# Numerical Algorithms for Visual Computing II

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## Assignment 9 (2 Exercises) - Pre-Condition Zero

### Exercise No. 1 – Getting symmetric (5+5 points)

The reverse Gauß-Seidel method is defined for

$$M_{\text{RGS}} := -(D+U)^{-1}L \quad (1)$$
$$N_{\text{RGS}} := (D+U)^{-1}. \quad (2)$$

1. Combine the standard and reverse Gauß-Seidel method in such a way that you receive a symmetric Gauß-Seidel method. Give the resulting matrices $M_{SGS}$ and $N_{SGS}$!

   Hint: Do an intermediate iteration step with standard Gauss-Seidel

   $$\tilde{x}_{m+1} = M_{\text{GS}}x_m + N_{\text{GS}}b \quad (3)$$

   and then use this result in the reverse Gauss-Seidel method, i.e.

   $$x_{m+1} = M_{\text{RGS}}\tilde{x}_{m+1} + N_{\text{RGS}}b \quad (4)$$

2. Derive analogously a symmetric SOR method (SSOR), i.e. a symmetric Gauss-Seidel method with an additional $\omega$-Parameter in the form

   $$x_{m+1} = M_{\text{SGS}}(\omega)x_m + M_{\text{SGS}}(\omega)b. \quad (5)$$

   Consider the fact, that the choice $\omega = 1$ gives you again the symmetric Gauss-Seidel method.

### Exercise No. 2 – Preposterous in Hilbert's space (2+3+5+10 points)

Consider a standard problem of numerical analysis. A Hilbert matrix $A \in \mathbb{R}^{n\times n}$ is defined as

$$A = (a_{i,j})_{i,j=1}^{n} \quad (6)$$

with

$$a_{i,j} := \frac{1}{i+j-1} \qquad 1 \le i \le n\,; 1 \le j \le n. \quad (7)$$

Also given is a vector $b \in \mathbb{R}^n$ that is defined as

$$b_i = \sum_{j=1}^{n} a_{ij} \qquad 1 \leq i \leq n. \tag{8}$$

In the following you should solve the equation $Ax = b$.

Take into account the following. For a real, symmetric matrix, the condition number can be determined by $\lambda_1/\lambda_n$, where $\lambda_1$ and $\lambda_n$ are the largest and smallest eigenvalue, respectively.

1. What is the real solution of this equation? Compute by hand!

2. Determine the condition numbers for different Hilbert matrices of size $2, 3, 5, 10, 15, 25$. What effect would you expect from an iterative process if you would solve the given problem for an arbitrarily large size $n$?

3. Solve the linear system of equations described by $Ax = b$ by use of the CG-method for sizes $2, 3, 5, 10, 15, 25$. Compute the error of your result with respect to the real solution in the maximum norm.

4. Solve the linear system of equations described by $Ax = b$ by use of the PCG-method for sizes $2, 3, 5, 10, 15, 25$. Use the Preconditioning with the symmetric Gauß-Seidel method that you derived in exercise 1. Compute the error of your result with respect to the real solution in the maximum norm.