Numerical Algorithms for Visual Computing II

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Assignment 1 (4 Exercises) – Oh, PDEs

Exercise No. 1 – Types of PDEs (4×2=8 points)

Consider a second-order PDE in two variables as an equation of the form

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G,$$

where A, B, C, D, E, F, G can be constants or given functions of x and y. It can be shown that parabolic equations satisfy the property $B^2 - 4AC = 0$, hyperbolic equations $B^2 - 4AC > 0$ and elliptic equations $B^2 - 4AC < 0$.

Categorise the following differential equations with respect to the order, (non-) linearity and the type of the PDE:

- $u_t = u_{xx}$
- $u_{tt} = u_{xx}$
- $u_{xx} + u_{yy} = 0$
- $xu_x + yu_y + u^2 = 0$

Exercise No. 2 – Taylor expansions and difference schemes (4+4=8 points)

Compute

- the Taylor expansion,
- the local truncation error

for an approximation of the second derivative approximation of u, i.e. $u''(x)$, by making use (only) of the mesh points $(j+2)\Delta x$, $j\Delta x$ and $(j-2)\Delta x$.

Exercise No. 3 – Big $\mathcal{O}$ I (4×2=8 points)

Let $h \in \mathbb{R}$ and $p, q \in \mathbb{N}$. Prove the validity (or show the non-validity) of the following assertions, also assuming $p < q$:

$$\mathcal{O}(h^p) + \mathcal{O}(h^q) = \mathcal{O}(h^p), \quad (1a)$$
$$\mathcal{O}(h^p) \cdot \mathcal{O}(h^q) = \mathcal{O}(h^{p+q}), \quad (1b)$$
$$\mathcal{O}(h^p) - \mathcal{O}(h^p) = \mathcal{O}(h^p), \quad (1c)$$
$$\frac{1}{\mathcal{O}(h^p)} \neq \mathcal{O}\left(\frac{1}{h^p}\right). \quad (1d)$$

Exercise No. 4 – Big $\mathcal{O}$ II (3×2=6 points)

Let the following functions be given, with $h \in \mathbb{R}$.

$$
\begin{aligned}
a(h) &= h + h^2 + 10^{20} h^3 \\
b_1(h) &= h + h^2 + 10^{20} h^3 + 10^{-100} h^4 \\
b_2(h) &= -h - h^2 + 10^{20} h^3 + 10^{-100} h^4.
\end{aligned}
$$

Write down the results of the following operations in terms of $\mathcal{O}(h^k)$, $k \in \mathbb{N}$. Give each time a reason for your answer.

1. $a(h) \cdot b_1(h)$
2. $b_1(h) - a(h)$
3. $b_1(h) + b_2(h)$