The following example shows the effect of an expansion move

α-expansion move for the labels α, β, γ. *Authors:* Boykov/Veksler/Zabih.

Graph Cuts (8)

| M	I
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Cycles and Iterations

◆ *Cycles:* Since both moves can only be applied to one or two sets of labels, cycles are performed to improve the performance

- Swap moves are applied to all pairwise combinations of labels
- Expansion moves are applied to all labels

◆ *Attention:* Only moves are accepted that decrease the energy

◆ *Iterations:* Cycles are iterated as long as one move in a cycle decreases the energy

The actual graph cut required in each move exceeds the contents of COPCV. More information can be obtained from any book on graph theory.

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Results (1)

Results for the Tsukuba Sequence (Ortho-Parallel Camera Setup)
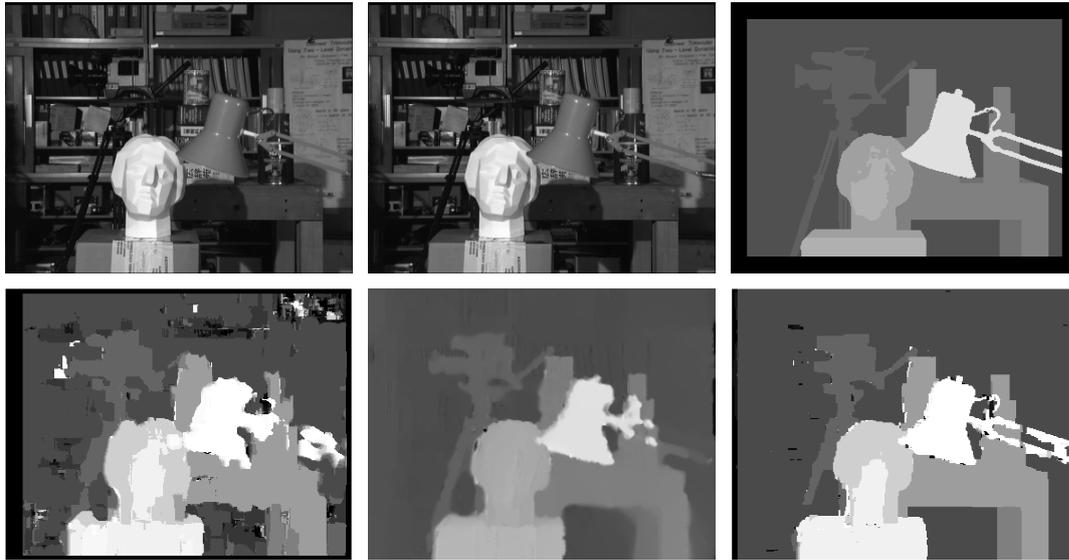
Results for the Tsukuba stereo pair (Middlebury stereo evaluation). **(a) Upper Left:** Left frame. **(b) Upper Center:** Right Frame. **(c) Upper Right:** Ground Truth. **(d) Lower Left:** Correlation technique. **(e) Lower Center:** Method of Slesareva *et al.* (*Author:* N. Slesareva). **(f) Lower Right:** Graph Cuts. (*Authors:* Boykov/Veksler/Zabih).

Results (2)

Comparison in Terms of the Bad Pixels Measure (BP)

◆ Qualitative Evaluation for the Tsukuba Stereo Pair

Technique	BP
Normalised Cross Correlation (NCC)	9.76%
Variational Methods	3.78%
Graph Cuts (GC)	1.86%

Graph Cuts have a very high spatial resolution (sharp discontinuities). However, their solution is discrete and thus produces staircasing effects.

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Results (3)

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Results for the Corridor Stereo Pair (Ortho-Parallel Camera Setup)
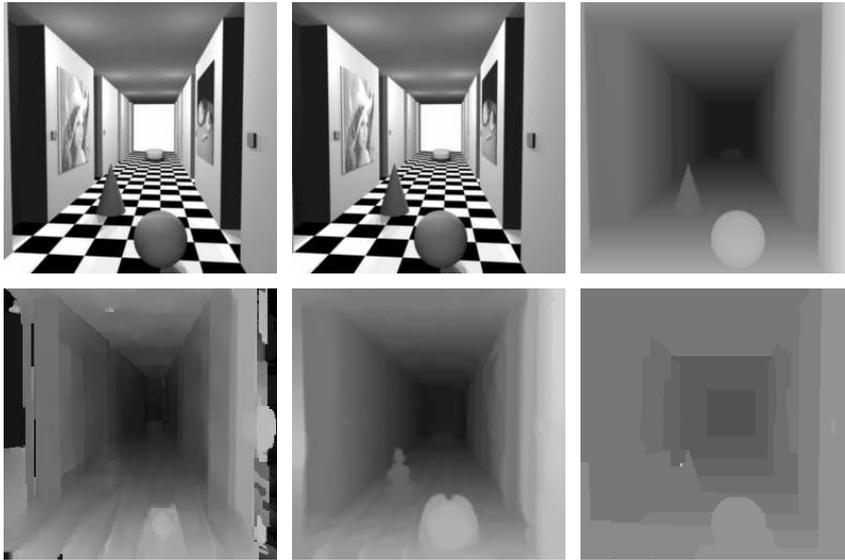
Results for the Corridor stereo pair (University of Bonn). **(a) Upper Left:** Left frame. **(b) Upper Center:** Right Frame. **(c) Upper Right:** Ground Truth. **(d) Lower Left:** Correlation technique. **(e) Lower Center:** Method of Slesareva *et al.* (*Author:* N. Slesareva). **(f) Lower Right:** Graph Cuts. (*Author:* N. Slesareva).

Summary (1)

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Summary

◆ The epipolar constraint can be integrated by means of a reparameterisation based on a directional decomposition into any variational optic flow technique

◆ Alternatively, if the camera system is calibrated, one can perform a rectification and use simpler algorithms that just focus on a line search in x-direction.

◆ These methods include

- Block matching/correlation techniques that are very fast due to the reduced search space (can be efficiently implemented in hardware)

- Variational approaches that explicitly consider the search space reduction in the model (can be seen as special instances of the general stereo model)

- Discrete variational approaches that roughly discretise the solution space but allow for a good optimisation even in the non-convex case using graph cuts

Summary (2)

Literature

◆ L. Alvarez, R. Deriche, J. Sánchez, J. Weickert:
Dense disparity map estimation respecting image derivatives: a PDE and scale-space based approach.
In *Journal of Visual Communication and Image Represenation*, Vol. 13, No. 1–2, pp 3–21, 2002.
(integration of the epipolar constraint in to variational methods)

◆ Y. Boykov, O. Veksler, R. Zabih:
Fast approximate energy minimization via graph cuts.
In *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 23, No. 11, pp 1222–1239, 2001.
(introduction to graph cuts for discrete energy minimisation)

◆ O. Faugeras, Q.-T. Luong:
The Geometry of Multiple Images.
MIT Press, 2001.
(book on projective geometry and stereo reconstruction)

◆ A. Fusiello, E. Trucco, A. Verri:
A compact algorithm for rectification of stereo pairs.
In *Machine Vision and Applications*, Vol. 12, pp. 16–22, 2000.
(simple algorithm for image rectification)

◆ H. Hirschmüller, P.R. Innocent, J. Garibaldi:
Real-time correlation-based stereo vision with reduced border errors.
In *International Journal of Computer Vision*, Vol. 47, No. 1–3, pp. 229–246, 2002.
(fast implementation of a correlation based stereo technique)

Summary (3)

Literature

◆ D. Scharstein, R. Szeliski:
A taxonomy and evaluation of dense two-frame stereo correspondence algorithms.
In *International Journal of Computer Vision*, Vol. 47, No. 1–3, pp. 7–42, 2002.
(overview on two-frame stereo techniques for an ortho-parallel camera setting)

◆ N. Slesareva, A. Bruhn, J. Weickert:
Optic flow goes stereo: a variational approach for estimating discontinuity-preserving dense disparity maps.
In *Pattern Recognition*, Lecture Notes in Computer Science, Vol. 3663, pp. 33–40, Springer, Berlin, 2005.
(article on variational stereo reconstruction, Brox et al. with epipolar constraint)

Assignment 9

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Assignment 9

Programming Exercise 1 (Fundamental Matrix Estimation)

You can download the file copcv09_ex09.tgz from the web page

    http://www.mia.uni-saarland.de/Teaching/copcv09.shtml

To unpack these data, use `tar xzvf copcv09_ex09.tgz`.

1. Supplement the routine `compute_fundamental_matrix_TLS` in the library `fundmatrix.c` with missing code so that it implements a total least squares for estimating the fundamental matrix. In order to compile your programme please use the contained makefile. The compiled programme is then executed by

    `./frontend <input_image1.pgm> <input_image2.pgm> <zoom_ratio> [ground_truth.F]`

   where the integer parameter `zoom_ratio` is in general set to 1. The use of a ground truth file `ground_truth.F` is optional and triggers the computation of the average angular error (AAE).

2. Use the provided image pairs `tsu1.pgm` and `tsu2.pgm` (ortho-parallel camera setup) and `jav1.pgm` and `jav2.pgm` (converging camera setup) to estimate the corresponding fundamental matrices. This can be done by pressing $F7$ after you computed a flow field. The created file containing the estimated fundamental matrix is denoted by `matrix_e.fm`.

Assignment 9

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Programming Exercise 2 (Visualisation)

3. In order to visualise the quality of the estimated fundamental matrices and in order to compare them to the correct matrices you can use the programme `view_epi` that can be compiled via the provided makefile. The compiled programme is then executed by

    `./view_epi <input_image1.pgm> <input_image2.pgm> <matrix_truth> <matrix_est> <zoom_ratio>`

   The ground truth matrices for the Tsukuba and the Javier image pair are given by the files `tsu_t.fm` and `jav_t.fm`, respectively. Using the mouse or the cursor you can select any point in the left or right image and draw the corresponding epipolar line in the other one. This way you can check the quality of the estimated matrix by selecting characteristic points in both frames.