Numerical Algorithms for Visual Computing II

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Released: 17.11.2008
Assigned to: Tutorial at 27.11.2008

Assignment 3 (2 Exercises) – Analytic Stuff

Exercise No. 1 – The 1-D Spectacle

In this exercise we consider 1-D boundary value problems.

(a) We consider the 1-D Laplace equation over the interval $I := (0,1)$ within the BVP

$$\begin{cases} u''(x) &= 0\,, \quad x \in I\,, \\ u(0) &= k_1\,, \\ u(1) &= k_2\,. \end{cases} \tag{1}$$

Compute the exact solution. **(5 pts)**

(b) Do the following problems seem to be reasonable? Justify your answer.

(b1) $u'(x) = 1$ for $x \in (0,1)$, $u(0) = 2$ **(1 pt)**

(b2) $u'(x) = 3$ for $x \in (1,2)$, $u(1) = 4$, $u(2) = 7$ **(1 pt)**

(b3) $u''(x) = 0$ for $x \in (1,2)$, $u(1) = 4$ **(1 pt)**

(b4) $u''(x) = 0$ for $x \in (1,2)$, $u(1) = 4$, $u'(1) = 1$ **(1 pt)**

(b5) $u''(x) = 0$ for $x \in (1,2)$, $u(1) = 4$, $u'(1) = 1$, $u''(1) = 1$ **(1 pt)**

Exercise No. 2 – Connecting analytical and numerical stuff

Consider the following problem:

$$\text{Find } u(x) \text{ minimising} \quad \int_0^1 \left[u'(x)\right]^2 \, dx \quad \text{for} \quad u(0) = u_L\,, u(1) = u_R\,. \tag{2}$$

(a) Discretise u' by use of forward differences, and derive a discrete formulation of the integral, making use of the discrete values $u_0, \ldots, u_{N+1}$. (Hint: This gives a sum.) **(5 pts)**

(b) Setting the first derivatives with respect to the unknowns $u_1, \ldots, u_N$ to zero, a necessary condition for a minimiser of the sum is obtained. Compute the arising system of equations. (Hint: This means to compute N equations, one for each partial derivative $\partial/\partial u_k$, $k = 1, \ldots, N$.) **(5 pts)**

(c) Does the system from (b) have a unique solution? For answering, use the system matrix. **(5 pts)**

(d) The inner part of the computational domain is determined by the indices $1, \ldots, N$. Can you relate for these indices the condition from (b) to a differential equation? Is this differential equation reasonable for the original purpose? Give a detailed discussion. **(5 pts)**