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Summer Term 2009

Example Questions for Correspondence Problems in Computer Vision

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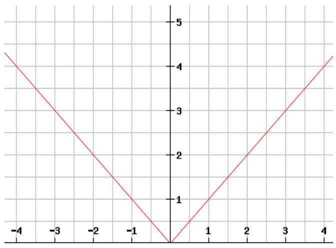
1. Explain the idea behind robust data terms and sketch a function that could be used for robustification. Why should separate robustification be applied in the case of two non-correlated (independent) constancy assumptions?

(2 + 1 + 1 p.)

▶ *Robust data terms use sub-quadratic penaliser functions to reduce the influence of outliers on the overall results (compared to the standard quadratic setting). In the Euler-Lagrange equations they lead to a downweighting of those locations where the constancy assumptions are violated and the smoothness term takes over.*

▶ *A typical sub-quadratic function used for the robustification of the data term is the regularised linear penaliser* $\Psi(s^2) = \sqrt{s^2 + \varepsilon^2}$ *with* ε *being very small.*



▶ *Since in the case of independent constancy assumptions some may be fulfilled while others are violated, only those assumptions that are actually violated should be weighted down in the Euler-Lagrange equations. This behaviour is realised by the concept of separate robustification.*

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2. Let a colour sequence $\mathbf{f}(x, y, t) = (R(x, y, t), G(x, y, t), B(x, y, t))^\top$ be given, where $(x, y)^\top$ is the location within a rectangular image domain Ω and $t \geq 0$ denotes time. You know that typical image sequences for your application contain shadow and shading, as well as multiplicative changes of the overall intensity. Moreover, you expect large displacements. Write down an energy functional to compute the optic flow $(u(x, y, t), v(x, y, t))^\top$ that addresses the aforementioned problems. Use a discontinuity-preserving smoothness term of your choice. Why is an image-driven smoothness terms not recommendable in this context? Explain!

(4 + 1 p.)

▶ *Let us first derive a suitable invariant with respect to global multiplicative changes (overall illumination) and local multiplicative changes (shadow/shading). Such an expression is for instance given by the ratio between two of the given colour channels such as*

$$p = \frac{R}{G} \; .$$

▶ *If we know that large displacement will occur, we should use the corresponding data term without linearisation. Moreover, since robustness is not required, we apply the standard quadratic regulariser. Thus, we obtain the data term*

$$\Big(p(x+u, y+v, t+1) - p(x,y,t)\Big)^2 \; .$$

▶ *As discontinuity-preserving smoothness term of our choice we select an isotropic flow-driven regulariser*

$$\Psi\Big(|\nabla u|^2 + |\nabla v|^2\Big) \; ,$$

where $\Psi(s^2) = \sqrt{s^2 + \varepsilon^2}$ *with* $\varepsilon = 10^{-3}$.

▶ *Combining all terms we obtain the following energy functional to compute the desired flow* $(u(x,y,t), v(x,y,t))^\top$*:*

$$E(u,v) = \int_\Omega \Big(p(x+u, y+v, t+1) - p(x,y,t)\Big)^2 + \alpha \; \Psi\Big(|\nabla u|^2 + |\nabla v|^2\Big) \; dx \; dy \; .$$

▶ *In the context of local multiplicative illumination changes, intensity edges may occur due to shadow boundaries. Image-driven methods would erroneously respect such edges and reduce the smoothness at those locations.*

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3. What are the Essential matrix and the Fundamental matrix describing? How are they related mathematically? Name one property that they share and one property that is different for both of them.

(2 + 1 + 2 p.)

▶ *While the Fundamental matrix describes the relation between two views in homogeneous 2-D pixel coordinates, the Essential matrix describes the relation between two views in 2-D homogeneous camera coordinates.*

▶ *The Fundamental matrix* $\mathcal{F}$ *and the Essential matrix* $\mathcal{E}$ *are related via the following equation*

$$\mathcal{F} = \mathcal{A}_2^{-\top} \mathcal{E} \mathcal{A}_1^{-1} \; ,$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are the intrinsic matrices of the first and the second camera, respectively.

▶ *One can select one of the following properties that are shared by both matrices:*

- *both matrices have size* 3×3
- *both matrices are singular*
- *both matrices have rank 2*
- *both matrices are only defined up to a scaling*

One can select one of the following properties that are different for both matrices:

- *two different vs. two identical singular values*
- *seven vs. five degrees of freedom*
- *intrinsic parameters required/not required*

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4. What is a joint probability density function? How can it be computed? What can it be used for?

(2 + 1 + 1 p.)

▶ *A joint probability density function describes for all possible values a and b of two signals/images the probability that we observe in a first signal/image a value a and at the same location in the second signal/image the value b (cooccurrence of the two events).*

▶ *It can be computed by counting the coocurences of all events, normalising the resulting joint histogram and smoothing it with a predefined Gaussian kernel (to obtain a continuous formulation).*

▶ *In medical image registration it can be used to compute the mutual information - a statistical measure between two grey value distributions that allows to register images from different image acquisition methods.*

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5. Name two smoothness terms for variational methods based on second-order-regularization and state their field of application. Write down the formula for one of them and explain the underlying concept.

(2 + 2 + 1 + 1 p.)

▶ *The two second-order smoothness terms discussed in the lecture are given by:*

- *Curvature-based regularisation*
- *Second-order div-curl regularisation*

▶ *The corresponding fields of application are given by:*

- *Medical image registration*
- *Particle image velocimetry*

▶ *One can select one of the following two definitions:*

- *curvature-based regularisation:*

$$S(u,v) = \alpha \left(|\Delta u|^2 + |\Delta v|^2) \right) \ .$$

- *second-order div-curl regularisation:*

$$S(u,v) = \alpha \left| \nabla \mathrm{div} \begin{pmatrix} u \\ v \end{pmatrix} \right|^2 + \beta \left| \nabla \mathrm{curl} \begin{pmatrix} u \\ v \end{pmatrix} \right|^2 \ ,$$

or equivalently:

$$S(u,v) = \alpha \left| \nabla \nabla \cdot \begin{pmatrix} u \\ v \end{pmatrix} \right|^2 + \beta \left| \nabla \nabla \times \begin{pmatrix} u \\ v \end{pmatrix} \right|^2 \ .$$

▶ *One can select the corresponding explanation:*

- *The curvature-based regulariser considers all flow fields to be smooth that have the following (affine) parameterisation:*

$$\begin{aligned} u &= \; = s_1 xy + ax + by + c \ , \\ v &= \; = s_2 xy + dx + ey + f \ , \end{aligned}$$

- *The second-order div-curl regulariser assumes the divergence*

$$\mathrm{div} \begin{pmatrix} u \\ v \end{pmatrix} = u_x + v_y$$

and the vorticity

$$\mathrm{curl} \begin{pmatrix} u \\ v \end{pmatrix} = v_x - u_y$$

to be smooth, i.e. it allows constant rotations and a constant change of the divergence by means of sinks and sources.

6. Check which of the following statements A–F are true, and which are false.

(1 p. for each correct "true" or "false" answer, -1 p. for each incorrect "true" or "false" answer, 0 p. for each unanswered statement. Negative total numbers of points are adjusted to 0*.)*

A: The spatiotemporal variant of the method of Lucas and Kanade requires to solve a 3×3 linear system of equation in each point.

▶ **False.** *As in the spatial case, one has to solve a* 2×2 *linear system of equations. However, the integration domain of the structure tensor entries is spatiotemporal.*

B: Multigrid methods are based on correction steps using the residual equation.

▶ **True.** *This statement is correct.*

C: In contrast to image-driven regularizers, flow-driven smoothness terms yield oversegmentation artifacts in highly textured regions.

▶ **False.** *Image-driven smoothness terms are prone to artifacts in highly textured regions. Flow-driven smoothness terms, however, do not use any image information and thus do not suffer from this drawback.*

D: The epipolar constraint relies on the assumption that the baseline and the two optical rays form an orthogonal system.

▶ **False.** *The epipolar constraint assumes that the baseline and the two optical rays are coplanar. Consequently, at most two of them can be orthogonal but not all three.*

E: Mutual information cannot be used to robustify variational optical flow methods with respect to global additive illumination changes.

▶ **False.** *Global additive illumination changes result in a constant shift of all grey values (shift of the histogram). Moreover, all mappings are unique. Consequently, such changes can be dealt with very well using mutual information.*

F: Integrating the incompressibility constraint (divergence-free-constraint) into a variational method yields non-linear Euler-Lagrange equations per construction.

▶ **False.** *The incompressibility constraint is a linear combination of first order derivatives of the flow (*$v_x - u_y = 0$*). Consequently, the resulting Euler-Lagrange equations can be linear, if a quadratic penaliser function is used.*