Lecture 6:
Optic Flow IV
Advanced Data and Smoothness Terms

Contents

The Combined-Local-Global Approach

How Can We Improve the Robustness under Noise?

- *Idea:* Consider fixed local neighbourhood instead of single pixels in the data term
- *Example:* Integrate Lucas/Kanade method as data term into Horn/Schunck
  (Bruhn/Weickert/Schnörr 2002)

$$E(\mathbf{w}) = \int_\Omega \underbrace{\mathbf{w}^\top J_\rho \, \mathbf{w}}_{\text{data term}} + \alpha \underbrace{\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}} \, dx \, dy$$

  where $J_\rho = K_\rho * J$ is the structure tensor (convolved motion tensor).

- *New:* **Combined-local-global** (CLG) approach as two-in-one-method
  - for $\rho \to 0$ one obtains global approach (Horn and Schunck)
  - for $\alpha \to 0$ one obtains local method (Lucas and Kanade)

The Combined-Local-Global Approach (2)

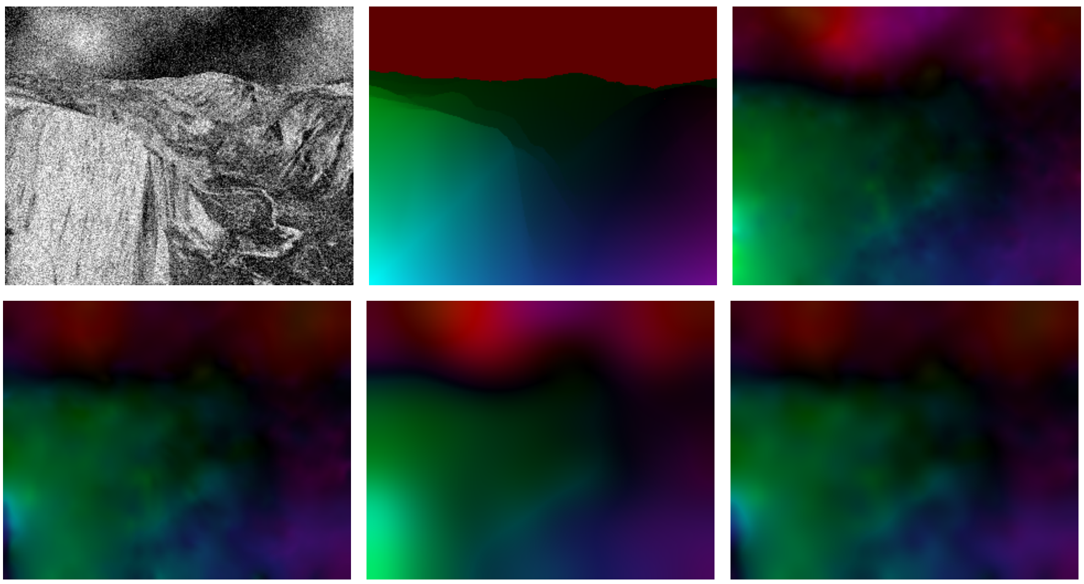
Advantages and Shortcomings of the Combined-Local-Global Approach

- Advantages
  - combines advantages of local and global methods
  - robust under noise as local approaches (neighbourhood information)
  - dense flow fields as global techniques (filling-in-effect)
  - can be written in motion tensor notation
  - can be combined with all kind of constancy assumptions (cf. last lecture)
  - computationally not more expensive than original approach
- Drawbacks
  - additional parameter (integration scale ρ)
  - relatively small impact (only small improvements)

The Combined-Local-Global Approach (3)

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for the CLG Horn and Schunck Method under Noise $\sigma_{\mathrm{n}} = 40$



Results for the Yosemite Sequence with clouds (L. Quam). **(a) Upper Left:** Frame 8 with Gaussian noise $\sigma_{\mathrm{n}} = 40$. **(b) Upper Center:** Ground truth. **(c) Upper Right:** Grey value ($\sigma = 2.4$, $\alpha = 4100$). **(d) Lower Left:** Gradient ($\sigma = 4.2$, $\alpha = 55$). **(e) Lower Center:** Grey value + CLG ($\sigma = 2.4$, $\rho = 17.6$, $\alpha = 2000$). **(f) Lower Right:** Gradient + CLG ($\sigma = 4.2$, $\rho = 15.4$, $\alpha = 26$).

Results for the CLG Horn and Schunck Method under Noise $\sigma_{\mathrm{n}} = 40$

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Horn and Schunck	(gradient constancy)	18.00°
Horn and Schunck + CLG	**(gradient constancy)**	**17.42°**
Horn and Schunck	(grey value constancy)	16.80°
Horn and Schunck + CLG	**(grey value constancy)**	**15.82°**

- Constancy assumptions on image derivatives are more sensitive to noise (without noise the gradient constancy outperforms the grey value constancy).
- The concept of local integration gives better results in all cases.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Robust Data Terms

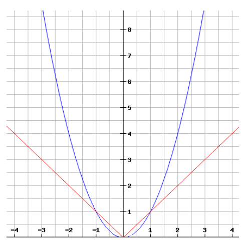
How Can We Improve the Robustness with Respect to Outliers?

- *Idea:* Reduce influence of outliers by using non-quadratic data terms
- *Example:* Replace quadratic data term by linear one (cf. SAD block matching)
  (Black/Anandan 1991, Bruhn/Weickert/Schnörr 2005)

$$E(\mathbf{w}) = \int_{\Omega} \underbrace{\Psi\left(\mathbf{w}^{\top} J\, \mathbf{w}\right)}_{\text{data term}} + \alpha \underbrace{\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}} \, dx\, dy$$

where $\Psi(s^2) = \sqrt{s^2 + \epsilon^2}$ is the regularised L_1 norm with small $\epsilon > 0$.

- *Properties:* In general $\Psi(s^2)$ should be **positive**, **increasing**, **sub-quadratic** and **strictly convex** to allow for a unique solution in the case of data terms with linearised constancy assumptions.



Robust Data Terms (2)

How Can We Extend this Idea to Multiple Constancy Assumptions?

- *Idea:* Use correlation between constancy assumptions as criterion
- *Example 1:* **Joint Robustification** if assumptions are correlated (e.g. RGB)
  (Brox/Bruhn/Papenberg/Weickert 2004)

$$E(\mathbf{w}) = \int_\Omega \underbrace{\Psi\Big(\sum_{i=1}^{n} \lambda_i\, \mathbf{w}^\top J_i\, \mathbf{w}\Big)}_{\text{data term}} + \alpha \underbrace{\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}} \, dx\, dy$$

- *Example 2:* **Separate Robustification** if assumptions can be fulfilled independently, such as e.g. the grey value and the gradient constancy
  (Bruhn/Weickert 2005)

$$E(\mathbf{w}) = \int_\Omega \underbrace{\sum_{i=1}^{n} \lambda_i\, \Psi\left(\mathbf{w}^\top J_i\, \mathbf{w}\right)}_{\text{data term}} + \alpha \underbrace{\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}} \, dx\, dy$$

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Robust Data Terms (3)

Minimisation with Nonquadratic Data Term

- *Example:* Let us consider the Horn and Schunck method with robust data term

$$E(\mathbf{w}) = \int_\Omega \Psi\left(\mathbf{w}^\top J\, \mathbf{w}\right) + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx\, dy$$

where $\Psi(s^2) = \sqrt{s^2 + \epsilon^2}$. The corresponding Euler-Lagrange equations read

$$\begin{aligned} 0 &= \Psi'\left(\mathbf{w}^\top J\, \mathbf{w}\right)\left(J_{11}u + J_{12}v + J_{13}\right) - \alpha\Delta u\, , \\ 0 &= \Psi'\left(\mathbf{w}^\top J\, \mathbf{w}\right)\left(J_{12}u + J_{22}v + J_{23}\right) - \alpha\Delta v\, . \end{aligned}$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

- *New:* Due to the factor $\Psi'\left(\mathbf{w}^\top J\, \mathbf{w}\right)$ with $\Psi'(s^2) = \frac{\partial \Psi(s^2)}{\partial s^2} = \frac{1}{2\sqrt{s^2+\epsilon^2}}$ these equations are **nonlinear** in u and v. Unreliable locations are weighted down.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Robust Data Terms (4)

Minimisation with Nonquadratic Data Term

- *Discretisation:* One obtains the following **nonlinear system of equations**

$$0 = [\Psi']_{i,j} \left([J_{11}]_{i,j}\, u_{i,j} + [J_{12}]_{i,j}\, v_{i,j} + [J_{13}]_{i,j}\right) - \alpha \sum_{l \in x,y} \sum_{\mathcal{N}_l(i,j)} \frac{u_{\tilde{i},\tilde{j}} - u_{i,j}}{h_l^2}$$

$$0 = [\Psi']_{i,j} \left([J_{12}]_{i,j}\, u_{i,j} + [J_{22}]_{i,j}\, v_{i,j} + [J_{23}]_{i,j}\right) - \alpha \sum_{l \in x,y} \sum_{\mathcal{N}_l(i,j)} \frac{v_{\tilde{i},\tilde{j}} - v_{i,j}}{h_l^2}$$

for $i = 1, ..., N$ and $j = 1, ..., M$. Thereby the nonlinear factor reads

$$[\Psi']_{i,j} = \Psi'\left(w_{i,j}^\top\, J_{i,j}\, w_{i,j}\right) = \Psi'\left(\begin{pmatrix} u_{i,j} \\ v_{i,j} \\ 1 \end{pmatrix}^\top \begin{pmatrix} [J_{11}]_{i,j} [J_{12}]_{i,j} [J_{13}]_{i,j} \\ [J_{12}]_{i,j} [J_{22}]_{i,j} [J_{23}]_{i,j} \\ [J_{13}]_{i,j} [J_{23}]_{i,j} [J_{33}]_{i,j} \end{pmatrix} \begin{pmatrix} u_{i,j} \\ v_{i,j} \\ 1 \end{pmatrix}\right) .$$

- *Problem:* So far we have only dealt with linear system of equations. How can we solve such a nonlinear system of equations?

Robust Data Terms (5)

Solving the Nonlinear System

- *Idea:* Solve nonlinear system as series of linear systems
- *Example:* Fixed point iteration, where the nonlinear expressions (here $[\Psi']_{i,j}$) are evaluated at the **old time step**. This requires to solve the linear equations system

$$0 = [\Psi']^k{}_{i,j} \left([J_{11}]_{i,j}\, u_{i,j}^{k+1} + [J_{12}]_{i,j}\, v_{i,j}^{k+1} + [J_{13}]_{i,j}\right) - \alpha \sum_{l \in x,y} \sum_{\mathcal{N}_l(i,j)} \frac{u_{\tilde{i},\tilde{j}}^{k+1} - u_{i,j}^{k+1}}{h_l^2}$$

$$0 = [\Psi']^k{}_{i,j} \left([J_{12}]_{i,j}\, u_{i,j}^{k+1} + [J_{22}]_{i,j}\, v_{i,j}^{k+1} + [J_{23}]_{i,j}\right) - \alpha \sum_{l \in x,y} \sum_{\mathcal{N}_l(i,j)} \frac{v_{\tilde{i},\tilde{j}}^{k+1} - v_{i,j}^{k+1}}{h_l^2}$$

for $i = 1, ..., N$ and $j = 1, ..., M$ at each fixed point iteration.

- *Properties:* This so-called **lagged nonlinearity method**
  - yields for non-constant images linear systems that are positive definite
  - requires only a very inexact solution for each linear system to converge

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Robust Data Terms (6)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

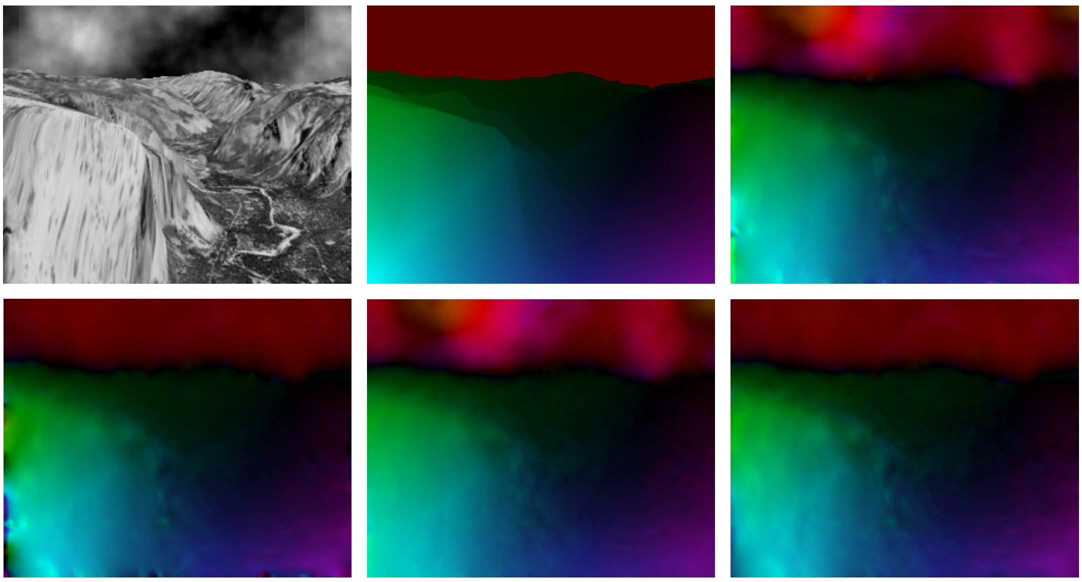
Advantages and Shortcomings of Robust Data Terms

- Advantages
  - improve the results with respect to outliers and noise
  - can be combined with all constancy assumptions
  - can be combined with the CLG approach
  - can be solved as series of linear systems (small implementation effort)
- Drawbacks
  - computationally expensive (require solving nonlinear equation system)
  - unlike block matching restricted to strictly convex functions (convergence)
  - small structures may disappear (considered as outliers)

Robust Data Terms (7)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for the Robust Horn and Schunck Method

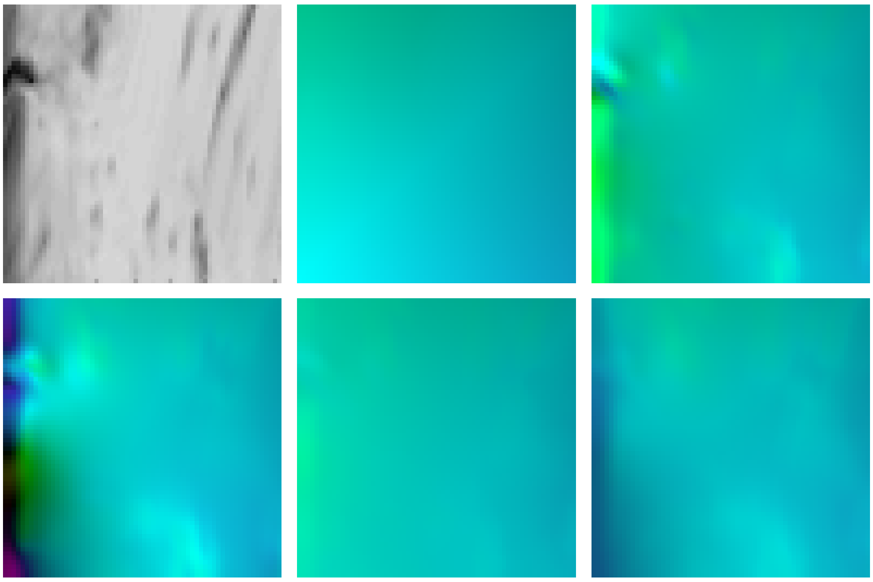


Results for the Yosemite Sequence with clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Ground truth. **(c) Upper Right:** Grey value constancy ($\sigma = 1.4$, $\alpha = 470$) **(d) Lower Left:** Gradient magnitude constancy ($\sigma = 1.9$, $\alpha = 14$). **(e) Lower Center:** Robust grey value constancy ($\sigma = 1.4$, $\alpha = 190$). **(f) Lower Right:** Robust gradient constancy ($\sigma = 2.1$, $\alpha = 400$).

Robust Data Terms (8)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for the Robust Horn and Schunck Method (Zoom Lower Left Corner)



Results for the Yosemite Sequence with clouds (L. Quam) – Zoom. **(a) Upper Left:** Frame 8. **(b) Upper Center:** Ground truth. **(c) Upper Right:** Grey value constancy ($\sigma = 1.4$, $\alpha = 470$) **(d) Lower Left:** Gradient magnitude constancy ($\sigma = 1.9$, $\alpha = 14$). **(e) Lower Center:** Robust grey value constancy ($\sigma = 1.4$, $\alpha = 190$). **(f) Lower Right:** Robust gradient constancy ($\sigma = 2.1$, $\alpha = 400$).

Robust Data Terms (9)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for the Robust Horn and Schunck Method

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Horn and Schunck	(gradient constancy)	7.12°
Robust Horn and Schunck	**(gradient constancy)**	**7.07°**
Horn and Schunck	(grey value constancy)	5.91°
Robust Horn and Schunck	**(grey value constancy)**	**5.21°**

- Robust data terms in particular useful for higher oder derivatives (enhanced sensitivity of such assumptions w.r.t to outliers and noise).
- Better results due to a better handling of boundaries (large amount of pixels that enter or leave the scene $\rightarrow$ outliers).

Image-Driven Smoothness Terms

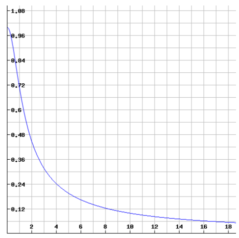
How Can We Preserve Discontinuities in the Motion Field?

- *Idea 1:* **Image-driven Isotropic** $\to$ Reduce smoothness totally at image edges
- *Example:* Using the weighting function $g(s^2) = \frac{1}{2\sqrt{s^2+\epsilon^2}}$ with $\epsilon > 0$ one obtains *(Alvarez/Esclarín/Lefébure and Sánchez 1999)*

$$E(\mathbf{w}) = \int_\Omega \underbrace{\mathbf{w}^\top J\,\mathbf{w}}_{\textbf{data term}} + \underbrace{\alpha\, g(|\nabla f|^2)\,(|\nabla u|^2 + |\nabla v|^2)}_{\textbf{smoothness term}}\,dx\,dy$$

where the spatial image gradient $|\nabla f|^2$ serves as fuzzy edge detector.

- *Properties:* In general the weighting function $g(s^2)$ should be **positive** and **decreasing**. Often the derivative of a strictly convex function is used, i.e. $g(s^2) = \Psi'(s^2)$ with $\Psi(s^2)$ as before.



How Can We Preserve Discontinuities in the Motion Field?

- *Idea 2:* **Image-driven Anisotropic** $\to$ Reduce smoothness only **across** edges
- *Example:* Penalising only the projection of the flow along the edge yields *(Nagel/Enkelmann 1986)*

$$E(\mathbf{w}) = \int_\Omega \underbrace{\mathbf{w}^\top J\,\mathbf{w}}_{\textbf{data term}} + \underbrace{\alpha\,(\nabla u^\top D(\nabla f)\nabla u + \nabla v^\top D(\nabla f)\nabla v)}_{\textbf{smoothness term}}\,dx\,dy$$

where the projection matrix **along the edge** (orthogonal to the gradient) reads

$$D(\nabla f) = \underbrace{\frac{1}{|\nabla f|^2 + 2\epsilon^2}}_{\substack{\text{normalisation}\\ (\operatorname{tr} D)^{-1}}} \left(\underbrace{\begin{pmatrix} f_y^2 & -f_x f_y \\ -f_x f_y & f_x^2 \end{pmatrix}}_{\substack{\text{projection}\\ \nabla f^\perp \nabla f^{\perp\top}}} + \epsilon^2 \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\substack{\text{regularisation}\\ \epsilon > 0}} \right).$$

- Deviations from smoothness **across the edge** not penalised (edges are preserved).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Image-Driven Smoothness Terms (3)

Minimisation with Image-Driven Smoothness Terms

- *Generic Form:* The Euler-Lagrange equations for image-driven methods read

$$
\begin{aligned}
0 &= J_{11}u + J_{12}v + J_{13} - \alpha \operatorname{div}\left(D\ \nabla u\right),\\
0 &= J_{12}u + J_{22}v + J_{23} - \alpha \operatorname{div}\left(D\ \nabla v\right)
\end{aligned}
$$

  with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

- *Notation:* The corresponding **diffusion tensors** D are given by the 2×2 matrices

Homogeneous (Horn/Schunck)	$\rightarrow$	$D = I$
Image-driven Isotropic	$\rightarrow$	$D = g(\lvert\nabla f\rvert^2)\, I$
Image-driven Anisotropic	$\rightarrow$	$D = D(\nabla f)$

- *As Before:* The Euler-Lagrange equations of image-driven methods are **linear**, since the associated diffusions tensors do not depend on the unknowns u and v.

Image-Driven Smoothness Terms (4)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Minimisation with Image-Driven Smoothness Terms

- *Discretisation:* Since the anisotropic approach requires discretisations with eight neighbours for the smoothness term, we restrict ourselves to the isotropic case
- *Step 1*: The discretisation of the divergence expression $\operatorname{div}\left(g(\lvert\nabla f\rvert^2)\nabla u\right)$ reads

$$
\begin{aligned}
\operatorname{div}\left(g \nabla u\right) &= (g u_x)_x + (g u_y)_y\\
&\approx \frac{(g u_x)_{i+\frac12,j} - (g u_x)_{i-\frac12,j}}{2(\frac12 h_x)} + \frac{(g u_y)_{i,j+\frac12} - (g u_y)_{i,j-\frac12}}{2(\frac12 h_y)}\\
&\approx \frac{\frac{g_{i+1,j}+g_{i,j}}{2}\left(\frac{u_{i+1,j}-u_{i,j}}{2(\frac12 h_x)}\right) - \frac{g_{i,j}+g_{i-1,j}}{2}\left(\frac{u_{i,j}-u_{i-1,j}}{2(\frac12 h_x)}\right)}{2(\frac12 h_x)}\\
&\quad + \frac{\frac{g_{i,j+1}+g_{i,j}}{2}\left(\frac{u_{i,j+1}-u_{i,j}}{2(\frac12 h_y)}\right) - \frac{g_{i,j}+g_{i,j-1}}{2}\left(\frac{u_{i,j}-u_{i,j-1}}{2(\frac12 h_y)}\right)}{2(\frac12 h_y)}\\
&= \frac{g_{i+1,j}+g_{i,j}}{2}\left(\frac{u_{i+1,j}-u_{i,j}}{h_x^2}\right) - \frac{g_{i,j}+g_{i-1,j}}{2}\left(\frac{u_{i,j}-u_{i-1,j}}{h_x^2}\right)\\
&\quad + \frac{g_{i,j+1}+g_{i,j}}{2}\left(\frac{u_{i,j+1}-u_{i,j}}{h_y^2}\right) - \frac{g_{i,j}+g_{i,j-1}}{2}\left(\frac{u_{i,j}-u_{i,j-1}}{h_y^2}\right).
\end{aligned}
$$

Image-Driven Smoothness Terms (5)

Minimisation with Image-Driven Smoothness Terms

- *Step 2:* The discrete variant of the nonlinear weight $g(|\nabla f|^2)$ is given by

$$g_{i,j} = g(|\nabla f|^2_{i,j}) = g([J_{11}]_{i,j} + [J_{22}]_{i,j}) \, .$$

- The discrete Euler-Lagrange equations for the isotropic image-driven approach can finally be written as

$$\begin{aligned} 0 &= [J_{11}]_{i,j}\, u_{i,j} + [J_{12}]_{i,j}\, v_{i,j} + [J_{13}]_{i,j} - \alpha \sum_{l \in x,y} \sum_{(\tilde{i},\tilde{j}) \in \mathcal{N}_l(i,j)} \frac{g_{\tilde{i},\tilde{j}} + g_{i,j}}{2} \left(\frac{u_{\tilde{i},\tilde{j}} - u_{i,j}}{h_l^2} \right) \\ 0 &= [J_{12}]_{i,j}\, u_{i,j} + [J_{22}]_{i,j}\, v_{i,j} + [J_{23}]_{i,j} - \alpha \sum_{l \in x,y} \sum_{(\tilde{i},\tilde{j}) \in \mathcal{N}_l(i,j)} \frac{g_{\tilde{i},\tilde{j}} + g_{i,j}}{2} \left(\frac{v_{\tilde{i},\tilde{j}} - v_{i,j}}{h_l^2} \right) \end{aligned}$$

  for $i = 1, ..., N$ and $j = 1, ..., M$.

- *Properties:* The resulting **linear system of equations**
  - has the same structure as the one for the original Horn and Schunck method
  - has for non-constant images a positive definite system matrix (convergence)

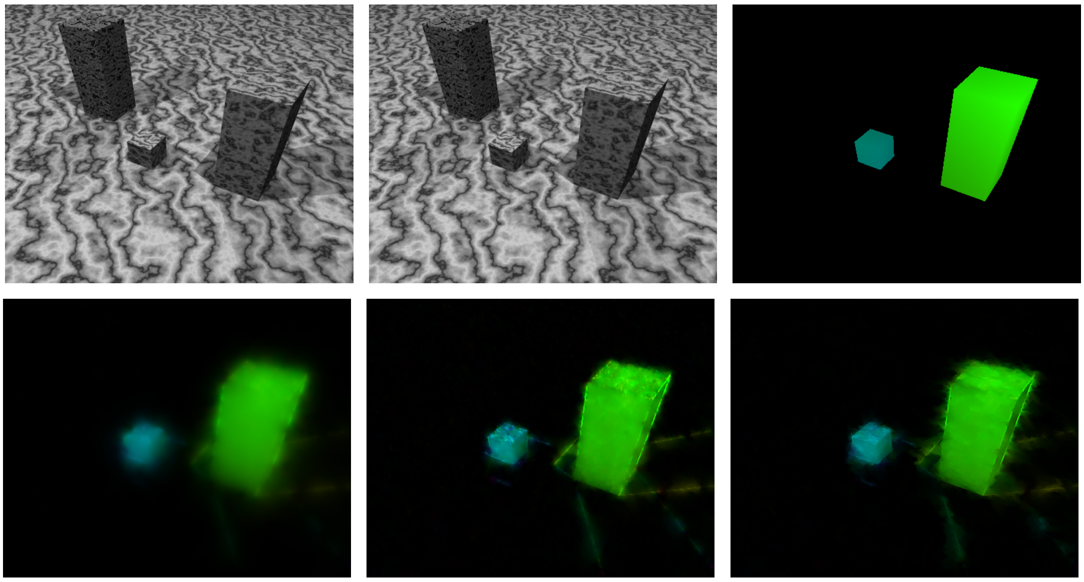
Image-Driven Smoothness Terms (6)

Advantages and Shortcomings of Image-Driven Smoothness Term

- Advantages
  - allow to respect discontinuities in the displacement field
  - still require to solve a linear system of equations
  - can be combined with any data term/any constancy assumption
- Drawbacks
  - spatial image gradient that serves edge detector is sensitive to noise
  - anisotropic case with boundary conditions difficult to implement
  - image edges are not always motion discontinuities
    (oversegmentation of the flow field in heavily textured regions)

Image-Driven Smoothness Terms (7)

Results for Image-Driven Smoothness Terms

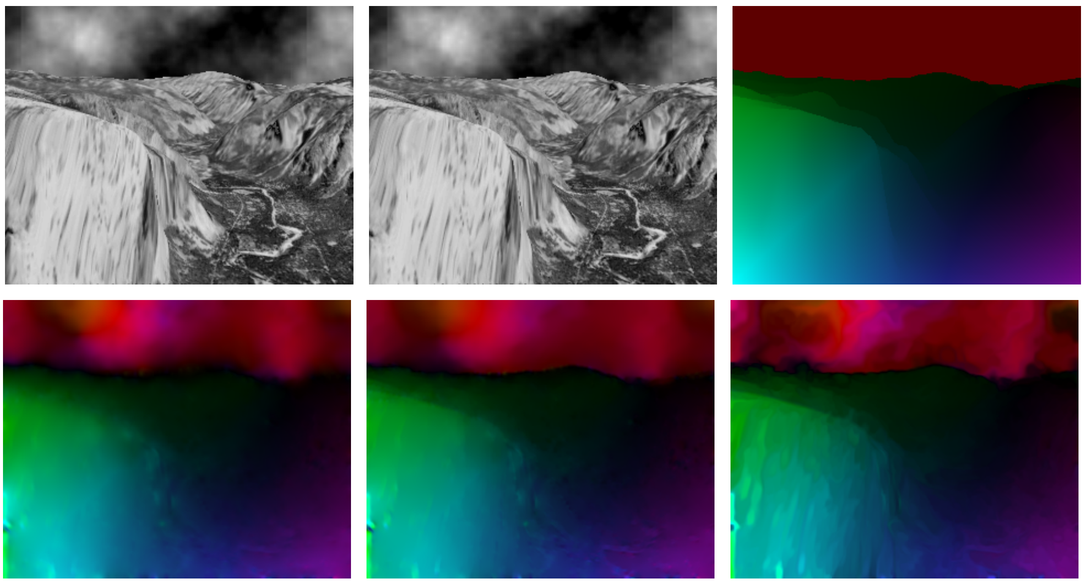


Results for the New Marble Sequence (H.-H. Nagel). **(a) Upper Left:** Frame 150. **(b) Upper Center:** Frame 151. **(c) Upper Right:** Ground truth. **(d) Lower Left:** Homogeneous regularisation. **(e) Lower Center:** Image-driven isotropic regularisation. **(f) Lower Right:** Image-driven anisotropic regularisation.

Image-Driven Smoothness Terms (8)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for Image-Driven Smoothness Terms



Results for the Yosemite Sequence with Clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Frame 9. **(c) Upper Right:** Ground truth. **(d) Lower Left:** Homogeneous regularisation ($\sigma = 1.3$, $\alpha = 500$). **(e) Lower Center:** Image-driven isotropic regularisation ($\sigma = 1.2$, $\alpha = 2700$). **(f) Lower Right:** Image-driven anisotropic regularisation ($\sigma = 1.3$, $\alpha = 4500$).

Results for Image-Driven Smoothness Terms

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Horn and Schunck	(homogeneous)	7.12°
Horn and Schunck	**(image-driven isotropic)**	**6.44°**
Horn and Schunck	**(image-driven anisotropic)**	**6.28°**

- Improvements at motion boundaries compared to homogeneous regularisation.
- Can be combined with any constancy assumption/any data term

How can we overcome the oversegmentation problem ?

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Flow-Driven Smoothness Terms

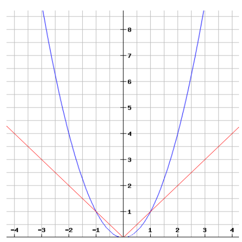
How Can We Overcome the Oversegmentation Problem?

- *Idea 1:* **Flow-driven Isotropic** $\rightarrow$ Allow piecewise constant flow fields
- *Example:* Make discontinuities more attractive by using linear penaliser
(Schnörr 1994, Weickert/Schnörr 2001)

$$E(\mathbf{w}) = \int_\Omega \underbrace{\mathbf{w}^\top J\,\mathbf{w}}_{\text{data term}} + \alpha\, \underbrace{\Psi\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}}\, dx\, dy$$

where $\Psi(s^2) = \sqrt{s^2 + \epsilon^2}$ is the regularised L_1 norm with small $\epsilon > 0$.

- *Properties:* In general $\Psi(s^2)$ should be **positive**, **increasing**, **sub-quadratic** and **strictly convex** to allow for a unique solution in the case of data terms with linearised constancy assumptions.



Flow-Driven Smoothness Terms (2)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

How Can We Overcome the Oversegmentation Problem?

- *Idea 2:* **Flow-driven Anisotropic** $\rightarrow$ Allow directed piecewise constant flow fields
- *Example:* Adapt smoothness to evolving flow structure $\nabla u \nabla u^\top + \nabla v \nabla v^\top$
(Weickert/Schnörr 2001)

$$E(\mathbf{w}) = \int_\Omega \underbrace{\mathbf{w}^\top J\, \mathbf{w}}_{\text{data term}} + \alpha \underbrace{\operatorname{tr} \Psi\left(\nabla u \nabla u^\top + \nabla v \nabla v^\top\right)}_{\text{smoothness term}} dx\, dy$$

where $\Psi(s^2) = \sqrt{s^2 + \epsilon^2}$ is the regularised L_1 norm with small $\epsilon > 0$.

- *New:* Requires application of scalar function Ψ to matrix A. This comes down to keeping the eigenvectors of A and applying Ψ only to its eigenvalues:

$$\Psi(A) = (\mathbf{e}_1, \mathbf{e}_2) \begin{pmatrix} \Psi(\lambda_1) & 0 \\ 0 & \Psi(\lambda_2) \end{pmatrix} (\mathbf{e}_1, \mathbf{e}_2)^\top .$$

- *Properties:* $\Psi(s^2)$ should be chosen as for the image-driven isotropic case.

Flow-Driven Smoothness Terms (3)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Minimisation with Flow-Driven Smoothness Terms

- *Generic Form:* The Euler-Lagrange equations for flow-driven methods read

$$\begin{aligned} 0 &= J_{11} u + J_{12} v + J_{13} - \alpha \operatorname{div}\left(D\, \nabla u\right), \\ 0 &= J_{12} u + J_{22} v + J_{23} - \alpha \operatorname{div}\left(D\, \nabla v\right) \end{aligned}$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

- *Notation:* The corresponding **diffusion tensors** D are given by the 2×2 matrices

Homogeneous (Horn/Schunck)	$\rightarrow$	$D = I$
Flow-driven Isotropic	$\rightarrow$	$D = \Psi'(\lvert\nabla u\rvert^2 + \lvert\nabla v\rvert^2)\, I$
Flow-driven Anisotropic	$\rightarrow$	$D = \Psi'\left(\nabla u \nabla u^\top + \nabla v \nabla v^\top\right)$

- *New:* The Euler-Lagrange equations of flow-driven methods are **nonlinear**, since the associated diffusions tensors depend on u and v – via $\Psi'(s^2) = \frac{1}{2\sqrt{s^2+\epsilon^2}}$.

Minimisation with Flow-Driven Smoothness Terms

- *Discretisation:* As in the image-driven case we only consider the isotropic case
- The discrete Euler-Lagrange equations for the isotropic flow-driven approach read

$$0 = [J_{11}]_{i,j}\, u_{i,j} + [J_{12}]_{i,j}\, v_{i,j} + [J_{13}]_{i,j} - \alpha \sum_{l\in x,y} \sum_{(\tilde{i},\tilde{j})\in\mathcal{N}_l(i,j)} \frac{[\Psi']_{\tilde{i},\tilde{j}} + [\Psi']_{i,j}}{2} \left(\frac{u_{\tilde{i},\tilde{j}} - u_{i,j}}{h_l^2}\right)$$

$$0 = [J_{12}]_{i,j}\, u_{i,j} + [J_{22}]_{i,j}\, v_{i,j} + [J_{23}]_{i,j} - \alpha \sum_{l\in x,y} \sum_{(\tilde{i},\tilde{j})\in\mathcal{N}_l(i,j)} \frac{[\Psi']_{\tilde{i},\tilde{j}} + [\Psi']_{i,j}}{2} \left(\frac{v_{\tilde{i},\tilde{j}} - v_{i,j}}{h_l^2}\right)$$

for $i = 1, ..., N$ and $j = 1, ..., M$.

- *Nonlinearities:* The nonlinear expressions $[\Psi']_{i,j}$ are thereby computed as

$$[\Psi']_{i,j} = \Psi'\big(|\nabla u|^2_{i,j} + |\nabla v|^2_{i,j}\big) = \Psi'\big([u_x]^2_{i,j} + [u_y]^2_{i,j} + [v_x]^2_{i,j} + [v_y]^2_{i,j}\big)$$

with $[u_x]_{i,j}$, $[u_y]_{i,j}$, $[v_x]_{i,j}$ and $[v_y]_{i,j}$ being approximated via central differences.

M I A

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Solving the Nonlinear System

- *Idea:* Solve nonlinear system as series of linear systems (cf. robust data terms)
- *Example:* Fixed point iteration, where the nonlinear expressions (here $[\Psi']_{i,j}$) are evaluated at the **old time step**. This requires to solve the linear equations system

$$0 = [J_{11}]_{i,j}\, u^{k+1}_{i,j} + [J_{12}]_{i,j}\, v^{k+1}_{i,j} + [J_{13}]_{i,j} - \alpha \sum_{l\in x,y} \sum_{(\tilde{i},\tilde{j})\in\mathcal{N}_l(i,j)} \frac{[\Psi']^k_{\tilde{i},\tilde{j}} + [\Psi']^k_{i,j}}{2} \left(\frac{u^{k+1}_{\tilde{i},\tilde{j}} - u^{k+1}_{i,j}}{h_l^2}\right)$$

$$0 = [J_{12}]_{i,j}\, u^{k+1}_{i,j} + [J_{22}]_{i,j}\, v^{k+1}_{i,j} + [J_{23}]_{i,j} - \alpha \sum_{l\in x,y} \sum_{(\tilde{i},\tilde{j})\in\mathcal{N}_l(i,j)} \frac{[\Psi']^k_{\tilde{i},\tilde{j}} + [\Psi']^k_{i,j}}{2} \left(\frac{v^{k+1}_{\tilde{i},\tilde{j}} - v^{k+1}_{i,j}}{h_l^2}\right)$$

for $i = 1, ..., N$ and $j = 1, ..., M$ at each fixed point iteration.

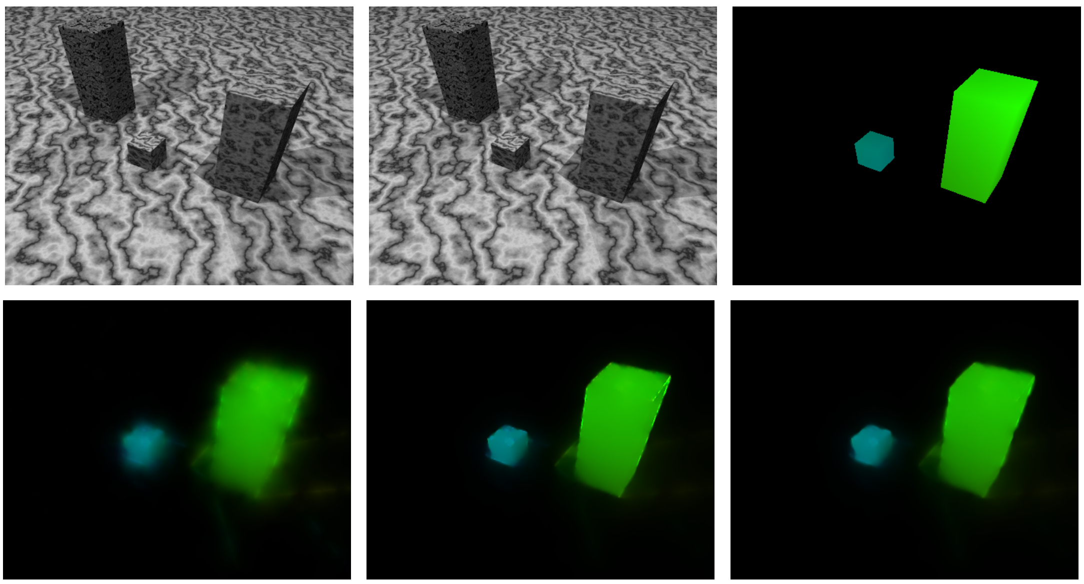
- *Properties:* This so-called **lagged nonlinearity method**
  - yields for non-constant images linear systems that are positive definite
  - requires only a very inexact solution for each linear system to converge

Advantages and Shortcomings of Flow-Driven Smoothness Term

- Advantages
  - allow to respect discontinuities in the displacement field
  - do not suffer from oversegmentation artifacts
  - can be combined with any data term/any constancy assumption
- Drawbacks
  - require to solve a nonlinear system of equations
  - anisotropic case with boundary conditions difficult to implement

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for Flow-Driven Smoothness Terms

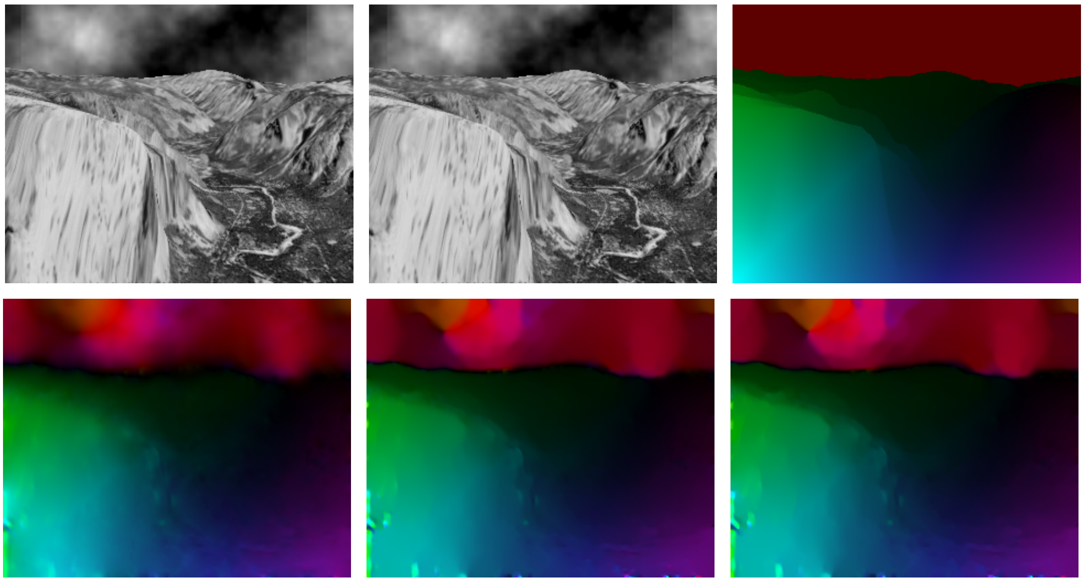


Results for the New Marble Sequence (H.-H. Nagel). **(a) Upper Left:** Frame 150. **(b) Upper Center:** Frame 151. **(c) Upper Right:** Ground truth. **(d) Lower Left:** Homogeneous regularisation. **(e) Lower Center:** Flow-driven isotropic regularisation. **(f) Lower Right:** Flow-driven anisotropic regularisation.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Flow-Driven Smoothness Terms (8)

Results for Flow-Driven Smoothness Terms



Results for the Yosemite Sequence with Clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Frame 9. **(c) Upper Right:** Ground truth. **(d) Lower Left:** Homogeneous regularisation ($\sigma = 1.3$, $\alpha = 500$). **(e) Lower Center:** Flow-driven isotropic regularisation ($\sigma = 1.3$, $\alpha = 42$). **(f) Lower Right:** Flow-driven anisotropic regularisation ($\sigma = 1.3$, $\alpha = 44$).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Flow-Driven Smoothness Terms (9)

Results for Flow-Driven Smoothness Terms

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Horn and Schunck	(homogeneous)	7.12°
Horn and Schunck	(image-driven isotropic)	6.44°
Horn and Schunck	**(flow-driven anisotropic)**	**6.42°**
Horn and Schunck	**(flow-driven isotropic)**	**6.32°**
Horn and Schunck	(image-driven anisotropic)	6.28°

- Less oversegmentation artifacts at heavily textured regions
- Often better than image-driven regularisation (depends on the image sequence)
- Can be combined with any constancy assumption/any data term

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Overview of Diffusion-Inspired Regularisers

- Classification of different smoothness terms
  (Weickert/Schnörr 2001)

Regularisation	Smoothness Term
homogeneous (*Horn/Schunck 1981*)	$\sum_{i=1}^{2} \lvert\boldsymbol{\nabla}_2 \boldsymbol{u}_i\rvert^2$
image-driven, isotropic (*Alvarez et al. 1999*)	$g(\lvert\boldsymbol{\nabla}_2 \boldsymbol{f}\rvert^2) \sum_{i=1}^{2} \lvert\boldsymbol{\nabla}_2 \boldsymbol{u}_i\rvert^2$
image-driven, anisotropic (*Nagel 1983*)	$\sum_{i=1}^{2} \boldsymbol{\nabla}_2 \boldsymbol{u}_i^\top \boldsymbol{D}(\boldsymbol{\nabla}_2 \boldsymbol{f})\, \boldsymbol{\nabla}_2 \boldsymbol{u}_i$
flow-driven, isotropic (*Schnörr 1994*)	$\boldsymbol{\Psi}\Big(\sum_{i=1}^{2} \lvert\boldsymbol{\nabla}_2 \boldsymbol{u}_i\rvert^2\Big)$
flow-driven, anisotropic (*Weickert/Schnörr 2001*)	$\mathrm{tr}\, \boldsymbol{\Psi}\Big(\sum_{i=1}^{2} \boldsymbol{\nabla}_2 \boldsymbol{u}_i \boldsymbol{\nabla}_2 \boldsymbol{u}_i^\top\Big)$

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Spatiotemporal Extensions

- *Idea:* Consider more than two frames for the computation
- *Example 1:* Extending the method of Horn and Schunck to the temporal domain
  (Nagel 1990)

$$E(\mathbf{w}) = \int_{\Omega\times T} \underbrace{\mathbf{w}^\top J\, \mathbf{w}}_{\text{data term}} + \alpha \underbrace{\left(|\nabla_3 u|^2 + |\nabla_3 v|^2\right)}_{\text{smoothness term}}\, dx\, dy\, dt\,.$$

  - **spatiotemporal** integration range $\Omega \times T$
  - **spatiotemporal** smoothness assumption using $\nabla_3 u = (u_x, u_y, u_t)$

- *Example 2:* Extending the flow-driven isotropic method to the temporal domain allows also **temporal discontinuities**
  (Weickert/Schnörr 2001)

$$E(\mathbf{w}) = \int_{\Omega\times T} \underbrace{\mathbf{w}^\top J\, \mathbf{w}}_{\text{data term}} + \alpha \underbrace{\Psi\left(|\nabla_3 u|^2 + |\nabla_3 v|^2\right)}_{\text{smoothness term}}\, dx\, dy\, dt\,.$$

Spatiotemporal Extensions (2)

Minimisation with Spatiotemporal Smoothness Terms

- The Euler-Lagrange equations for spatiotemporal methods are given by

$$\begin{aligned} 0 &\overset{!}{=} F_u - \frac{\partial}{\partial x} F_{u_x} - \frac{\partial}{\partial y} F_{u_y} - \frac{\partial}{\partial t} F_{u_t} \ , \\ 0 &\overset{!}{=} F_v - \frac{\partial}{\partial x} F_{v_x} - \frac{\partial}{\partial y} F_{v_y} - \frac{\partial}{\partial t} F_{v_t} \end{aligned}$$

  with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla_3 u = 0$ and $\mathbf{n}^\top \nabla_3 v = 0$.

- For the spatiotemporal variant of the Horn and Schunck method they thus read

$$\begin{aligned} 0 &= J_{11} u + J_{12} v + J_{13} - \alpha \Delta_3 u \ , \\ 0 &= J_{12} u + J_{22} v + J_{23} - \alpha \Delta_3 v \end{aligned}$$

  with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla_3 u = 0$ and $\mathbf{n}^\top \nabla_3 v = 0$.

- *New:* **Spatiotemporal** Laplacian $\Delta_3 u = u_{xx} + u_{yy} + u_{tt}$ instead of a spatial one.

Spatiotemporal Extensions (3)

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Discrete Euler-Lagrange Equations

- The discrete Euler-Lagrange equations for the spatiotemporal method of Horn and Schunck can be written as

$$\begin{aligned} 0 &= [J_{11}]_{i,j,k}\, u_{i,j,k} + [J_{12}]_{i,j,k}\, v_{i,j,k} + [J_{13}]_{i,j,k} - \alpha \sum_{l \in x,y,t} \ \sum_{(\tilde{i},\tilde{j},\tilde{k}) \in \mathcal{N}_l(i,j,k)} \frac{u_{\tilde{i},\tilde{j},\tilde{k}} - u_{i,j,k}}{h_l^2} \\ 0 &= [J_{12}]_{i,j,k}\, u_{i,j,k} + [J_{22}]_{i,j,k}\, v_{i,j,k} + [J_{23}]_{i,j,k} - \alpha \sum_{l \in x,y,t} \ \sum_{(\tilde{i},\tilde{j},\tilde{k}) \in \mathcal{N}_l(i,j,k)} \frac{v_{\tilde{i},\tilde{j},\tilde{k}} - v_{i,j,k}}{h_l^2} \end{aligned}$$

  for $i = 1, ..., N$, $j = 1, ..., M$ and $k = 1, ..., Z$ (Z denotes the number of frames).

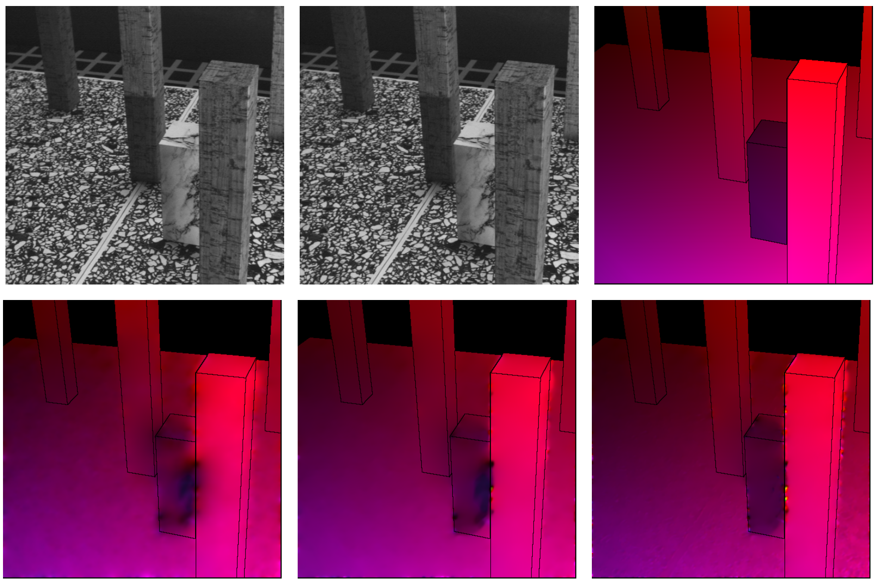
  - here, $\mathcal{N}_l(i,j,k)$ denotes the set of neighbours of pixel i,j,k in direction of axis l (assuming six direct neighbours, i.e. two in each direction)
  - as the spatial method these equations constitute a **linear equations system**; this time however w.r.t. the $2N \times M \times Z$ unknowns $u_{i,j,k}$ and $v_{i,j,k}$.
  - for non-constant images the matrix of the linear system is positive definite (convergence of Gauß-Seidel method)

Advantages and Shortcomings of Spatiotemporal Smoothness Terms

- Advantages
  - allow to use more then two frames
  - can make use of spatiotemporal presmoothing techniques
  - are more robust due to spatiotemporal filling-in-effect
  - can detect sub-pixel motion more accurately
  - any smoothness term can be extended to the temporal domain
  - can be combined with any data term/any constancy assumption
  - computationally hardly more expensive per frame (less than 30%)
- Drawbacks
  - not real-time suitable due to delayed en-block computation (require frames from the future to determine current frame)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

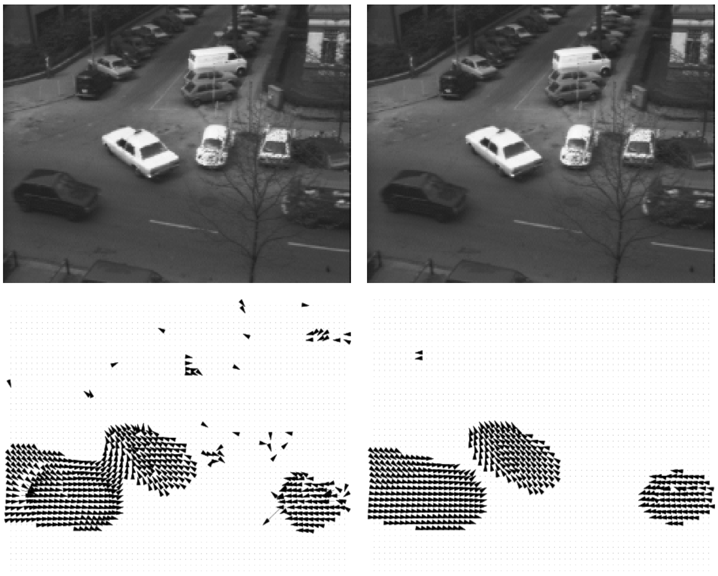
Results for Spatiotemporal Smoothness Terms



Results for the Old Marble Sequence (H.-H. Nagel). **(a) Upper Left:** Frame 16. **(b) Upper Center:** Frame 17. **(c) Upper Right:** Ground truth. **(d) Lower Left:** Homogeneous 2-D. **(e) Lower Center:** Flow-driven isotropic 2-D. **(f) Lower Right:** Flow-driven isotropic 3-D.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for Spatiotemporal Smoothness Terms



Results for the Taxi Sequence (H.-H. Nagel). **(a) Upper Left:** Frame 10. **(b) Upper Right:** Frame 11. **(c) Lower Left:** Homogeneous 2-D. **(d) Lower Right:** Homogeneous 3-D. *Author: J. Weickert*

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Results for Spatiotemporal Smoothness Terms

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		Frames	AAE
Horn and Schunck	(homogeneous)	2	7.12°
Horn and Schunck	(homogeneous)	3	6.83°
Horn and Schunck	(homogeneous)	4	6.54°
Horn and Schunck	(homogeneous)	5	6.56°
Horn and Schunck	**(homogeneous)**	**6**	**6.23°**
Horn and Schunck	(homogeneous)	8	6.31°
Horn and Schunck	(homogeneous)	10	6.30°
Horn and Schunck	(homogeneous)	12	6.25°

- The quality improves if more than two frames are used
- The number of frames should not be chosen too large (completely different types of motion may appear)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Summary (1)

Summary

- Locally integrating data terms improve the robustness under noise (CLG)
- Robust data terms yield a better performance in the presence of outliers
- Adaptive smoothness terms allow the preservations of motion discontinuities
  - image-driven smoothness terms adapt to image edges
  - flow-driven smoothness terms adapt to evolving flow edges
  - diffusion tensor notation as motion tensor equivalent for smoothness term
- Spatiotemporal smoothness terms allow to consider more than two frames
- Nonquadratic data and smoothness terms can be implemented using the lagged nonlinearity method (if the underlying penaliser functions are strictly convex)

Summary (2)

Literature

- M. J. Black, P. Anandan:
  Robust dynamic motion estimation over time.
  In *Proc. 1991 IEEE Conference on Computer Vision and Pattern Recognition*, pp. 292-302
  IEEE Computer Society Press, 1991.
  (robust data term, adaptive back-in-time smoothness term)
- T. Brox, A. Bruhn, N. Papenberg, J. Weickert:
  High accuracy optical flow estimation based on a theory for warping.
  In T. Pajdla, J. Matas (Eds.): *Computer Vision – ECCV 2004*
  Lecture Notes in Computer Science, Springer, Vol. 3024, pp. 25–36, 204.
  (joint robustification, high accuracy method)
- A. Bruhn, J. Weickert:
  Towards ultimate motion estimation: Combining highest accuracy with real-time performance.
  In *Proc. 2005 IEEE International Conference on Computer Vision*, Vol. 1, pp. 749–755
  IEEE Computer Society Press, 2005.
  (separate robustification, high accuracy method)
- A. Bruhn, J. Weickert, C. Schnörr:
  Lucas/Kanade Meets Horn/Schunck: Combining local and global optic flow methods. In *International Journal of Computer Vision*, Vol. 61, No. 3, pp. 211–231, 2005.
  (description of the combined-local-global approach)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Literature

- H.-H. Nagel:
  Extending the 'oriented smoothness constraint' into the temporal domain
  and the estimation of derivatives of optical flow.
  In O. Faugeras (Ed.): *Computer Vision – ECCV '90*
  Lecture Notes in Computer Science, Springer, Vol. 427, pp. 139–148, 1990.
  (spatio-temporal image-driven smoothness term)
- J. Weickert, C. Schnörr:
  Variational optic flow computation with a spatio-temporal smoothness constraint.
  In *Journal of Mathematical Imaging and Vision*, Vol. 14, No. 3, pp. 245–255, 2001.
  (spatio-temporal flow-driven smoothness term)
- J. Weickert, C. Schnörr:
  A theoretical framework for convex regularizers in PDE-based computation of image motion.
  In *International Journal of Computer Vision*, Vol. 45, No. 3, pp. 245–264, 2001.
  (overview of diffusion-inspired smoothness terms)

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Assignment 5

Programming Exercise (Gradient Constancy)

You can download the file copcv09_ex05.tgz from the web page

`http://www.mia.uni-saarland.de/Teaching/copcv09.shtml`

To unpack these data, use `tar xzvf copcv09_ex05.tgz`.

1. Supplement the routine `compute_motion_tensor()` in the C programme `horn_schunck.c` with missing code so that it computes the motion tensor of the *gradient constancy assumption*. You can approximate the required second order derivatives f_{xx}, f_{xy}, f_{yy}, f_{xt}, and f_{yt} by first computing f_x, f_y, and f_t and then applying a simple central difference scheme for additional $x-$ and $y-$ derivatives (without averaging). In order to compile your programme please use the contained makefile. The compiled programme is then executed by

   `./frontend <input_image1.pgm> <input_image2.pgm> <zoom_ratio> [ground_truth.F]`

   where the integer parameter `zoom_ratio` is in general set to 1. The use of a ground truth file `ground_truth.F` is optional and triggers the computation of the average angular error (AAE).
2. Use the provided image pair `yos1.pgm` and `yos2.pgm` to optimise your results with respect to the average angular error (AAE).

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Assignment 5

Programming Exercise (Backward Registration)

Use the same code for the second task.

3. Supplement the routine `backward_registration()` in the same C programme with missing code so that it compensates the second image by a given flow field.
4. You can use this routine by pressing $F8$ after computing a displacement field. The motion compensated second frame ist then written out as file `frame2_bw.pgm`. Use the Linux command

   `animate <frame1.pgm> <frame2_bw.pgm`

   to visually compare the first and the motion compensated second visually for your best results for the sequences `yos1.pgm` and `yos2.pgm` as well as `rhein1.pgm` and `rhein2.pgm`. Do the results make sense in both cases?