

Diffusion Tensor Imaging: An Introduction

Seminar "Processing of Matrix-Valued Images"

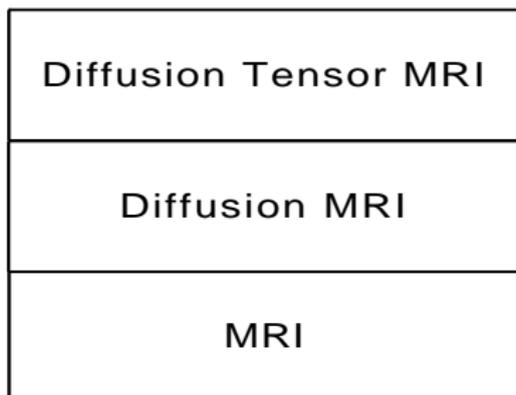
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Motivation

- We want to help patients who suffer from a disease like a stroke, epilepsy, multiple sclerosis, dementia and other white-matter diseases.
- Optimally, the measurements should be non-invasive.
- We want to discuss the following techniques:

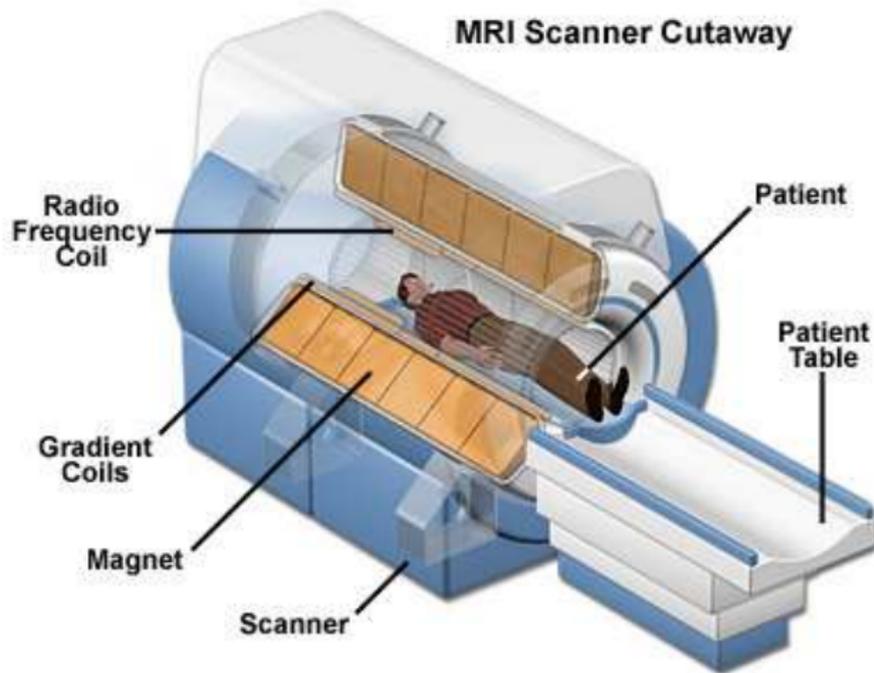


Magnetic Resonance Imaging



Modern 3 tesla clinical MRI scanner. Source: *Wikipedia*.

Magnetic Resonance Imaging (2)

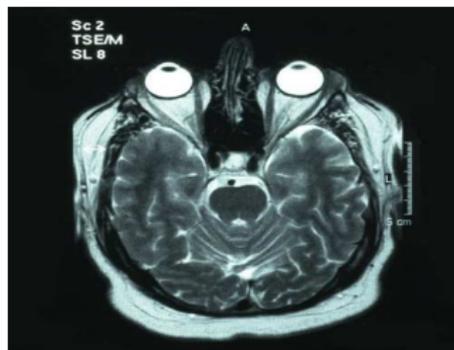


Schematic of a MRI scanner. Source:

<http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/>

Magnetic Resonance Imaging (3)

- MRI (Kernspintomographie) is a medical imaging technique used to visualize the structure and the function of the body.
- Uses a *strong magnetic field* instead of radiation.
- Measures *displacements of water molecules* using gradients in different directions. Usually six (or even four) directions are used.
- Better soft tissue contrast than for example for CT.



Left: CT image of the head. Eyes can be clearly seen. **Right:** Approximately the same image, but measured with MRI. Source: *Wikipedia*.

Diffusion Magnetic Resonance Imaging

- Diffusion Magnetic Resonance Imaging measures the *diffusion* of water molecules.
- In the case of an *isotropic medium*, water molecules move randomly according to Brownian motion.
- In biological tissues, diffusion can be *anisotropic*.

Reminder:

- Anisotropy: the property of being directionally dependent.
- Isotropy: the opposite of anisotropy.

The Diffusion Tensor

- In an *isotropic* medium, the diffusion can be fully described by a single (scalar) parameter. The *attenuation* A is described as follows:

$$A = \exp(-bD)$$

where

- b : "b-factor" which characterizes the magnetic field gradient pulses (timing, amplitude, shape),
- D : diffusion coefficient.

The Diffusion Tensor (2)

- In an *anisotropic* medium, one requires a tensor instead of a single coefficient. The tensor is *symmetric* and looks like this:

$$\mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

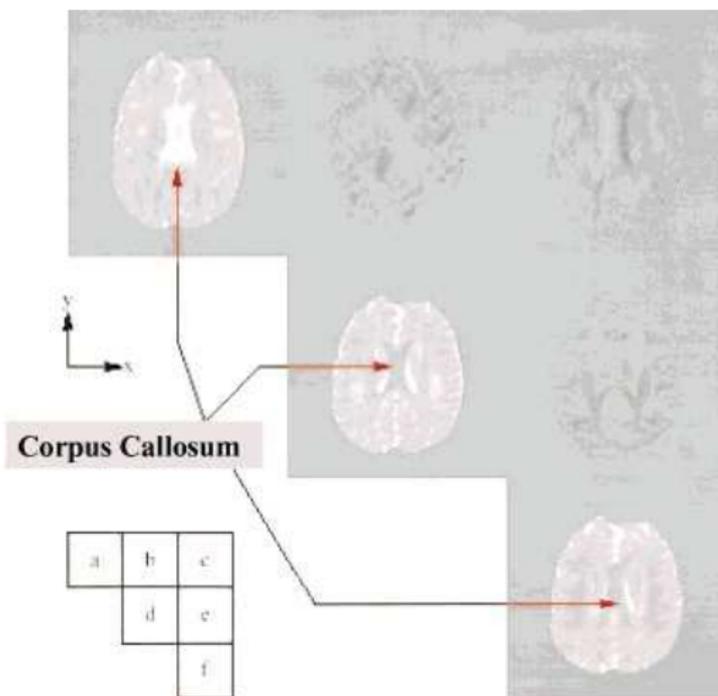
where D_{xx} is the diffusivity in x , D_{yy} is the diffusivity in y and D_{zz} is the diffusivity in z . \mathbf{D} is a positive definite 3×3 matrix.

- The *attenuation* A is described as follows:

$$A = \exp \left(- \sum_{i=x,y,z} \sum_{j=x,y,z} b_{i,j} D_{i,j} \right)$$

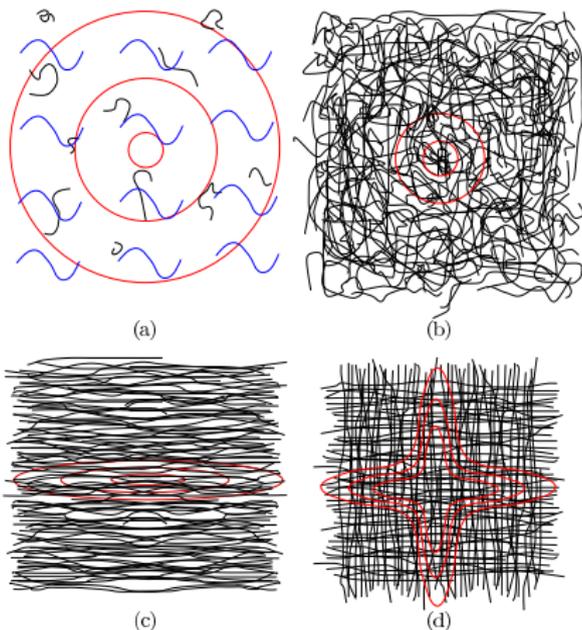
where \mathbf{b} is now a matrix that replaces the "b-factor".

The Diffusion Tensor (3)



The diffusion tensor, axial slices. Diffusivity along x , y and z , shown in D_{xx} (a), D_{yy} (d) and D_{zz} (f) images. Authors: Le Bihan et al.

The Diffusion Tensor (4)



Four microstructures found in the brain. Black lines are barriers to the movement of water molecules. Red shapes indicate expected diffusion. **(a)** fluid-filled region, **(b)** isotropic grey matter, **(c)** and **(d)** white matter with one and two dominant fiber orientations. Author: Daniel C. Alexander.

Diffusion Tensor MRI

- Let p be a probability density function of particle displacements \mathbf{x} over a fixed time t .
- Diffusion Tensor MRI computes the diffusion tensor on the assumption that p is a zero-mean trivariate Gaussian distribution:

$$G(\mathbf{x}; \mathbf{D}, t) = ((4\pi t)^3 \det(\mathbf{D}))^{-\frac{1}{2}} \exp\left(-\frac{\mathbf{x}^T \mathbf{D}^{-1} \mathbf{x}}{4t}\right).$$

- This indicates the probability density for a water molecule to move from $\mathbf{x} = 0$ at time $t = 0$ to the point \mathbf{x} in time t .
- We get the *Stejskal-Tanner formula* which gives the following:

$$S(\mathbf{q}) = S_0 \cdot \exp\left(-t\mathbf{q}^T \mathbf{D} \mathbf{q}\right)$$

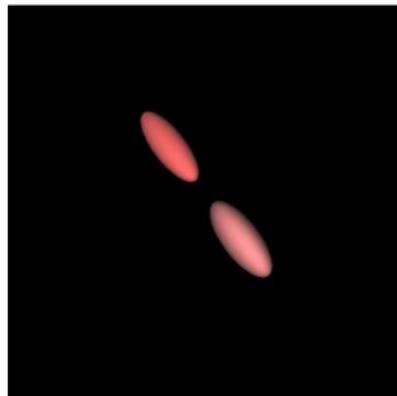
where S_0 is the signal strength of an unweighted measurement and $S(\mathbf{q})$ is the signal strength under a gradient field in direction \mathbf{q} .

Diffusion Tensor MRI (2)

- It is difficult to interpret raw tensor data.
- Any positive definite 3×3 matrix can be represented by an ellipsoid. DT-MRI depicts the diffusion tensor \mathbf{D} by ellipsoids.

How to interpret ellipsoids?

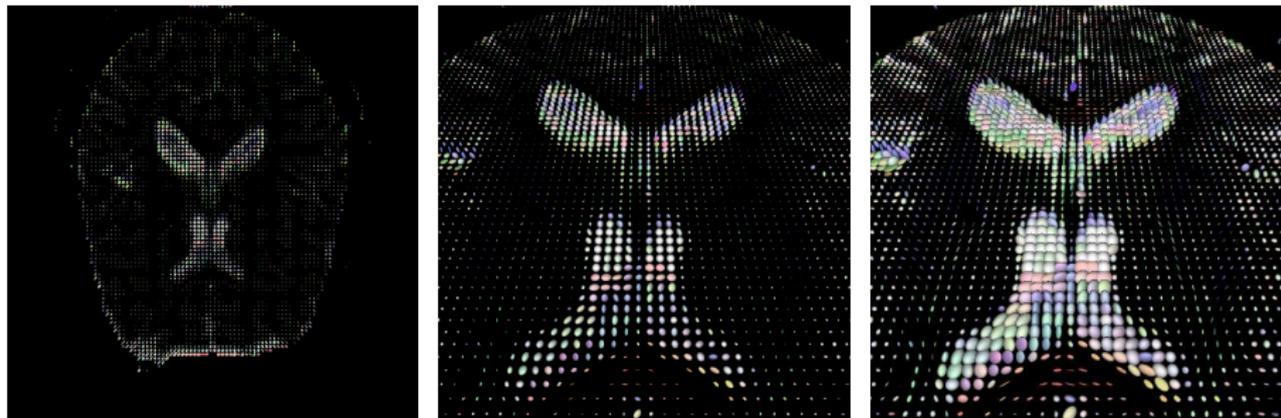
- An ellipsoid's principal axes point into the directions of the eigenvectors of \mathbf{D} and have the eigenvalues of \mathbf{D} as their length.
- So it is a *3-D representation* of the *diffusion distance* in a given *diffusion time* T_d .
- The *eccentricity* (Exzentrizität) of the ellipsoid provides information about the *degree of anisotropy* and its *symmetry*. Isotropic diffusion would be seen as a sphere.



Two ellipsoids.

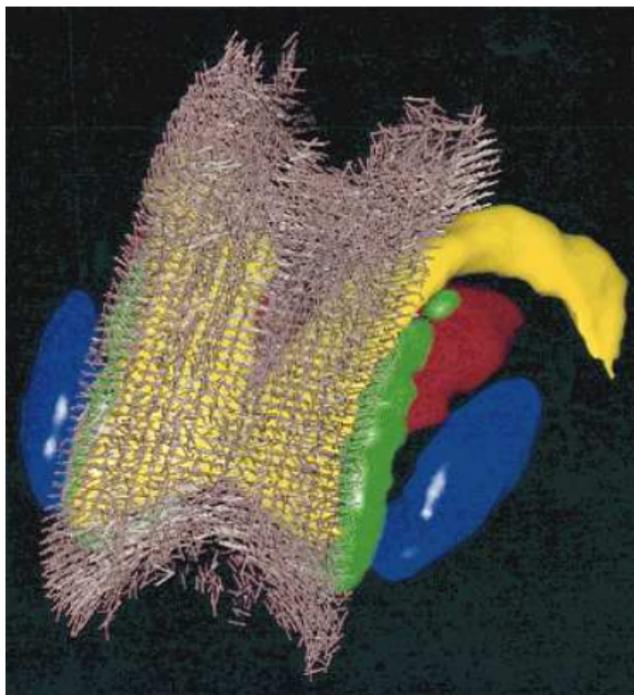
(Visualisation software written by Stephan Didas, Luis Pizarro, and Stephan Zimmer.)

Example



Diffusion Tensors visualized by ellipsoids. **Left:** One brain slice of a human head on top view. **Middle:** The part where the corpus callosum is located. **Right:** Same view as before, but with higher scaling of the ellipsoids. This gives a better visual impression of what is isotropic and anisotropic diffusion. Images measured by the University of Eindhoven. (*Visualisation software written by Stephan Didas, Luis Pizarro, and Stephan Zimmer.*)

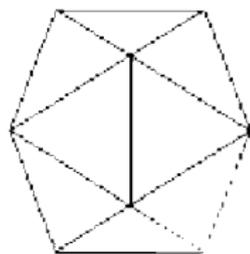
Another visualisation



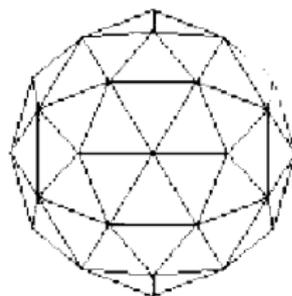
3-D visualisation of the diffusion tensor. Main eigen-values are shown as cylinders, the length of which is scaled with the degree of anisotropy. Corpus callosum fibers are displayed around ventricles, thalami, putamen, and caudate nuclei. Authors: Le Bihan et al.

How many scanning directions?

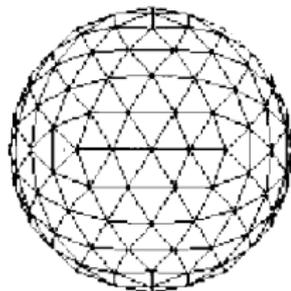
12 directions



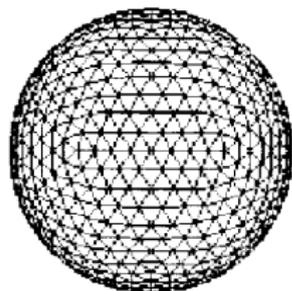
42 directions



162 directions



642 directions



Schemes for directional sampling. Initial sampling schemes consisted of six (or even four) directions of measurement. Progress is made in scanning space more uniformly along many directions. Authors: Le Bihan et al.

Applications

- *Fibers in the brain* can be detected because water molecules preferably move along them. This allows to diagnose
 - brain ischemia (Blutleere)
 - strokes (Schlaganfälle)
 - multiple sclerosis
 - schizophrenia
 - epilepsy
 - dementia

and to study **brain connectivity**.

- One can also have a look at *fibers in the body*, for example muscle fibers. Typical organs would be the tongue or the heart. However, in this case the images suffer from strong respiratory motion artifacts.

Summary

- Diffusion Tensor MRI is now a routine clinical technique. It can detect anisotropies.
- It is a non-invasive, harmless measurement technique to get images of the brain.
- The image depicts the diffusion of water molecules, which is important for many applications, namely fiber tracking or brain connectivity.
- Diffusion tensors can be visualized by ellipsoids.

References

- Denis Le Bihan et al., *Diffusion Tensor Imaging: Concepts and Applications*. Journal of Magnetic Resonance Imaging 13, p. 534-546, 2001.
- Daniel C. Alexander, *An Introduction to Computational Diffusion MRI: the Diffusion Tensor and Beyond*. Department of Computer Science, University College London, p. 77-100, 2006.
- <http://de.wikipedia.org/wiki/Diffusions-Tensor-Bildgebung> (*Wikipedia article on DT-MRI (in german). Very good article, better than the English one.*)
- http://en.wikipedia.org/wiki/Magnetic_resonance_imaging (*Wikipedia article on Magnetic Resonance Imaging.*)
- <http://de.wikipedia.org/wiki/Magnetresonanztomographie> (*Wikipedia article on Magnetic Resonance Imaging (in german).*)

Thanks for your attention!
Feel free to ask questions.