

Correspondence Problems in Computer Vision

Andrés Bruhn

Faculty of Mathematics and Computer Science
Saarland University, Saarbrücken

bruhn@mia.uni-saarland.de
www.mia.uni-saarland.de/bruhn

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www.mia.uni-saarland.de/Teaching/copcv09.shtml

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Organisational Issues (1)

Organisational Issues

Welcome to this class!

Please do not hesitate to pose questions anytime (in English or German).

What Type of Lecture is It?

- ◆ 2 hours classroom lectures, 2 hour tutorials (6 ECTS points)
- ◆ specialised class (Spezialvorlesung) in visual computing and computer science
- ◆ or an applied mathematics class
- ◆ qualifies for starting a master or diploma thesis in our group (in particular in the field of computer vision)

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Organisational Issues (2)



What Prerequisites are Required?

- ◆ undergraduate mathematics (e.g. Mathematik für Informatiker I-III)
- ◆ elementary C knowledge (for the programming assignments)
- ◆ passive knowledge of English (solutions to exams can be submitted in German)
- ◆ specific knowledge on partial differential equations, image processing or computer vision is useful (but not required)

Tutorials

- ◆ programming and theoretical assignments alternating each week
- ◆ assignment can be done in groups of up to three students
- ◆ one group: **Thursday 4-6 PM** (consensus?)
- ◆ supervised by myself
- ◆ start: **next week** → for more details see web page

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Organisational Issues (3)



Grading Policy

- ◆ bilingual oral exam at the end of semester
 - second chance at the end of the teaching-free period only for those who failed or did not attend the first exam
- ◆ in order to qualify for the exam you must
 - regularly attend the tutorials (at least 10 out of 12 tutorials) → tutorials are an essential part of this lecture!

Registration and Lecture Notes

- ◆ please register for the tutorial group as of today, April 29, 4 PM
www.mia.uni-saarland.de/Teaching/copcv.shtml
closing date: Tuesday, May 5, 4 PM
- ◆ you can also find the lecture notes there after each lecture

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Contents and Goals (1)

What is It All About

- ◆ correspondence problems are the *key problems* in computer vision
- ◆ problem: identify corresponding structures in two or more views/images



- ◆ appear in the context of
 - image sequence analysis /tracking
 - stereo vision / structure-from-motion
 - medical image registration
 - particle image velocimetry
 - and many more ...

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Contents and Goals (2)

After this Class You Should Be Able

- ◆ to understand the challenges behind the different correspondence problems
- ◆ to understand the basic strategies for solving them (in particular you should know their advantages and shortcomings)
- ◆ to implement some of these algorithms
- ◆ to design a suitable method for each of the tasks
- ◆ to start a diploma or master thesis in our group

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Literature

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- ◆ A. Bruhn: *Variational Optic Flow Computation - Accurate Modelling and Efficient Numerics*. Ph.D. Thesis. Available from <http://www.mia.uni-saarland.de/bruhn/PhDThesis.pdf>. (*main reference, in particular for image sequence analysis*)
- ◆ O. Faugeras and Q.-T. Luong: *The Geometry of Multiple Images*. MIT Press, 2001. (*nice book on projective geometry and stereo reconstruction*)
- ◆ J. Modersitzki: *Numerical Methods for Image Registration*. Oxford Press, 2003. (*comparison of methods for medical image registration*)
- ◆ O. Scherzer (Ed.): *Mathematical Models for Registration and Applications to Medical Imaging*. Series on Mathematics in Industry, Vol. 10, Springer, New York, 2006. (*collection of papers on image registration techniques with one chapter on optic flow methods*)
- ◆ Many articles from journals and conferences

Main books will be available at the computer science / applied mathematics library. Each lecture provides specific references. Please ask me if you have no access to them.

Correspondence Problems in Computer Vision
Andrés Bruhn, Lecture 1

Lecture 1:

An Overview of
Correspondence Problems in Computer Vision

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Contents

1. What are Correspondence Problems?
2. Visualisation Aspects
3. Optical Flow Estimation
4. Stereo Matching
5. Medical Image Registration
6. Particle Image Velocimetry
7. Specifications/Desired Properties
8. Summary

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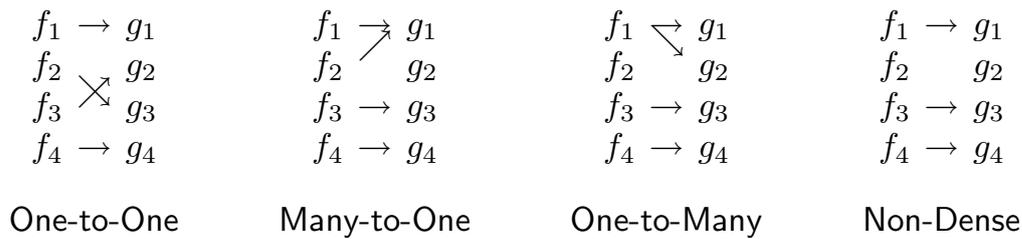
What are Correspondence Problems?

Given: Two sets of entities (e.g. $F = \{f_1, \dots, f_N\}$ and $G = \{g_1, \dots, g_N\}$).

Wanted: Correspondences *between* entities of the two sets.

Matching Scenarios

- Most prominent examples



- Depending on the applied matching criteria the same two sets can produce different correspondences. Given a certain task which are the best criteria?

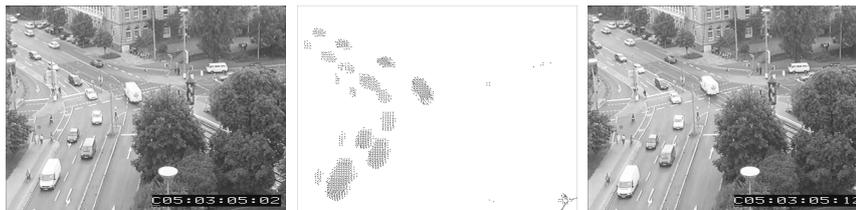
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Relation to Computer Vision

If we interpret images as sets of pixels, we obtain a computer vision problem:

Given: Two images $\mathbf{f} = (f_{1,1}, \dots, f_{N,M})^\top$ and $\mathbf{g} = (g_{1,1}, \dots, g_{N,M})^\top$

Wanted: Displacement field $\mathbf{u} = (\mathbf{u}_{1,1}^\top, \dots, \mathbf{u}_{N,M}^\top)^\top$ between corresponding pixels in both images, where $\mathbf{u}_{i,j} = (u_{i,j}, v_{i,j})^\top$.



- Problem:** Since pixel locations are discrete but the real world is continuous, the actual displacements are not restricted to integer values.

Visualisation Aspects

How to Visualise Displacement Vector Fields?

Given: Vector field $\mathbf{u} = (\mathbf{u}_{1,1}^\top, \dots, \mathbf{u}_{N,M}^\top)^\top$ with $\mathbf{u}_{i,j} = (u_{i,j}, v_{i,j})^\top$

Vector Plot

- ◆ *Idea:* Visualise displacement vectors $\mathbf{u}_{i,j}$ directly



Pro: - shows both direction and magnitude of vector

Con: - requires subsampling of vector field (only each n -th vector shown)

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Magnitude Plot

- ◆ *Idea:* Visualise magnitude $|\mathbf{u}_{i,j}| = \sqrt{u_{i,j}^2 + v_{i,j}^2}$ of each vector as grey value



Pro: - shows magnitude for **each** displacement vector

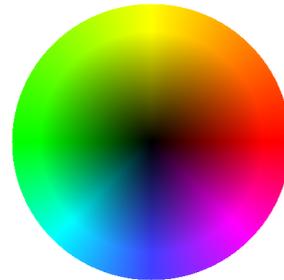
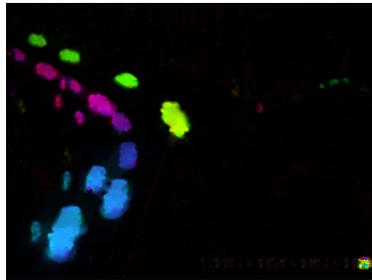
Con: - discards directional information

How can we combine the advantages of vector and magnitude plots?

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Colour Plot

- ◆ *Idea:* Visualise direction as colour and magnitude as brightness
 → Use polar coordinates for $\mathbf{u}_{i,j}$ with radius $r_{i,j}$ and angle $\phi_{i,j}$



Pros: - shows both direction and magnitude for each displacement vector

Cons: - difficult to interpret exactly

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Optic Flow Estimation

What is the Optic Flow Problem?

Given: Two consecutive images of an image sequence (→ one camera, different time).

Wanted: Motion field between both frames.

Example for Optic Flow Estimation



Dancing Lecturer Sequence. (a) Left: Frame 1. (b) Centre: Optic Flow. (c) Right: Frame 2.

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Static Camera

- ◆ Traffic Sequence Analysis
(Papenberg/Bruhn/Brox/Didas/Weickert 2006)



Karl Wilhelm Straße (H.-H. Nagel)



Ettlinger Tor (H.-H. Nagel)

- ◆ *Problem:* In general, the motion in image sequences can be arbitrary. If the type of the motion is known, additional constraints can be incorporated.

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Moving Camera

- ◆ Camera Mounted on a Moving Vehicle
(Mileva/Bruhn/Weickert 2007)



DIPLODOC image sequence. (a) Left: Frame 205. (b) Right: Frame 207.

- ◆ *Problem:* In outdoor sequences illumination changes are a frequent source of problems. Moreover, shadows and specular reflections may occur.

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Fields of Application

- ◆ Robot navigation
- ◆ Driver assistance systems
- ◆ Video compression
- ◆ Surveillance/Tracking
- ◆ Structure-from-motion

Challenges in Video Sequences

- ◆ Arbitrary motion
- ◆ Varying illumination (e.g. automatic brightness adjustment)
- ◆ Shadows and shading
- ◆ Camera noise
- ◆ Occlusions
- ◆ Appearing and disappearing objects

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Stereo Matching

What is the Stereo Problem?

Given: Two different views of the same scene (→ two cameras, same time).

Wanted: Displacement field between both views (disparity map).

Example for Stereo Matching



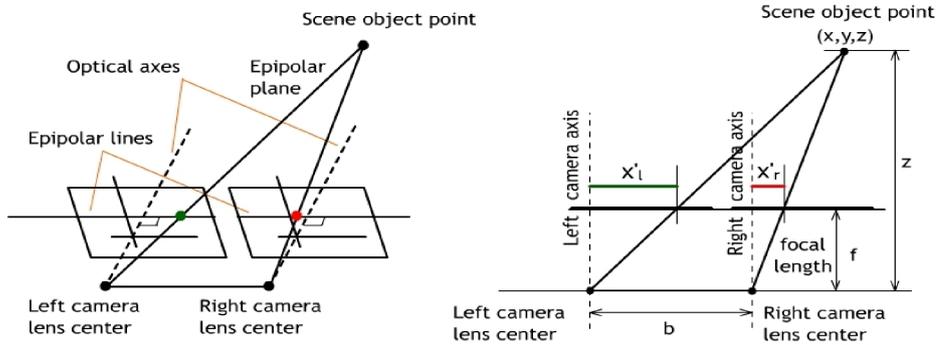
Javier Sánchez stereo pair. (a) **Left:** Left image. (b) **Centre:** Disparity map. (c) **Right:** Right image.

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Stereo Matching (2)

From Matching to Reconstruction

- ◆ Relation between Disparity and Depth



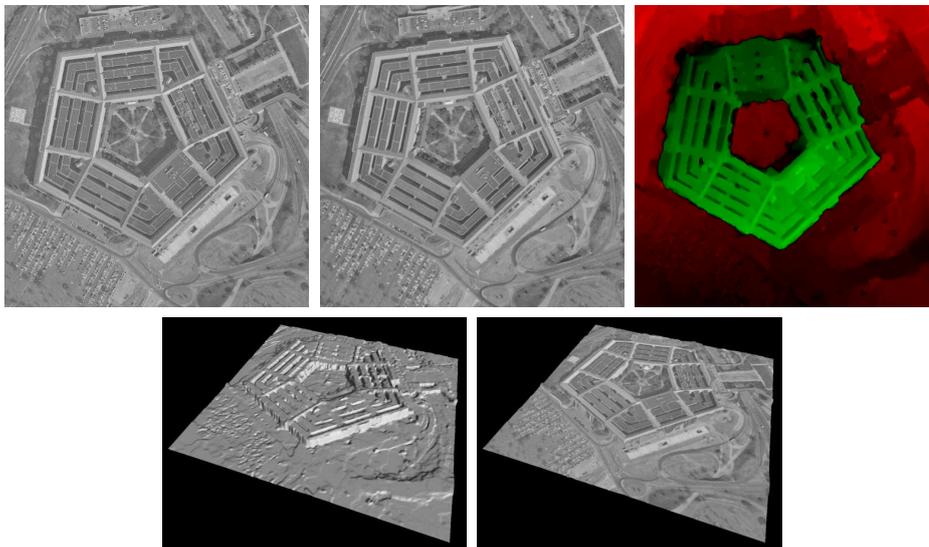
(a) Left: Ortho-parallel setting (3-D view). (b) Right: Top view. Author: N. Slesareva (2005).

- ◆ *Problem:* Displacements in stereo matching cannot be arbitrary like in motion estimation. They have to satisfy certain geometric constraints.
- ◆ *Attention:* Stereo matching is an essential step for stereo reconstruction. The reconstruction itself, however, is more closely related to computer graphics.

Stereo Matching (3)

Reconstruction Example I

- ◆ Reconstruction of the Pentagon
(Slesareva/Bruhn/Weickert 2005)



Pentagon image pair (CMU database). (a) Top Left: Left frame. (b) Top Right: Right frame. (c) Bottom Left: Reconstruction with illumination. (d) Bottom Right: Reconstruction with texture.

Reconstruction Example II

- ◆ Face Reconstruction
(Valgearts/Bruhn/Weickert 2008)



Pascal Gwosdek image pair. (a) **Left:** Left frame. (b) **Centre:** Right frame. (c) **Right:** Reconstruction.

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Fields of Application

- ◆ Face recognition
- ◆ Free viewpoint stereo
- ◆ Archeology
- ◆ 3-D map generation
- ◆ Virtual reality
- ◆ Robot navigation

Challenges in Stereo Matching

- ◆ Very large displacements (wide baseline stereo)
- ◆ Rigid body motion (due to change of viewpoint)
- ◆ Multi-view integration
- ◆ Occlusions
- ◆ Camera calibration (if position and orientation of cameras is not known)

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Medical Image Registration

What is the Medical Image Registration Problem?

Given: Two medical images from the same or different modalities (e.g. CT, MRT).

Wanted: Realistic deformation field that allows to compare/combine information.

Example for a Typical Medical Image Registration Task



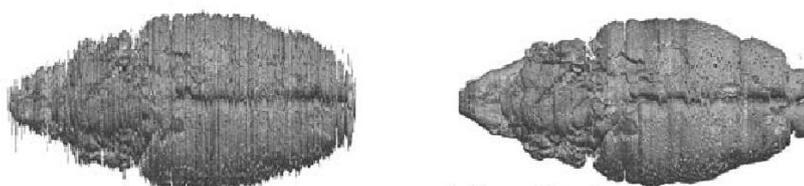
CT images of a human hand. **(a) Left:** Template Image. **(b) Centre:** Source image. **(c) Right:** Matched source image. *Author:* M. Martin-Fernández (2005).

Volume Reconstruction from Histologic Slices

- ◆ Registration of Histologic Slices
(Schmitt/Modersitzki/Heldmann/Wirtz/Fischer 2006)



(a) Left: Workplace. **(b) Centre:** Section. **(c) Right:** Slice scanner. *Author:* O. Schmitt (2003).



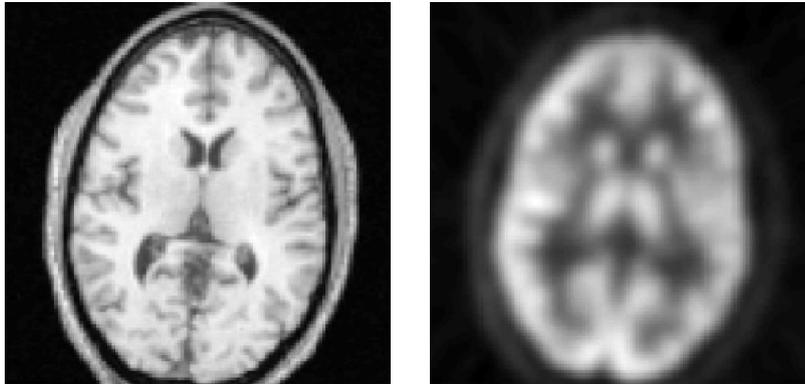
Rat brain. **(a) Left:** Before registration. **(b) Right:** After registration. *Author:* O. Schmitt (2003).

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Combining Information from Different Image Acquisition Methods

- ◆ Registration of MRT and PET Scans



Images of a human brain. (a) **Left:** MRT scan. (b) **Right:** PET scan. *Author:* W.-H. Liao (2003).

- ◆ *Problem:* How to register images that visualise completely different properties of the same object. Is there some common information that still allows registration?
- ◆ Specific constrains from information theory and elasticity theory available.

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Fields of Application

- ◆ Radiotherapy (e.g. tumor therapy, improved diagnosis)
- ◆ Pathology/Histology (e.g. slice registration)

Challenges in Medical Image Data

- ◆ Large displacements
- ◆ Deformations constrained to elastic motion
- ◆ Intensity differences
- ◆ Images may be created by different image acquisition methods
- ◆ Heavy noise (e.g. speckle noise in ultrasound images)
- ◆ Manually set displacements (landmarks) have to be respected

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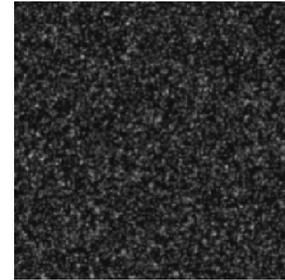
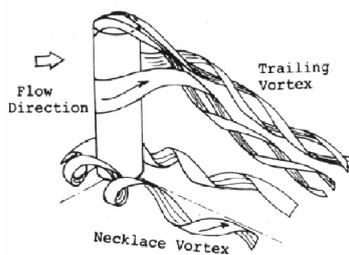
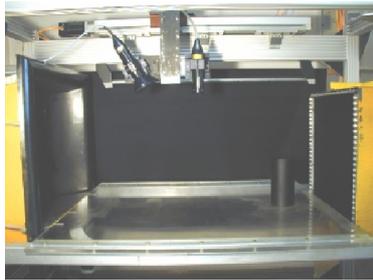
Particle Image Velocimetry

What is the PIV Problem?

Given: Two artificially created particle images from fluid dynamics.

Wanted: Complex motion field with turbulent flows.

How Are PIV Images Obtained?

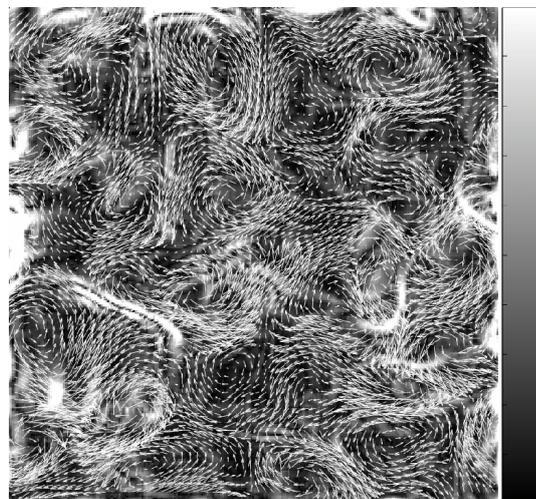
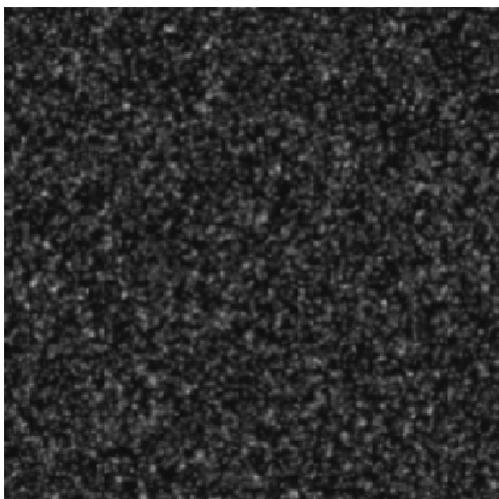


(a) **Left:** Experimental setting for studying the flow around a cylinder. (b) **Centre:** Schematic illustration of flow phenomena. (c) **Right:** Typical PIV image. *Authors:* P. Ruhnau (2006), T.Kawamura (1984).

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Fluid Dynamics

- ◆ Complex Flow Fields with Vortices
(*Ruhnau/Stahl/Schnörr 2006*)



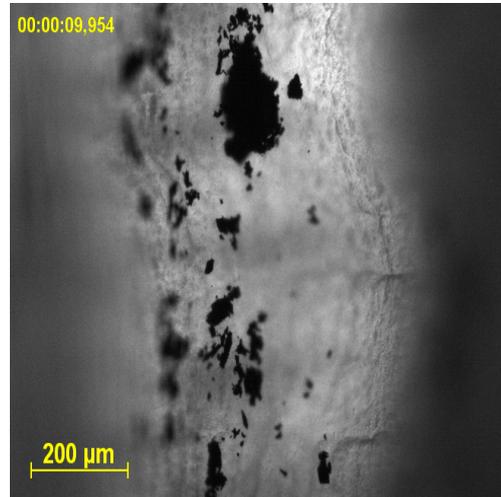
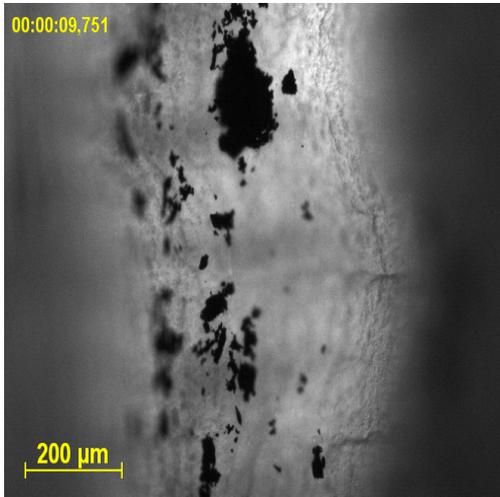
PIV Estimation. (a) **Left:** PIV Image. (b) **Right:** Corresponding flow field.

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Particle Image Velocimetry (3)

Pharmaceutical Technology

- ◆ Nano-Particles in Tracheal Tissue
(Kariger/Bruhn/Henning/Weickert/Lehr 2008)



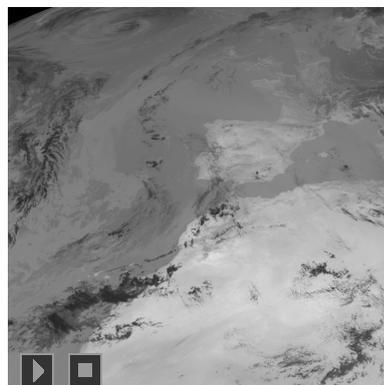
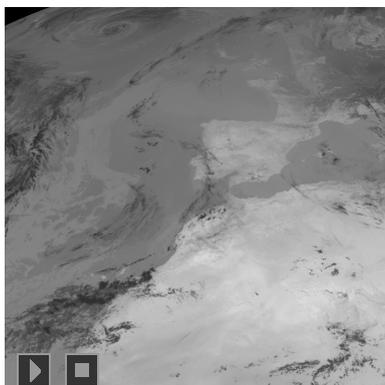
Analysis of Mucociliary Clearance. (a) Left: First Image. (b) Right: Second Image.

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Particle Image Velocimetry (4)

Climate Analysis

- ◆ Meteorologic Data from the Canary Islands
(Alvarez/Castano/Mazorra/Krissian/Gracia/Salgado/Sanchez 2007)



Estimation of Cloud Motion over the Canary Islands. (a) Left: Frame 1. (b) Right: Frame 9.

- ◆ Problem: Motion estimation from PIV images requires constraints that are in accordance with the underlying physical laws (e.g. Navier-Stokes equations).

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Particle Image Velocimetry (5)

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Fields of Application

- ◆ Aerodynamics (e.g. aeronautics, car industry)
- ◆ Fluid dynamics (e.g. process engineering, shipbuilding, biology)
- ◆ Thermodynamics (e.g. machine/chemical engineering)
- ◆ Meteorology (e.g. climate analysis, weather forecast)

Challenges in PIV Data

- ◆ Small particles but large displacements
- ◆ Non-rigid motion (guided by physical laws)
- ◆ Complex motion patterns such as vortices
- ◆ Incompressibility constraints

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Specifications/Desired Properties (1)

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Specifications

What characterises an algorithm that solves a correspondence problem?

- ◆ **Constraints on the data:**
→ Which types of features are used? (e.g. pixels, neighbourhoods, corners)
- ◆ **Constraints on the solution:**
→ Which types of transformations are allowed? (e.g. translations, rotations)
- ◆ **Cost function:**
→ How are deviations from the assumptions penalised?
- ◆ **Optimisation strategy:**
→ How is the optimal solution found? (e.g. global/local minimisation, greedy)

All these specifications have to be adapted w.r.t. a given correspondence problem. The better they are adapted, the more precise the solution will be.

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Desired Properties

What properties of a matching algorithm are desirable?

◆ Guarantees

- for a unique solution (optimal solution w.r.t. to assumptions can be found)
- for the stability of the algorithm (no critical configurations)

◆ High Quality

- by providing dense flow fields (one displacement per pixel)
- by integration of all available prior knowledge (e.g. specific constraints)
- by correct estimation of large displacements (e.g. multiscale strategies)
- by sub-pixel accuracy (results not restricted to integer values)
- by rotational invariance (algorithm does not favour any directions)

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Desired Properties (continued)

◆ Robustness

- under noise (e.g. Gaussian noise, salt-and-pepper noise)
- under varying illumination (e.g. additive and multiplicative changes)
- under model parameter variations (solution should change only slightly)
- with respect to appearing, occluded or vanishing objects

◆ Real-Time Performance

- by efficient numerical schemes (e.g. by multigrid techniques)
- by parallelisation strategies (e.g. by domain decomposition methods)

◆ Extensibility

- to multiple input images (e.g. multi-view stereo)
- to multi-channel image data (e.g. RGB colour images)

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Desired Properties (continued)

◆ Confidence Measures

- for the reliability of the data (a-priori confidence measures)
- for the reliability of the solution (a-posteriori confidence measures)

Evidently, there are many nice properties. In the ideal case all of them are fulfilled. Since the ideal case does not exist, one tries to include as many of them as possible.

Most of these properties will be addressed in this lecture.

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Summary

- ◆ Correspondence problem appear frequently in computer vision
- ◆ Each application requires own adaptations/strategies
- ◆ Depending on the application prior knowledge can/must be integrated
 - simplifies matching of corresponding points
 - prevents from finding wrong solutions
- ◆ Algorithms for finding correspondence are characterised by
 - the features they use
 - the type of transformation they allow
 - the cost function that is applied to rate each possible correspondence
 - the optimisation strategy they rely on to find the optimal solution (given the three previous specifications)
- ◆ There are many desirable properties that should be additionally addressed

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