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Lecture 5:

Optic Flow III
Advanced Constancy Assumptions, Large Motion

Contents

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Higher Order Constancy Assumptions

How Can We Improve the Performance under Varying Illumination?

- *Idea:* Consider constancy assumptions on image derivatives
- *Example:* Constancy of the **spatial image gradient** $\nabla_2 f = (f_x, f_y)^\top$:
  (Uras/Girosi/Verri/Torre 1988)

$$\begin{aligned} f_x(x,y,t) - f_x(x+u, y+v, t+1) &= 0\,, \\ f_y(x,y,t) - f_y(x+u, y+v, t+1) &= 0\,. \end{aligned}$$

  Linearisation yields the corresponding optic flow constraints (OFCs):

$$\begin{aligned} f_{xx}\, u + f_{xy}\, v + f_{xt} &= 0\,, \\ f_{yx}\, u + f_{yy}\, v + f_{yt} &= 0\,. \end{aligned}$$

- *New:* Two equations for two unknowns may already be **sufficient** to determine a local solution for the data term (→ aperture problem not always present).

Higher Order Constancy Assumptions (2)

The Horn and Schunck Method with Gradient Constancy

- Plugging the gradient constancy assumption as a quadratic data term into the method of Horn and Schunck yields the energy functional

$$E(u,v)=\int_\Omega \underbrace{(f_{xx}u+f_{xy}v+f_{xt})^2+(f_{yx}u+f_{yy}v+f_{yt})^2}_{\text{data term}}+\alpha\,\underbrace{(|\nabla u|^2+|\nabla v|^2)}_{\text{smoothness term}}\,dx\,dy\,.$$

- The corresponding Euler-Lagrange equations are given by

$$\begin{aligned} f_{xx}(f_{xx}u+f_{xy}v+f_{xt})+f_{yx}(f_{yx}u+f_{yy}v+f_{yt})-\alpha\Delta u &= 0\,,\\ f_{xy}(f_{xx}u+f_{xy}v+f_{xt})+f_{yy}(f_{yx}u+f_{yy}v+f_{yt})-\alpha\Delta v &= 0 \end{aligned}$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top\nabla u=0$ and $\mathbf{n}^\top\nabla v=0$.

- Looks complicated but can be implemented analogously to the original method (using the so-called **motion tensor notation**; see later in this lecture)

Higher Order Constancy Assumptions (3)

Can this Idea Be Extended to Higher Order Derivatives?

- *Example:* Constancy of the **spatial image Hessian** $\mathcal{H}_2 f=\begin{pmatrix} f_{xx} & f_{xy}\\ f_{xy} & f_{yy}\end{pmatrix}$:
(Papenberg/Brox/Bruhn/Didas/Weickert 2006)

$$\begin{aligned} f_{xx}(x,y,t)-f_{xx}(x+u,y+v,t+1) &= 0\,,\\ f_{xy}(x,y,t)-f_{xy}(x+u,y+v,t+1) &= 0\,,\\ f_{xy}(x,y,t)-f_{xy}(x+u,y+v,t+1) &= 0\,,\\ f_{yy}(x,y,t)-f_{yy}(x+u,y+v,t+1) &= 0\,. \end{aligned}$$

Linearisation yields four equations for two unknowns, at most three of them are linear independent (the ones with mixed derivatives are identical)

$$\begin{aligned} f_{xxx}\,u+f_{xxy}\,v+f_{xxt} &= 0\,,\\ f_{xyx}\,u+f_{xyy}\,v+f_{xyt} &= 0\,,\\ f_{xyx}\,u+f_{xyy}\,v+f_{xyt} &= 0\,,\\ f_{yyx}\,u+f_{yyy}\,v+f_{yyt} &= 0\,. \end{aligned}$$

- The higher the derivative order the more equations/constraints are obtained.

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Advantages and Shortcomings of Constancy Assumptions on Derivatives

- Advantages
  - invariant under global additive illumination changes (here given by c):

$$\big(f(x,y,t)+c\big)_x = f_x(x,y,t)\ .$$

  - aperture problem less distinct due to the use of multiple constraints

- Drawbacks
  - enhanced sensitivity under noise due to derivatives
  - information loss as prize for the illumination invariance
  - not suitable for spatially varying additive changes
    (in this case $c_x(x,y,t) \neq 0$)
  - not suitable for rotations due to **contained directional information**
    (implicit constancy assumption on the direction)

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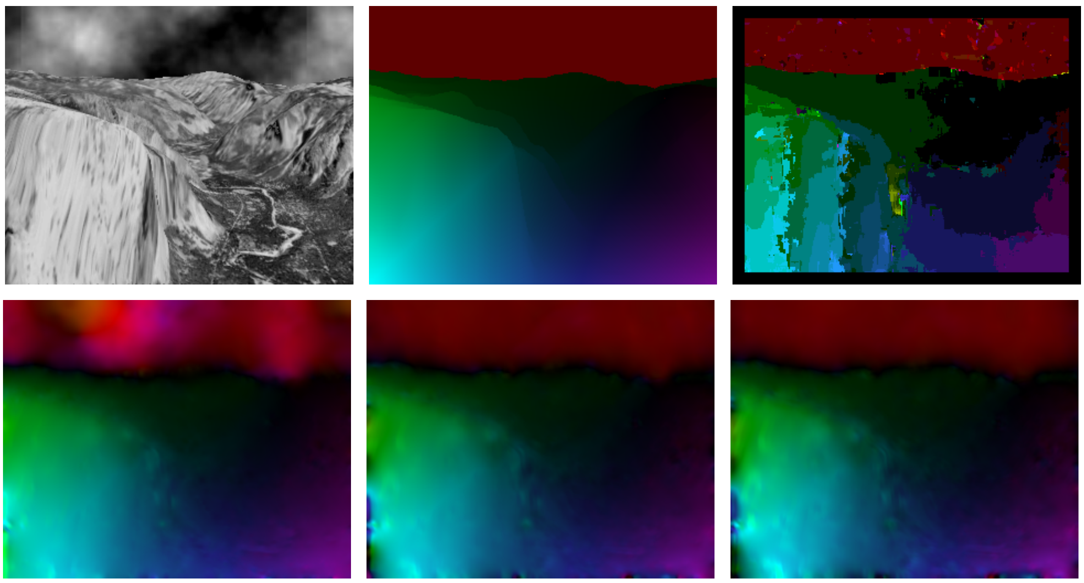
Results for Horn and Schunck with Constancy Assumptions on Derivatives



Results for the Yosemite Sequence with clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Ground truth. **(c) Upper Right:** NCC ($m=4$, $d=7$) **(d) Lower Left:** Grey value constancy ($\sigma=1.4$, $\alpha=470$). **(e) Lower Center:** Gradient constancy ($\sigma=2.1$, $\alpha=20$). **(f) Lower Right:** Hessian constancy ($\sigma=2.7$, $\alpha=1.8$).

Higher Order Constancy Assumptions (6)

Results for Horn and Schunck with Constancy Assumptions on Derivatives

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Normalised Cross Correlation (NCC)		21.84°
Block Matching + Subpixel (SSD)		21.46°
Bigün et al.		10.60°
Lucas/Kanade		8.79°
Horn and Schunck	(grey value constancy)	7.12°
Horn and Schunck	**(Hessian constancy)**	**6.46°**
Horn and Schunck	**(gradient constancy)**	**5.91°**

- Constancy assumptions on image derivatives can significantly improve the results under varying illumination. However, how can we handle fast rotational motion?

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Higher Order Constancy Assumptions (7)

How Can We Handle Fast Rotational Motion?

- *Idea:* Discard directional information and create motion invariant assumptions
- *Example:* Constancy of the **magnitude of the spatial image gradient**:
  (Papenberg/Bruhn/Brox/Didas/Weickert 2006)

$$|\nabla_2 f(x,y,t)| - |\nabla_2 f(x+u,y+v,t+1)| \quad = \quad 0\,.$$

Linearisation yields the corresponding optic flow constraint:

$$|\nabla_2 f|_x\, u + |\nabla_2 f|_y\, v + |\nabla_2 f|_t \quad = \quad 0$$

Evaluating the derivatives yields the expression:

$$\frac{f_x f_{xx} + f_y f_{xy}}{|\nabla_2 f|}\, u + \frac{f_x f_{xy} + f_y f_{yy}}{|\nabla_2 f|}\, v + \frac{f_x f_{xt} + f_y f_{yt}}{|\nabla_2 f|} \quad = \quad 0\,.$$

- Gives once again one equation for two unknowns (→ aperture problem)

Can this Idea Also Be Extended to Higher Order Derivatives?

- *Idea:* Eigenvalues are invariant under rotations, eigenvectors not
- *Example:* In the case of the Hessian two strategies possible:
  - Constancy of the **trace of the Hessian** (sum of eigenvalues):
    (Papenberg/Bruhn/Brox/Didas/Weickert 2006)

$$\mathrm{tr}\;(\mathcal{H}_2 f(x,y,t)) - \mathrm{tr}\;(\mathcal{H}_2 f(x+u,y+v,t+1)) \;=\; 0$$

  - Constancy of the **determinant of the Hessian** (product of eigenvalues):
    (Papenberg/Bruhn/Brox/Didas/Weickert 2006)

$$\det\,(\mathcal{H}_2 f(x,y,t)) - \det\,(\mathcal{H}_2 f(x+u,y+v,t+1)) \;=\; 0\,.$$

- Yields for each strategy one equation for two unknowns (→ aperture problem)
- The higher the derivatives order the more of such invariant expressions exist

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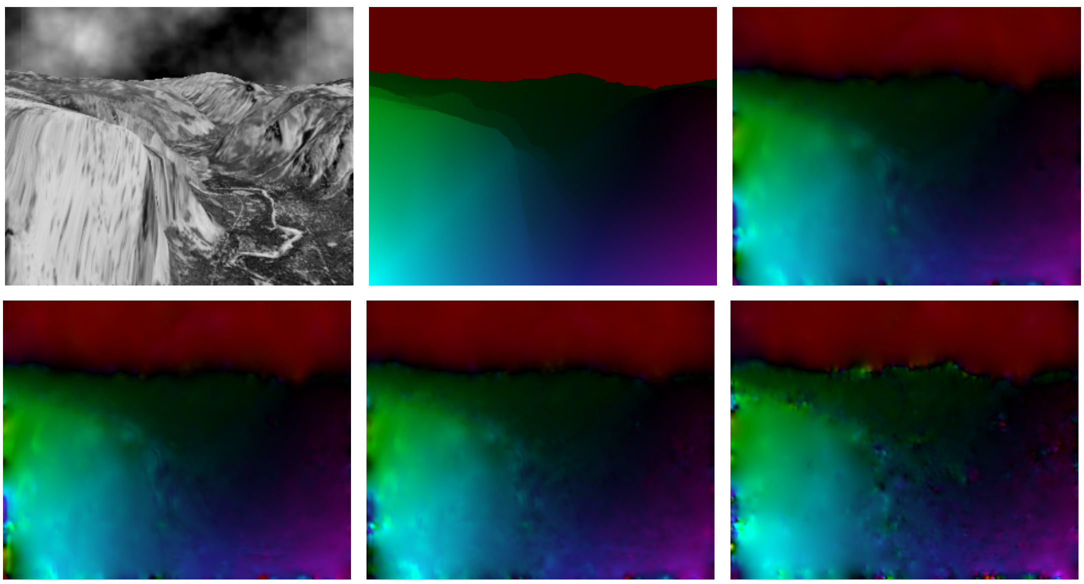
Results for Horn/Schunck with Motion Invariant Assumptions on Derivatives



Results for the Yosemite Sequence with clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Ground truth. **(c) Upper Right:** Grey value constancy ($\sigma = 1.4$, $\alpha = 470$) **(d) Lower Left:** Gradient magnitude constancy ($\sigma = 1.9$, $\alpha = 14$). **(e) Lower Center:** Hessian trace constancy ($\sigma = 2.5$, $\alpha = 3$). **(f) Lower Right:** Hessian determinant constancy ($\sigma = 3.0$, $\alpha = 0.1$).

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Higher Order Constancy Assumptions (10)

Results for Horn/Schunck with Motion Invariant Assumptions on Derivatives

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Normalised Cross Correlation (NCC)		21.84°
Block Matching + Subpixel (SSD)		21.46°
Bigün et al.		10.60°
Lucas/Kanade		8.79°
Horn and Schunck	**(Hessian determinant constancy)**	**8.10°**
Horn and Schunck	(grey value constancy)	7.12°
Horn and Schunck	(Hessian constancy)	6.46°
Horn and Schunck	**(gradient magnitude constancy)**	**6.37°**
Horn and Schunck	**(Hessian trace constancy)**	**6.18°**
Horn and Schunck	(gradient constancy)	5.91°

Higher Order Constancy Assumptions (11)

Overview of Higher Order Constancy Assumptions

- Directional Constancy Assumptions on Image Derivatives
  (*Uras/Girosi/Verri/Torre 1988, Papenberg/Bruhn/Brox/Didas/Weickert IJCV 2006*)

Constancy	Data Term	Motion Type
gradient	$\sum_{i=1}^{2} (\mathbf{w}^\top \nabla_3 f_{x_i})^2$	translational divergent slow rotational
Hessian	$\sum_{i=1}^{2}\sum_{j=1}^{2} (\mathbf{w}^\top \nabla_3 f_{x_i x_j})^2$	translational divergent slow rotational

- Motion Invariant Constancy Assumptions on Image Derivatives
  (*Papenberg/Bruhn/Brox/Didas/Weickert IJCV 2006*)

Constancy	Data Term	Motion Type
gradient magnitude	$(\mathbf{w}^\top \nabla_3 \lvert\nabla_2 f\rvert)^2$	**any**
Hessian trace	$(\mathbf{w}^\top \nabla_3 \operatorname{tr}(\mathcal{H}_2 f))^2$	**any**
Hessian determinant	$(\mathbf{w}^\top \nabla_3 \det(\mathcal{H}_2 f))^2$	**any**

The Motion Tensor Notation (1)

The Motion Tensor Notation

- *Idea:* Compact representation of combined data terms (*Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006*)
  - consider n constancy assumptions on $p_1,\ldots,p_n$ with weights $\gamma_1,\ldots,\gamma_n$

$$\sum_{i=1}^{n} \gamma_i \, (\mathbf{w}^\top \nabla_3 p_i)^2 = \sum_{i=1}^{n} \gamma_i \, (\mathbf{w}^\top \nabla_3 p_i \, \nabla_3 p_i^\top \, \mathbf{w}) = \mathbf{w}^\top \Big(\sum_{i=1}^{n} \gamma_i \, \nabla_3 p_i \, \nabla_3 p_i^\top \Big) \, \mathbf{w} = \mathbf{w}^\top \underbrace{J(\nabla_3 p_1, .., \nabla_3 p_n)}_{\text{Motion Tensor}} \, \mathbf{w}$$

  - single quadratic form with 3×3 positive semi-definite motion tensor J
  - framework for all previously presented data terms

The Motion Tensor Notation (2)

Advantages of the Motion Tensor Notation

- *Example:* The gradient constancy assumption yields the motion tensor

$$J(\nabla_3 f_x, \nabla_3 f_y) = \nabla_3 f_x \nabla_3 f_x^\top + \nabla_3 f_y \nabla_3 f_y^\top = \begin{pmatrix} f_{xx} f_{xx} + f_{yx} f_{yx} & f_{xy} f_{xx} + f_{yy} f_{yx} & f_{xt} f_{xx} + f_{yt} f_{yx} \\ f_{xx} f_{xy} + f_{yx} f_{yy} & f_{xy} f_{xy} + f_{yy} f_{yy} & f_{xt} f_{xy} + f_{yt} f_{yy} \\ f_{xx} f_{xt} + f_{yx} f_{yt} & f_{xy} f_{xt} + f_{yy} f_{yt} & f_{xt} f_{xt} + f_{yt} f_{yt} \end{pmatrix}.$$

- Advantages
  - works for all linearised constancy assumptions
  - only one implementation for all constancy assumptions required
  - rank analysis specifies degrees of freedom, i.e. if the aperture problem is locally present or not (cf. method of Lucas and Kanade)

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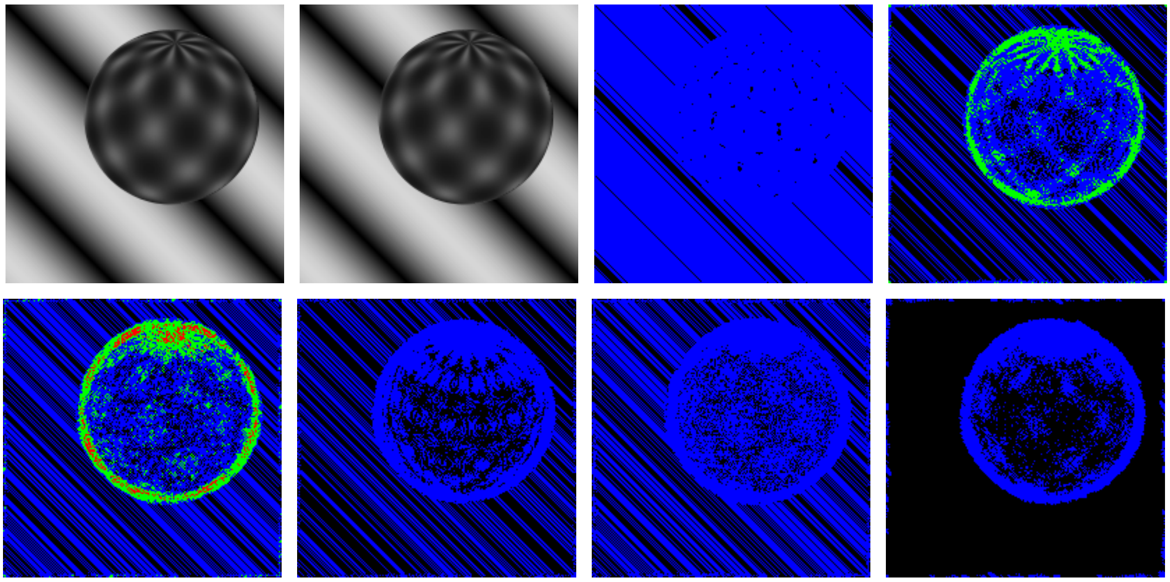
Visualisation of the Aperture Problem for Different Constancy Assumptions



Rotating Sphere sequence (Galvin *et al.*). **Black:** No information. **Blue:** Aperture problem. **Green:** Complete Flow. **Red:** Contradicting Assumptions. **Upper Row:** Frame 10, frame 11, grey value, gradient. **Lower Row:** Hessian, gradient magnitude, trace of the Hessian, determinant of the Hessian.

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Extension to RGB Colour Sequences

- *Idea:* Exploit all information in colour sequences (not only the brightness)
- Assume constancy of all three colour channels $(f_1, f_2, f_3)^\top = (R, G, B)^\top$:

$$\begin{aligned} f_1(x,y,t) - f_1(x+u, y+v, t+1) &= 0\,, \\ f_2(x,y,t) - f_2(x+u, y+v, t+1) &= 0\,, \\ f_3(x,y,t) - f_3(x+u, y+v, t+1) &= 0\,. \end{aligned}$$

Linearisation yields the following three optic flow constraints:

$$\begin{aligned} f_{1x}\,u + f_{1y}\,v + f_{1t} &= 0\,, \\ f_{2x}\,u + f_{2y}\,v + f_{2t} &= 0\,, \\ f_{3x}\,u + f_{3y}\,v + f_{3t} &= 0\,. \end{aligned}$$

- All three constraints should be fulfilled **jointly** by one displacement field

The Horn and Schunck Method for RGB Colour Sequences

- Plugging the RGB constancy assumption as a quadratic data term into the method of Horn and Schunck yields the energy functional
  (Ohta 1989, Barron/Klette 2002)

$$E(u,v)=\int_\Omega \underbrace{\sum_{i=1}^{3}(f_{i_x}u+f_{i_y}v+f_{i_t})^2}_{\text{data term}} + \alpha\,\underbrace{(|\nabla u|^2+|\nabla v|^2)}_{\text{smoothness term}}\;dx\,dy\;.$$

- Advantages
  - computes one joint displacement field for all three channels (no averaging)
  - can be written and implemented compactly using the motion tensor notation
  - qualitatively better results than for the brightness information only
  - can be extended in a straightforward way to colour image derivatives

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Photometric Invariants

- *Idea:* Derive illumination-robust invariants from RGB colour images
- Photometric invariants are based on the dichromatic reflection model
  (Shafer 1985)
  - This model describes the observed RGB colour $\mathbf{f}(\mathbf{x})=(R(\mathbf{x}),G(\mathbf{x}),B(\mathbf{x}))^\top$ at a certain location $\mathbf{x}=(x,y)^\top$ as the sum

$$\mathbf{f}(\mathbf{x})=\mathbf{f}_i(\mathbf{x})+\mathbf{f}_b(\mathbf{x}).$$

  - **interface reflection component** $\mathbf{f}_i(\mathbf{x})$ due to specularities or highlights
  - **body reflection component** $\mathbf{f}_b(\mathbf{x})$ due to the matte body (Lambertian)
- This model is often simplified by using two additional assumptions
  - uniform spectral illumination (white spectrum)
  - neutral interface reflection (NIR)

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Photometric Invariants (2)

Towards A Simplified Dichromatic Reflection Model

- *Simplification 1:* Uniform spectral illumination allows the factorisation

$$\mathbf{f}(\mathbf{x}) = e\, m_i(\mathbf{x})\, \widehat{\mathbf{f}}_i(\mathbf{x}) + e\, m_b(\mathbf{x})\, \widehat{\mathbf{f}}_b(\mathbf{x})\ .$$

  - overall illumination intensity e
  - geometrical reflection factors $m_i(\mathbf{x})$ and $m_b(\mathbf{x})$
  - reflectance colours $\widehat{\mathbf{f}}_i(\mathbf{x})$ and $\widehat{\mathbf{f}}_b(\mathbf{x})$

- *Simplification 2:* It moreover yields pure achromatic interface colours $\widehat{\mathbf{f}}_i$.
  - all three channels of $\widehat{\mathbf{f}}_i$ have equal contributions, i.e.

$$\widehat{R_i}(\mathbf{x}) = \widehat{G_i}(\mathbf{x}) = \widehat{B_i}(\mathbf{x}) =: w_i(\mathbf{x})\,.$$

  Here, w_i is some value that depends on the location $\mathbf{x} = (x, y)^\top$.

Photometric Invariants (3)

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Towards A Simplified Dichromatic Reflection Model

- *Simplification 3:* Neutral interface reflection (NIR)
  - the achromatic value $w_i(\mathbf{x})$ is independent from the location $\mathbf{x}$

- *Final Model:* Assuming uniform illumination and neutral boundary interface reflection one obtains the **simplified dichromatic model**. It is given by

$$\mathbf{f}(\mathbf{x}) = e\,\left(m_i(\mathbf{x})\, w_i\, \mathbf{1} + m_b(\mathbf{x})\, \widehat{\mathbf{f}}_b(\mathbf{x})\right)$$

  which can be formulated in terms of the three independent equations

$$\begin{aligned} R(\mathbf{x}) &= e\,\left(m_i(\mathbf{x})\, w_i + m_b(\mathbf{x})\, \widehat{R}_b(\mathbf{x})\right),\\ G(\mathbf{x}) &= e\,\left(m_i(\mathbf{x})\, w_i + m_b(\mathbf{x})\, \widehat{G}_b(\mathbf{x})\right),\\ B(\mathbf{x}) &= e\,\left(m_i(\mathbf{x})\, w_i + m_b(\mathbf{x})\, \widehat{B}_b(\mathbf{x})\right). \end{aligned}$$

Photometric Invariants (4)

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Classification of Photometric Invariants

- Three kinds of photometric invariants can be distinguished: Invariants under
  (van de Weijer/Gevers 2005)
  - **Global multiplicative illumination changes**
    ($\rightarrow$ expressions that eliminate e)
  - **Shadow and shading**
    ($\rightarrow$ expressions that eliminate e and $m_b(\mathbf{x})$ if $m_i(\mathbf{x}) = 0$)
  - **Highlights and specular reflections**
    ($\rightarrow$ expressions that eliminate e, $m_b(\mathbf{x})$ and $m_i(\mathbf{x})$)
- Three main concepts are used to design such photometric invariants
  (Mileva/Bruhn/Weickert 2007)
  - Log-Derivative transforms
  - Normalisation strategies
  - Conical/spherical transforms

Photometric Invariants (5)

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Log-Derivative Transforms

- *Idea:* Logarithmise image data before using assumptions on image derivatives
- *Example:* For the spatial image gradient one obtains the mapping
  (Mileva/Bruhn/Weickert 2007)

$$(R, G, B)^\top \mapsto \left((\ln R)_x, (\ln R)_y, (\ln G)_x, (\ln G)_y, (\ln B)_x, (\ln B)_y\right)^\top .$$

  - can be applied to any higher order constancy assumption
  - works also for grey value image sequences
- *Invariance Properties:*
  - invariant under global multiplicative illumination changes
  - invariant under shadow and shading (only if $m_b(\mathbf{x})$ varies smoothly)

Photometric Invariants (6)

Normalisation Strategies

- *Idea:* Normalise all channels to remove local scale information
- *Example:* Chromaticity spaces are given by the mapping
  (Golland/Bruckstein 1997)

$$(R,G,B)^\top \mapsto \left(\frac{R}{N},\frac{G}{N},\frac{B}{N}\right)^\top .$$

  - arithmetic chromaticity space for $N = \frac{1}{3}(R+G+B)$
  - geometric chromaticity space for $N = \sqrt[3]{RGB}$

- *Invariance Properties:*
  - invariant under global multiplicative illumination changes
  - invariant under shadow and shading

Photometric Invariants (7)

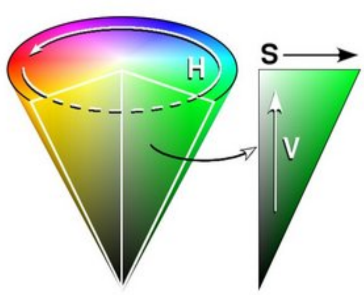
Conical/Spherical Transforms

- *Idea:* Transform RGB colour space into conical/speherical coordinates
- *Example 1:* The conical HSV colour space (hue, saturation, value) is given by
  (Golland/Bruckstein 1997)

$$(R,G,B)^\top \mapsto \left\{ \begin{array}{lcl} H & = & \left\{ \begin{array}{ll} \frac{G-B}{M-m} \times 60^\circ, & R \geq G,B, \\ (2+\frac{B-R}{M-m}) \times 60^\circ, & G \geq R,B, \\ (4+\frac{R-G}{M-m}) \times 60^\circ, & B \geq R,G, \end{array} \right. \quad (\mathrm{mod}\ 360^\circ)\,, \\ S & = & \frac{M-m}{M} \\ V & = & M\,. \end{array} \right.$$

where $M = \max(R,G,B)$ and $m = \min(R,G,B)$.

  - the hue denotes the pure colour
  - the saturation is the achromatic component
  - the value stands for denotes the actual brightness



Author: Wikipedia

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Conical/Spherical Transforms

- *Invariance Properties:*
  - the hue is invariant under global multiplicative illumination changes, shadow and shading as well as specular reflections (discards most information)
  - the saturation is invariant under global multiplicative illumination changes as well as shadow and shading
  - the value is no photometric invariant expression

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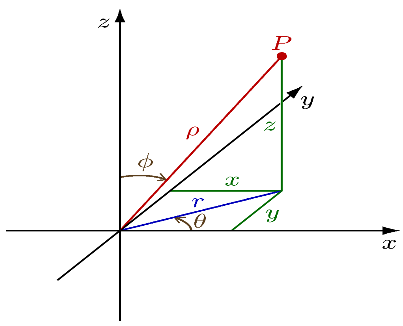
Conical/Spherical Transforms

- *Example 2:* The spherical $\rho\phi\theta$ colour space (radius,longitude,latitude) is given by *(van de Weijer/Gevers 2005)*

$$(R,G,B)^\top \mapsto \begin{cases} \rho &= \sqrt{R^2+G^2+B^2} \\ \theta &= \arctan\left(\frac{G}{R}\right) \\ \phi &= \arcsin\left(\frac{\sqrt{R^2+G^2}}{\sqrt{R^2+G^2+B^2}}\right) \end{cases}$$



Author: Wikipedia

- *Invariance Properties:*
  - longitude and latitude are invariant under global multiplicative illumination changes as well as shadow and shading
  - the radius is no photometric invariant expression

Photometric Invariants (10)

Advantages and Drawbacks of Photometric Invariants

- Advantages
  - exploit multiple measurements at each pixel (colour channels)
  - yield good performance under varying illumination
  - can be written in motion tensor notation

- Drawbacks
  - information loss as prize for the invariance
    (the higher the degree of invariance the more information is discarded)
  - only usable in the context of colour image sequences
    (apart from Log-Derivatives)

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Photometric Invariants (11)

Results for a Variational Method using Different Photometric Invariants

Results for the Street Sequence (Galvin *et al.*). **Upper Row:** Frame 10 and 11 with spatially varying multiplicative illumination, ground truth, Log-Gradient RGB constancy. **Lower Row:** normalised RGB constancy (arith.), normalised RGB constancy (geom.), Hue constancy, $\phi\theta$ constancy. *Author:* Y.Mileva.

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Results for a Variational Method using Different Photometric Invariants

- Qualitative Evaluation for the Street Sequences
  (with spatially varying multiplicative illumination changes)

Technique		AAE
Variational Method	(RGB constancy)	43.44°
Variational Method	**(Hue constancy)**	**4.28°**
Variational Method	**(Log-Gradient RGB constancy)**	**2.89°**
Variational Method	(Gradient RGB constancy)	2.64°
Variational Method	**(geom. normalised RGB constancy)**	**2.26°**
Variational Method	**(arithm. normalised RGB constancy)**	**2.22°**
Variational Method	**($\phi\theta$ constancy)**	**2.07°**

- Photometric invariants still give excellent results in cases where the RGB constancy assumption fails completely. Image derivatives also may work fine.

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Constancy Assumptions without Linearisation

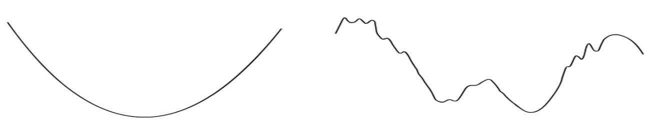
- *Idea:* Postpone the linearisation to the numerical scheme
- *Example:* Consider the grey value constancy assumption without linearisation

$$f(x,y,t) - f(x+u, y+v, t+1) = 0\ .$$

- The corresponding variant of the Horn and Schunck method is given by

$$E(u,v) = \int_\Omega \underbrace{\left(f(x,y,t) - f(x+u,y+v,t+1)\right)^2}_{\text{data term}} + \alpha \underbrace{\left(|\nabla u|^2 + |\nabla v|^2\right)}_{\text{smoothness term}} \, dx\, dy\ .$$

- *New:* This is a **nonconvex** energy functional with **multiple local minima**.



How Can We Minimise A Nonconvex Energy Functional ?

- *Standard:* Minimisation via the associated Euler-Lagrange equations given by

$$\begin{aligned} f_x(x{+}u, y{+}v, t{+}1)\big(f(x{+}u, y{+}v, t{+}1) - f(x,y,t)\big) - \alpha \Delta u &= 0\,, \\ f_y(x{+}u, y{+}v, t{+}1)\big(f(x{+}u, y{+}v, t{+}1) - f(x,y,t)\big) - \alpha \Delta v &= 0 \end{aligned}$$

with (reflecting) Neumann boundary conditions $\mathbf{n}^\top \nabla u = 0$ and $\mathbf{n}^\top \nabla v = 0$.

- *New:* Unlike in the convex case, in the nonconvex case there are
  - **multiple solutions** (for each minimum one)
  - **implicit equations** in u and v (unknown functions as arguments)

We need a suitable minimisation strategy → Warping

The Warping Strategy

- *Idea:* Incremental coarse-to-fine fixed point iteration
  - Fixed point iteration
  - Incremental computation
  - Coarse-to-fine strategy

- *Step 1:* We introduce the following fixed point iteration step

$$\begin{aligned} f_x(x{+}u^k, y{+}v^k, t{+}1)\big(f(x{+}u^{k+1}, y{+}v^{k+1}, t{+}1) - f(x,y,t)\big) - \alpha \Delta u^{k+1} &= 0\,, \\ f_y(x{+}u^k, y{+}v^k, t{+}1)\big(f(x{+}u^{k+1}, y{+}v^{k+1}, t{+}1) - f(x,y,t)\big) - \alpha \Delta v^{k+1} &= 0 \end{aligned}$$

  - **semi-implicit** in the data term (expressions from k and $k+1$)
  - **implicit** in the smoothness term (expressions only from $k+1$)

The Warping Strategy (2)

Incremental Computation and Linearisation

- *Step 2a:* We introduce an incremental computation by the splitting

$$
\begin{aligned}
u^{k+1} &= u^k + du^k \\
v^{k+1} &= v^k + dv^k
\end{aligned}
$$

  - known displacement (u^k, v^k) from old time step k
  - unknown displacement increment (du^k, dv^k) from new time step $k+1$

- *Step 2b:* We linearise the data term with respect to du^k and dv^k

$$
\begin{aligned}
f(x+u^{k+1}, y+v^{k+1}, t+1) &= f(x+u^k+du^k, y+v^k+dv^k, t+1) \\
&= f(x+u^k, y+v^k, t+1) \\
&\quad + f_x(x+u^k, y+v^k, t+1)\, du^k \\
&\quad + f_y(x+u^k, y+v^k, t+1)\, dv^k
\end{aligned}
$$

The Warping Strategy (3)

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Towards a Unified Motion Tensor Notation

- *Step 2c:* We thus obtain the linearised step at fixed point iteration k

$$
\begin{aligned}
0 = f_x(x+u^k, y+v^k, t+1)\big(& f_x(x+u^k, y+v^k, t+1)\, du^k \\
& + f_y(x+u^k, y+v^k, t+1)\, dv^k \\
& + f(x+u^k, y+v^k, t+1) - f(x,y,t)\big) - \alpha\Delta\big(u^k + du^k\big) \\
0 = f_y(x+u^k, y+v^k, t+1)\big(& f_x(x+u^k, y+v^k, t+1)\, du^k \\
& + f_y(x+u^k, y+v^k, t+1)\, dv^k \\
& + \underbrace{f(x+u^k, y+v^k, t+1) - f(x,y,t)}_{\approx f_t}\big) - \alpha\Delta\big(v^k + dv^k\big)
\end{aligned}
$$

This step can be rewritten in **motion tensor notation**

$$
\begin{aligned}
0 &= J_{11}^k\, du^k + J_{12}^k\, dv^k + J_{13}^k - \alpha\Delta u^k - \alpha\Delta du^k \,, \\
0 &= J_{12}^k\, du^k + J_{22}^k\, dv^k + J_{23}^k - \alpha\Delta v^k - \alpha\Delta dv^k \,.
\end{aligned}
$$

These linear equations have a unique solution (related to a **convex problem!**)

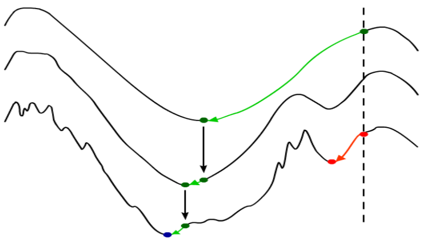
The Warping Strategy (4)

The Unified Motion Tensor Notation

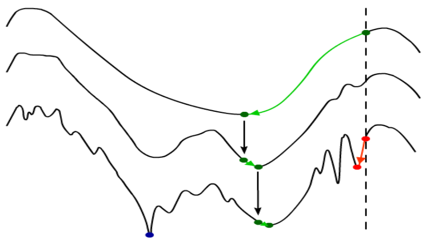
- *New:* Use frames $f(x,y,t)$ and $f(x+u^k, y+v^k, t+1)^\top$ to compute the motion tensor (compensated by the already computed motion) at fixed point step k

$$J^k = \nabla_3 f(x+u^k, y+v^k, t+1)\ \nabla_3 f(x+u^k, y+v^k, t+1)^\top\ .$$

- *Step 3*: Embed fixed point iteration into a coarse-to-fine strategy
  - start from a coarse level and successively refine the resolution at each fixed point iteration (refinement factor η) $\rightarrow$ **avoids large local minima**



global minimum found



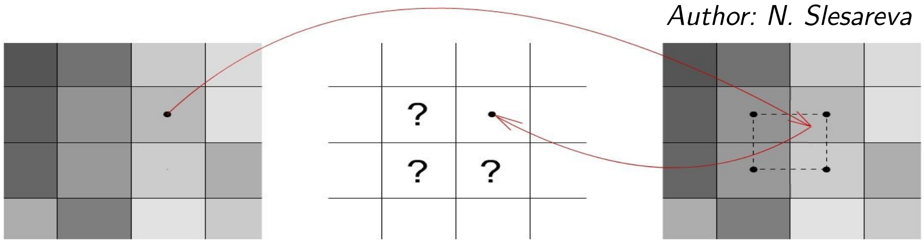
good local minimum found

The Warping Strategy (5)

Backward Registration for Motion Compensation

- Compute $f(x+u^k, y+v^k, t+1)$ by compensating second frame $f(x,y,t+1)$ by already computed motion (u^k, v^k):

Author: N. Slesareva



$f(x,y,t)$ $\quad$ $f(x+u^k, y+v^k, t+1)$ $\quad$ $f(x,y,t+1)$

- Use linear interpolation to determine subpixel values

$$\begin{aligned}
\left[f(x+u^k, y+v^k, t+1)\right]_{i,j} \quad = \quad & (1-\epsilon_u)(1-\epsilon_v)\ f_{(i+\bar{u}),(j+\bar{v}),t+1} \\
& +(\quad \epsilon_u)(1-\epsilon_v)\ f_{(i+\bar{u})+1,(j+\bar{v}),t+1} \\
& +(1-\epsilon_u)(\quad \epsilon_v)\ f_{(i+\bar{u}),(j+\bar{v})+1,t+1} \\
& +(\quad \epsilon_u)(\quad \epsilon_v)\ f_{(i+\bar{u})+1,(j+\bar{v})+1,t+1}
\end{aligned}$$

where $u = \bar{u} + \epsilon_u$ and $v = \bar{v} + \epsilon_v$ with integer displacements $\bar{u}$ and $\bar{v}$ and subpixel displacements ϵ_u and ϵ_v.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

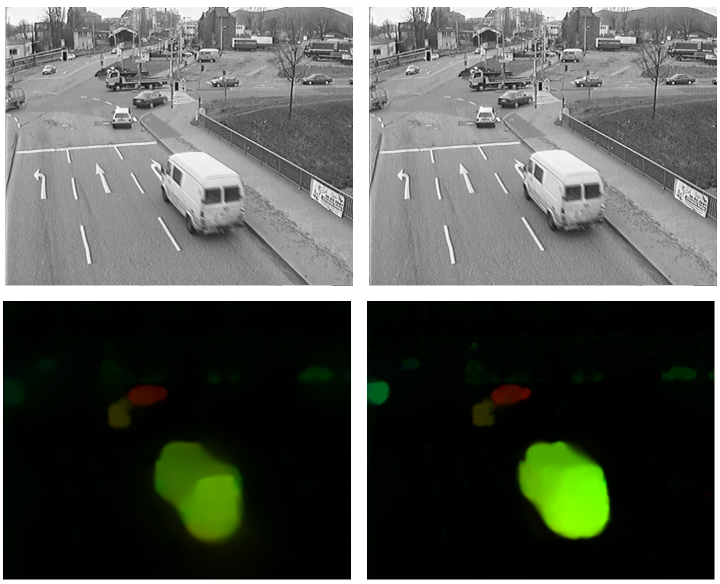
M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

The Summary of the Warping Strategy

- **Nonconvex Optimisation Problem**
  - linearisation of constancy assumptions in the numerical scheme
    ($\rightarrow$ approximation by a **series of convex** optimisation problems)
  - embed solution process in a coarse-to-fine hierarchy
    ($\rightarrow$ avoids local minima)
  - solve convex problem with unique solution et each hierarchy level
- **Warping (Hierarchical Incremental Fixed Point Iteration)**
  - downsample image data, solve problem at coarse scale
  - use this flow field at next finer scale to determine the **difference problem**:
    $\rightarrow$ **warp second image** in order to compensate for this estimated motion
  - solve difference problem (with compensated image data) at finer scale
  - continue until finest scale reached
  - sum up optic flow contributions from all scales

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

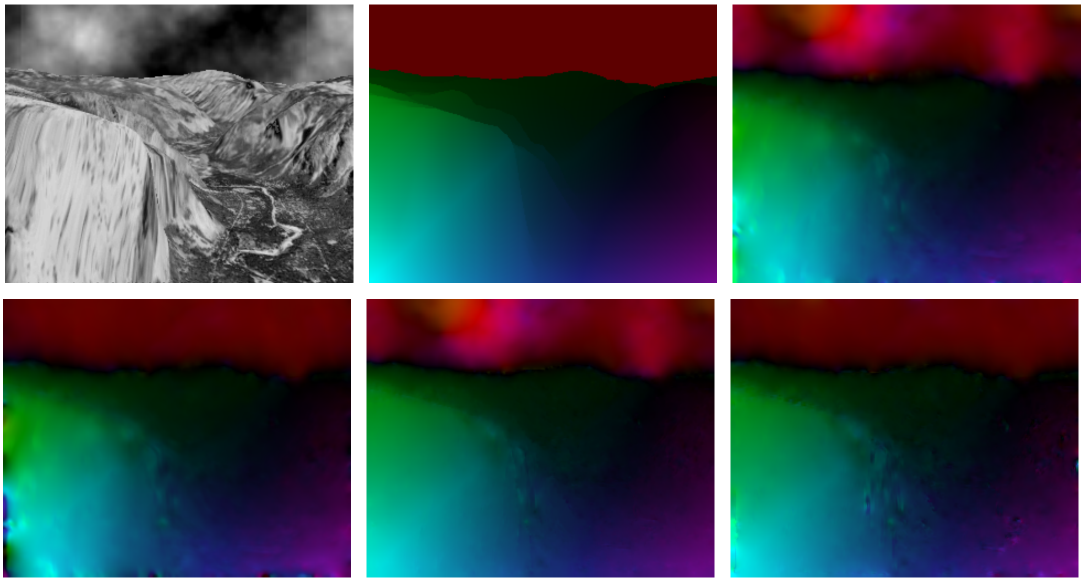
Results for a Variational Approach with Warping



Results for the Rheinhafen sequence (H.-H. Nagel). **(a) Upper Left:** Frame 1130. **(b) Upper right:** Frame 1131. **(c) Lower Left:** Grey value constancy without warping. **(d) Lower Right:** Grey value constancy with warping.

The Warping Strategy (8)

Results for the Horn and Schunck Method with Warping



Results for the Yosemite Sequence with clouds (L. Quam). **(a) Upper Left:** Frame 8. **(b) Upper Center:** Ground truth. **(c) Upper Right:** Grey value constancy ($\sigma = 1.4$, $\alpha = 470$). **(d) Lower Left:** Gradient constancy ($\sigma = 2.1$, $\alpha = 20$). **(e) Lower Center:** Grey value constancy + warping ($\sigma = 0.7$, $\alpha = 560$, $\eta = 0.5$). **(f) Lower Right:** Gradient constancy + warping ($\sigma = 1.5$, $\alpha = 45$, $\eta = 0.5$).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

The Warping Strategy (9)

Results for the Horn and Schunck Method with Warping

- Qualitative Evaluation for the Yosemite Sequence with Clouds

Technique		AAE
Normalised Cross Correlation (NCC)		21.84°
Block Matching + Subpixel (SSD)		21.46°
Bigün et al.		10.60°
Lucas/Kanade		8.79°
Horn and Schunck	(grey value constancy)	7.12°
Horn and Schunck + warping	**(grey value constancy)**	**6.12°**
Horn and Schunck	(gradient constancy)	5.91°
Horn and Schunck + warping	**(gradient constancy)**	**5.14°**

- Combinations of the warping technique with all other concepts are possible (derivatives, photometric invariants, etc.).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Summary

- Suitable constancy assumptions may significantly improve the performance
- Varying illumination can be handled by
  - higher order data terms (for grey value image sequences)
  - photometric invariants (for colour image sequences)
- Large displacements require constancy assumptions without linearisation
  - nonconvex energy functionals (multiple local minima, no uniqueness)
  - sophisticated optimisation techniques (coarse-to-fine warping)
- Motion tensors allow a compact notation as well as a simple implementation
  - for linearised constancy assumptions (in the functional)
  - for original constancy assumptions (in the numerics)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

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  (photometric invariants for the Lucas/Kanade method)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Assignment 4

Theoretical Exercise 1 (Affine Horn and Schunck)

- In order to improve the performance of the method of Horn and Schunck, derive a variant of the original energy functional with affine parameterisation

$$\mathbf{w} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{pmatrix} .$$

  Make use of the motion tensor notation. What are suitable models for the data and for the smoothness term? Be careful not to reinvent the original method in disguise!

- Derive the Euler–Lagrange equations that must be satisfied by each minimiser of the novel energy functional from (a). How many Euler-Lagrange equations are there in total?

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

Assignment 4

Theoretical Exercise 2 (Motion Tensors)

- Derive the entries of the motion tensor for the assumption that the determinant of the Hessian remains constant over time. Are there cases where the aperture problem does not appear?

- Extend the previous motion tensor to RGB colour images. Does the situation with respect to the aperture problem changes compared to the constancy assumption from (a)?