Seminar "Processing of Matrix-Valued Images"
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Diffusion Tensor MRI: An Introduction

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1 Introduction

Nowadays, we have very good medical techniques to image for example bones. But when it comes to the brain, classical imaging techniques like X-Ray or CT don't help any more, especially if one is interested in good soft tissue contrast. Another aspect every medical application focusses on is the fact that it should be harmless to the patient. In contrast to that, we can consider for example CT measurements which are not harmless to the patient, because they use radiation to image parts of the body.

The question now is: What is so interesting about Magnetic Resonance Imaging and its extensions?

2 The way to Diffusion Tensor MRI

2.1 Magnetic Resonance Imaging

Magnetic Resonance Imaging is a medical imaging technique which is used to visualise the structure and the function of the body. This happens at a microscopic scale well beyond the usual image resolution. In contrast to the measurements that use radiation to obtain their images, Magnetic Resonance Imaging uses a strong magnetic field which is harmless to the patient's body. The powerful magnetic field is used to align the nuclear magnetisation of hydrogen atoms in water in the body. Radiofrequency fields change constantly the alignment of the magnetisation (spin) which makes the hydrogen atoms generate a magnetic field detectable by a scanner. By using other magnetic fields, one can alter the obtained signal to build up enough information to reconstruct an image of the body.

2.2 Diffusion Magnetic Resonance Imaging

Diffusion Magnetic Resonance Imaging is not interested in the displacement of water molecules in the brain but in their diffusion. When it comes to diffusion we have to distinguish two main cases, namely isotropic and anisotropic diffusion. Anisotropic diffusion means that the diffusion is directionally dependent where isotropic diffusion means that it is directionally independent.

An isotropic medium could be for example some liquid or a gas. Here diffusion is equally likely in every direction. To be more precise: Water molecules move randomly according to Brownian motion which is the random movement of particles suspended in a liquid or gas or the mathematical model used to describe such random movements.

In biological tissue one can also encounter the situation that the medium one is investigating is anisotropic. Now, water molecules cannot diffuse in every direction – they are bounded. As we will see later, anisotropic diffusion is a little bit more complicated to describe.

2.3 Diffusion Tensor Magnetic Resonance Imaging

Before we can understand what Diffusion Tensor MRI does, we have to understand what a diffusion tensor is.

2.3.1 The Diffusion Tensor

Now, we have a pretty good intuition what isotropic and anisotropic diffusion means. When it comes to isotropic diffusion, the whole diffusion process can mainly be described using one single coefficient, the diffusion coefficient. This is due to the fact that in an isotropic medium – as we already know – diffusion is equally likely in every single direction. To describe the attenuation in the process in a correct way one also has to consider the characteristics of the magnetic field gradient pulses (timing, amplitude, shape) which are represented by the so-called "b-factor". With this knowledge, we can now formulate the attenuation A in the isotropic case

$$A = \exp(-bD)$$

where b is the "b-factor" and D the diffusion coefficient.

In the anisotropic case it is more difficult to describe a diffusion process. This is due to the fact that in the anisotropic case diffusion is not equally likely in every direction any more: We have different diffusivities in the different directions x, y and z. To describe the diffusion process in a precise way we have to introduce the diffusion tensor which is a positive definite and symmetric 3×3 matrix:

$$\mathbf{D} = \begin{matrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{matrix}$$

where D_{xx} is the diffusivity in x, D_{yy} is the diffusivity in y and D_{zz} is the diffusivity in z, respectively. D_{xy} for example gives a correlation of the diffusivities in x and y and D_{xz} and D_{yz} the correlations of the diffusivities in the other directions, respectively.

Since the single coefficient for the diffusion from the isotropic case is now replaced by a matrix, we also have to make a slight change to the "b-factor" which is now also a matrix. The attenuation in the anisotropic case is now given as follows:

$$A = \exp\left(- \sum_{i=x,y,z} \sum_{j=x,y,z} b_{i,j} D_{i,j} \right).$$

One could also write this as follows:

$$A = \exp(-b_{xx}D_{xx} - b_{yy}D_{yy} - b_{zz}D_{zz} - 2b_{xy}D_{xy} - 2b_{xz}D_{xz} - 2b_{yz}D_{yz}).$$

2.3.2 Diffusion Tensor MRI

In the case of Diffusion Tensor MRI we now take into account a probability density function p of particle displacements $\mathbf{x}$ over a fixed time t to compute the diffusion tensor. The assumption is that p is a zero-mean trivariate Gaussian distribution G

$$p(\mathbf{x}) = G(\mathbf{x}; \mathbf{D}, t) = ((4\pi t)^3 \det(\mathbf{D}))^{-\frac{1}{2}} \exp\left(- \frac{\mathbf{x}^T \mathbf{D}^{-1} \mathbf{x}}{4t} \right).$$

which indicates the probability density for a water molecule to move from $\mathbf{x} = 0$ at time $t = 0$ to the point $\mathbf{x}$ in time t.

We can now formulate the Stejskal-Tanner formula which gives the loss in signal strength (attenuation) under a gradient field in direction $\mathbf{q}$:

$$S(\mathbf{q}) = S_0 \cdot \exp\left(-t\mathbf{q}^T \mathbf{D}\mathbf{q}\right)$$

where S_0 is the signal strength of an unweighted measurement.

2.3.3 Number of measurement directions

We have seen that the diffusion tensor is a symmetric 3×3 matrix. This leaves six degrees of freedom, namely the diagonal and the upper or lower diagonal part (for symmetry reasons it doesn't matter). At the beginning of Diffusion Tensor MRI only these six measurements plus one more for S_0 were used which were enough to compute the diffusion tensor. Today, more measurements, say n, are taken simply for the sake of precision (see figure 1). But now the problem arises that one has to solve n equations for $S(\mathbf{q})$, because one has more measurements than entries in the matrix. Often one uses the linear least-squares fit to solve for the $6 + 1$ unknowns, namely 6 for the diffusion tensor $\mathbf{D}$ and 1 for S_0.

2.3.4 Visualisation of Diffusion Tensor MRI data

The main focus in this introduction is not on the visualisation part, but it is nice to know that one can visualise raw diffusion data – which are not so easy to interpret – much more intuitive – at least for some applications – by exploiting the following fact: It is possible to visualise every positive definite 3×3 matrix by an ellipsoid. Since our diffusion tensor $\mathbf{D}$ is a positive definite 3×3 matrix

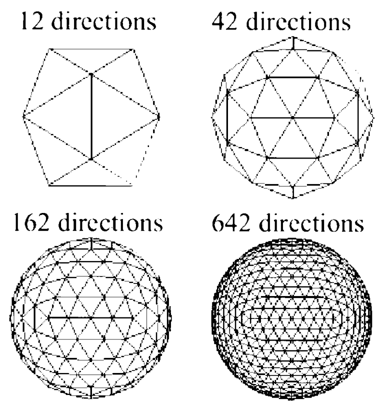

Figure 1: Schemes for directional sampling. Initial sampling schemes consisted of six directions of measurement. Progress is made in scanning space more uniformly along many directions. Authors: Le Bihan et al.

one can visualise it by an ellipsoid (see also figure 2). The question now is how shall we interpret these ellipsoids?

We know that the diffusion tensor **D** is a matrix, so we can calculate the eigenvectors and the corresponding eigenvalues. The ellipsoid's axes point into the direction of the eigenvectors and use the corresponding eigenvalues as their length.

With ellipsoids we have on the one hand side a simple representation for the diffusion tensor which on the other hand has also some disadvantages. An example would be in the case of fiber crossings, i.e. parts in the brain where two or three diffusion processes "cross" each other, the representation with ellipsoids would result in a planar-like shape (for two fibers) and a sphere-like shape (for three fibers) which would indicate an isotropic diffusion. In further talks in this seminar we will see more visualisations of the diffusion tensor.

3 Applications

We will have a short look at two main applications namely detecting fibers in the brain and detecting fibers in the body and tracking them ("tractography"). Considering fibers in the brain, we can have a look at water molecules moving along them. This allows us to diagnose deseases like brain ischemia, strokes, multiple sclerosis, schizophrenia, epilepsy or dementia and allows also to study brain connectivity.

Typical organs which one tries to image when considering fibers in the body would be the heart or the tongue. However, these images suffer from strong respiratory motion artifacts.

Besides these two examples there are a lot of other applications which we won't have a look at here.

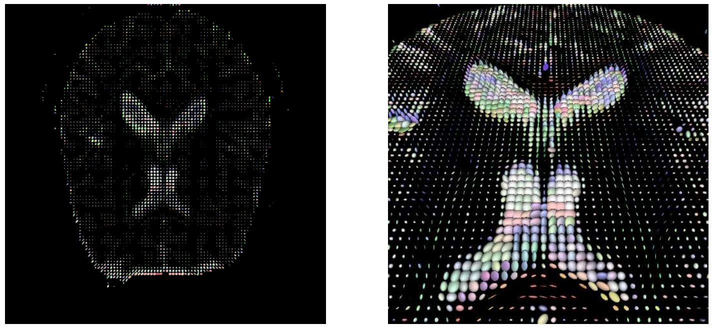

Figure 2: Diffusion Tensors visualized by ellipsoids. *Left:* One brain slice of a human head on top view. *Right:* The part where the corpus callosum is located with high scaling of the ellipsoids. This gives a better visual impression of what is depicted as isotropic and anisotropic diffusion. Images measured by the University of Eindhoven. *(Visualisation software written by Stephan Didas, Luis Pizarro, and Stephan Zimmer.)*

4 Summary

Diffusion Tensor Magentic Resonance Imaging is now a routine clinical technique which is non-invasive and harmless to the patient. It can detect anisotropies and depicts the diffusion of water molecules which is useful for many applications. Furthermore there is a nice visualisation method to visualise the diffusion tensor which is ellipsoids, but there exist much more methods which all have their advantages and disadvantages.

5 References

1. Denis Le Bihan et al., *Diffusion Tensor Imaging: Concepts and Applications*. Journal of Magnetic Resonance Imaging 13, p. 534-546, 2001.

2. Daniel C. Alexander, *An Introduction to Computational Diffusion MRI: the Diffusion Tensor and Beyond*. Department of Computer Science, University College London, p. 77-100, 2006.

3. `http://de.wikipedia.org/wiki/Diffusions-Tensor-Bildgebung`
   (Wikipedia article on DT-MRI (in german). Very good article, better than the English one.)

4. `http://en.wikipedia.org/wiki/Magnetic_resonance_imaging`
   (Wikipedia article on Magnetic Resonance Imaging.)

5. `http://de.wikipedia.org/wiki/Magnetresonanztomographie`
   (Wikipedia article on Magnetic Resonance Imaging (in german).)