Numerical Algorithms for Visual Computing II

PD Dr. Michael Breuß and M.Sc. Kai Uwe Hagenburg

Released: 20.01.2009
Assigned to: Tutorial at 26.01.2009

Assignment 7 (2 Exercises) - The X-file amidst the assignments

Exercise No. 1 – Linear $\overset{Paranormal..?}{\textbf{Parabolic}}$ Stuff (10 points)

Prove remark (iii) from paragraph §12.1: You have to show that for the diffusion tensor $D = I$, where I is the 2×2 identity matrix, the use of (12.14)-(12.16) within an Euler forward scheme with $\Delta x = \Delta y$ and $t = n\delta t$ yields the standard difference formula

$$\begin{aligned} u_{i,j}^{n+1} = u_{i,j}^{n} \quad &+ \quad \frac{\delta t}{\Delta x^2}(u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}) \\ &+ \quad \frac{\delta t}{\Delta y^2}(u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}). \end{aligned}$$

To this end, you may assume that $u_{i,j}$ is a central pixel of an image with an image size larger than 3×3 (i.e., you do not have to consider boundary conditions).

Exercise No. 2 – Creepy nonlinear $\overset{Paranormal..?}{\textbf{Parabolic}}$ Phenomenae (15+5 points)

Consider the Perona-Malik-model for nonlinear isotropic diffusion filtering

$$u_t \quad = \quad \text{div}(D\nabla u) \tag{1}$$

where D is a structure tensor of the form $D = g(|\nabla u|^2)I$, where I is the 2×2 identity matrix, with a diffusivity function

$$g(s^2) \quad = \quad \frac{1}{1 + \frac{s^2}{\lambda^2}}. \tag{2}$$

For our computations, you may assume that $\lambda = 1$ in the following.

1. Construct a numerical solver for the Perona-Malik-model. Thereby, follow the construction steps as described in §12. Show in detail the steps you take.
2. Define numerical boundary conditions that ensure that the average grey value of the image is conserved.