Lecture 24: Self-Snakes and Active Contours

Contents

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Self-Snakes

Motivation

- Linear diffusion
$$\partial_t u \;=\; \Delta u \;=\; \operatorname{div}(\nabla u)$$
does not allow edge enhancement.

- In order to overcome this problem, nonlinear diffusion filters have been introduced. They use a diffusivity function g that depends on the Gaussian-smoothed image gradient:
$$\partial_t u \;=\; \operatorname{div}\left(g(|\nabla u_\sigma|^2)\,\nabla u\right)$$

- Mean curvature motion
$$\partial_t u \;=\; |\nabla u|\,\operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)$$
does also not allow to enhance edges.

- Can MCM be modified in a similar way as linear diffusion ?

Self-Snakes (2)

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Self-Snakes

(Sapiro 1996)

- modifies mean curvature motion

$$\partial_t u \;=\; |\nabla u| \,\mathrm{div} \left(\frac{\nabla u}{|\nabla u|} \right)$$

with an edge-stopping function $g(|\nabla u_\sigma|^2)$:

$$\partial_t u \;=\; |\nabla u| \,\text{div} \left(g(|\nabla u_\sigma|^2) \, \frac{\nabla u}{|\nabla u|} \right)$$

- For the edge-stopping function g, one can use the same functions as for nonlinear diffusion, e.g.

$$g(|\nabla u_\sigma|^2) \;:=\; \frac{1}{1 + |\nabla u_\sigma|^2/\lambda^2}.$$

Self-Snakes (3)

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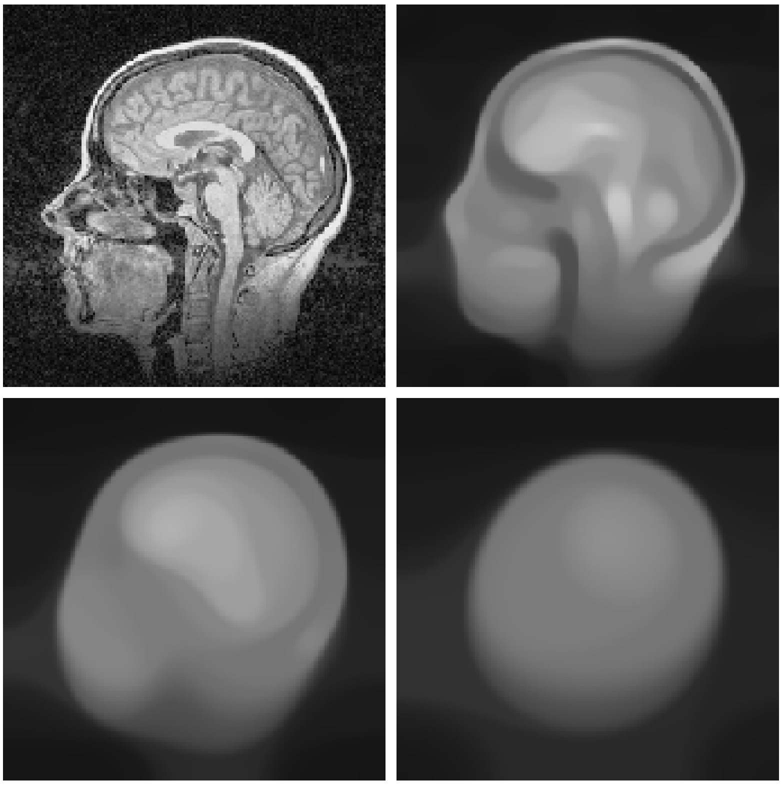
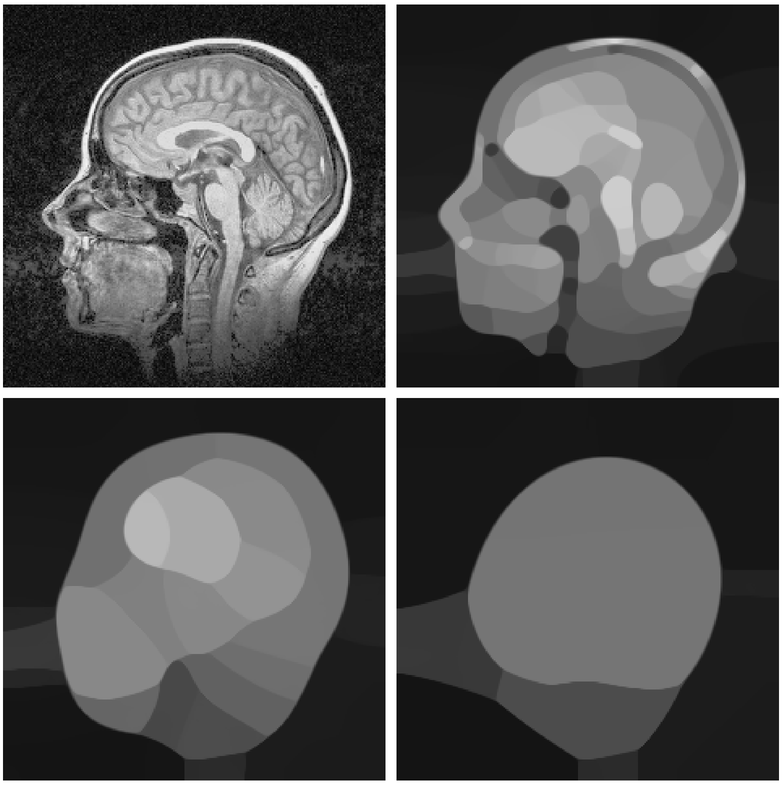


Mean curvature motion. (a) **Top left:** Original image, $\Omega = (0, 236)^2$. (b) **Top right:** $t = 70$. (c) **Bottom left:** $t = 275$. (d) **Bottom right:** $t = 1275$.

Self-Snakes (4)



Self-snakes ($\lambda = 3$, $\sigma = 1$). (a) **Top left:** Original image, $\Omega = (0, 236)^2$. (b) **Top right:** $t = 600$. (c) **Bottom left:** $t = 5000$. (d) **Bottom right:** $t = 40000$.

Self-Snakes (5)

Why Do Self-Snakes Allow Edge Enhancement ?

- By means of the product rule, self-snakes can be rewritten as

$$\partial_t u \;=\; g(|\nabla u_\sigma|^2)\,|\nabla u|\,\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right) \;+\; \nabla^\top(g(|\nabla u_\sigma|^2))\,\nabla u$$

- The red term is a *shock term*, responsible for *edge enhancement*.

- To understand this, consider the 1-D case with $\sigma = 0$.
Then the chain rule gives

$$\begin{aligned}\partial_x\left(g(u_x^2)\right)\,u_x &= g'(u_x^2)\,2u_x\,u_{xx}\,u_x \\ &= \underbrace{2\,g'(u_x^2)\,|u_x|\,|u_{xx}|}_{\le 0}\,\mathrm{sgn}(u_{xx})\,|u_x|.\end{aligned}$$

Note that $g' \le 0$ for decreasing g.

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- This resembles the 1-D shock filter (cf. Lecture 21)

$$u_t = -\mathrm{sgn}(u_{xx})\,|u_x|$$

which performs dilation $u_t = |u_x|$ around maxima $(u_{xx} < 0)$, and erosion $u_t = -|u_x|$ around minima $(u_{xx} > 0)$.

Geodesic Active Contours

(Caselles et al. 1995, Kichenassamy et al. 1995)

- interactive segmentation technique that shrinks a manually specified initial contour $\mathbf{c}_0$ towards an object boundary of some image h
- level set strategy: embed $\mathbf{c}_0$ as zero-level isophote in an image f
- evolve f under

$$\begin{aligned} \partial_t u &= |\nabla u|\,\mathrm{div}\left(g(|\nabla h_\sigma|^2)\,\frac{\nabla u}{|\nabla u|}\right) \\ u(t=0) &= f \end{aligned}$$

where the edge-stopping function g inhibits evolution at edges of h.

- extract evolving contour $\mathbf{c}$ as zero-level isophote of u

Geodesic Active Contours (2)

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Advantages of the Level Set Formulation:

- no explicit curve representation and parametrisation problems
- topological changes possible: curves can split

Geodesic Active Contours (3)

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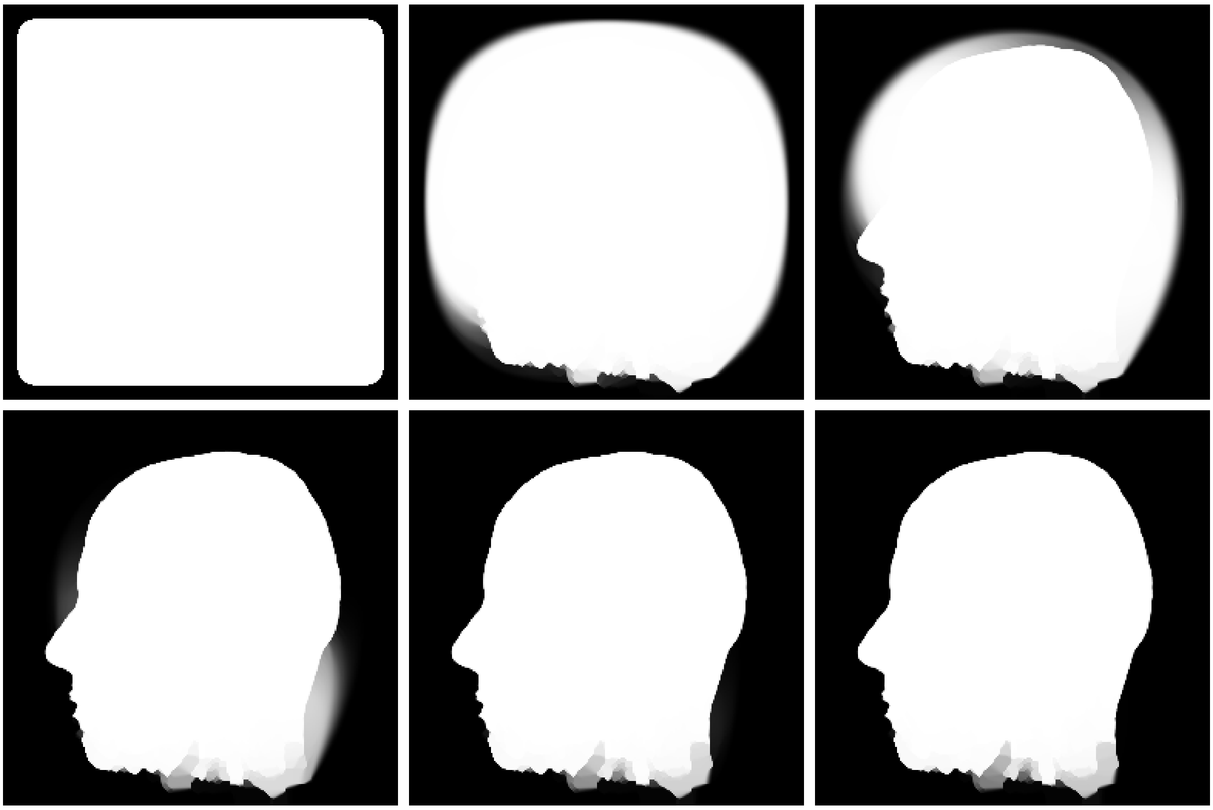
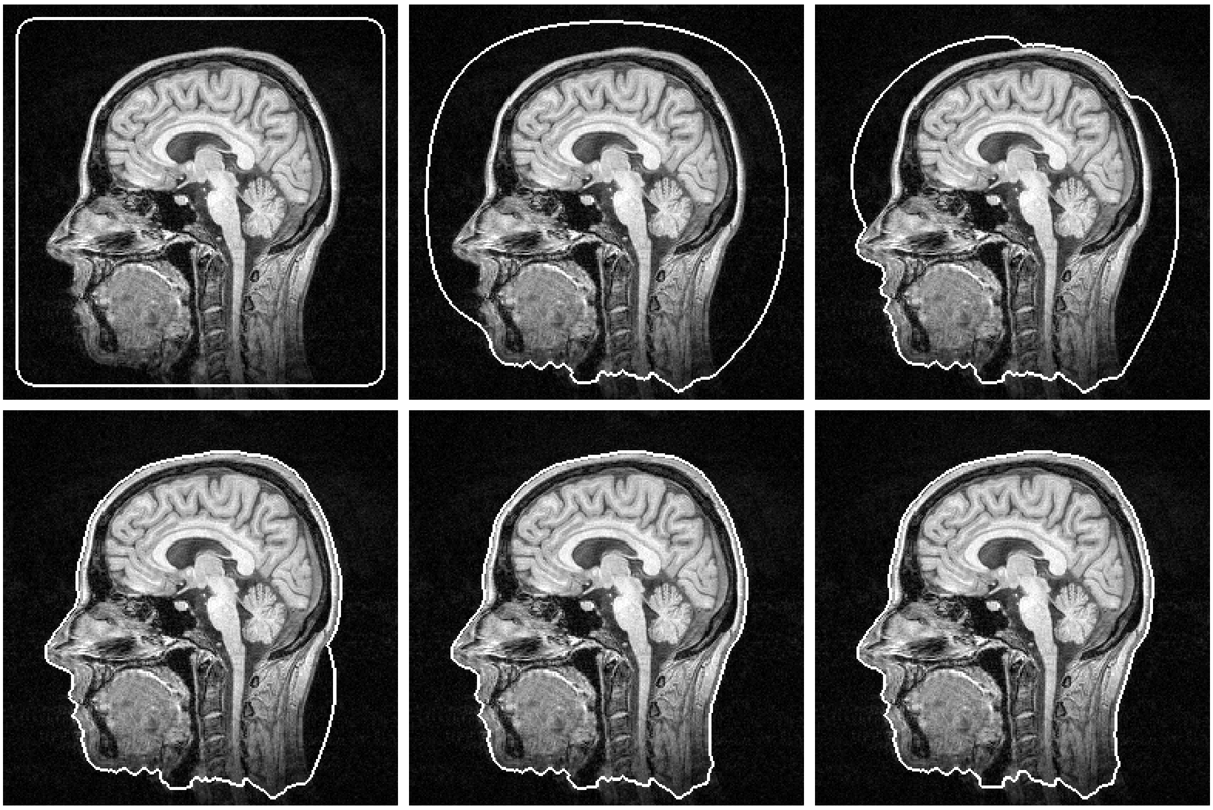


Temporal evolution of the geodesic active contour embedded as level set into the image f, $\Omega = (0, 256)^2$, $\lambda = 5$, $\sigma = 1$. **From top left to bottom right:** $t = 0$, 1500, 3000, 4500, 6000, 7500.

Geodesic Active Contours (4)



Temporal evolution of the geodesic active contour superimposed on the original image h, $\Omega = (0, 256)^2$, $\lambda = 5$, $\sigma = 1$. **From top left to bottom right:** $t = 0,\ 1500,\ 3000,\ 4500,\ 6000,\ 7500$.

Geodesic Active Contours (5)

Why Especially This Type of Evolution?

- rewrite image evolution

$$\partial_t u \;=\; |\nabla u| \operatorname{div}\left(g\left(|\nabla h_\sigma|^2\right) \frac{\nabla u}{|\nabla u|} \right)$$

  by means of the product rule:

$$\partial_t u \;=\; |\nabla u|\, g \operatorname{div}\left(\frac{\nabla u}{|\nabla u|} \right) + |\nabla u|\, \nabla g^\top \frac{\nabla u}{|\nabla u|}$$

- It evolves its isophotes under

$$\partial_t \mathbf{c} \;=\; g\, \kappa\, \mathbf{n} - \left(\nabla g^\top \mathbf{n}\right) \mathbf{n}$$

  where $\kappa = \frac{\det(\mathbf{c}_p, \mathbf{c}_{pp})}{|\mathbf{c}_p|^3}$ is the curvature and $\mathbf{n} = \frac{\mathbf{c}_p}{|\mathbf{c}_p|}$ the normal vector of $\mathbf{c}(p, t)$.

Geodesic Active Contours (6)

- Its steady state can be shown to be the Euler–Lagrange equation for

$$E_h(\mathbf{c}) := \oint_{\mathbf{c}(\rho)} \underbrace{g(|\nabla h_\sigma|^2)}_{\text{weight}} \underbrace{|\mathbf{c}_\rho(\rho)|\, d\rho}_{\text{arc-length}}.$$

- Minimising $E_h(\mathbf{c})$ means finding a curve of minimal length *(geodesic)* w.r.t. some image-induced metric: *geodesic snake, geodesic active contour*
- The curve is attracted by image edges, where the weight $g(|\nabla h_\sigma|^2)$ is small.
- Thus, the curve evolution has nontrivial steady-states for $t \to \infty$:
  The snake becomes stationary after some time
- Initialisation does matter: convergence to the next *local* minimum

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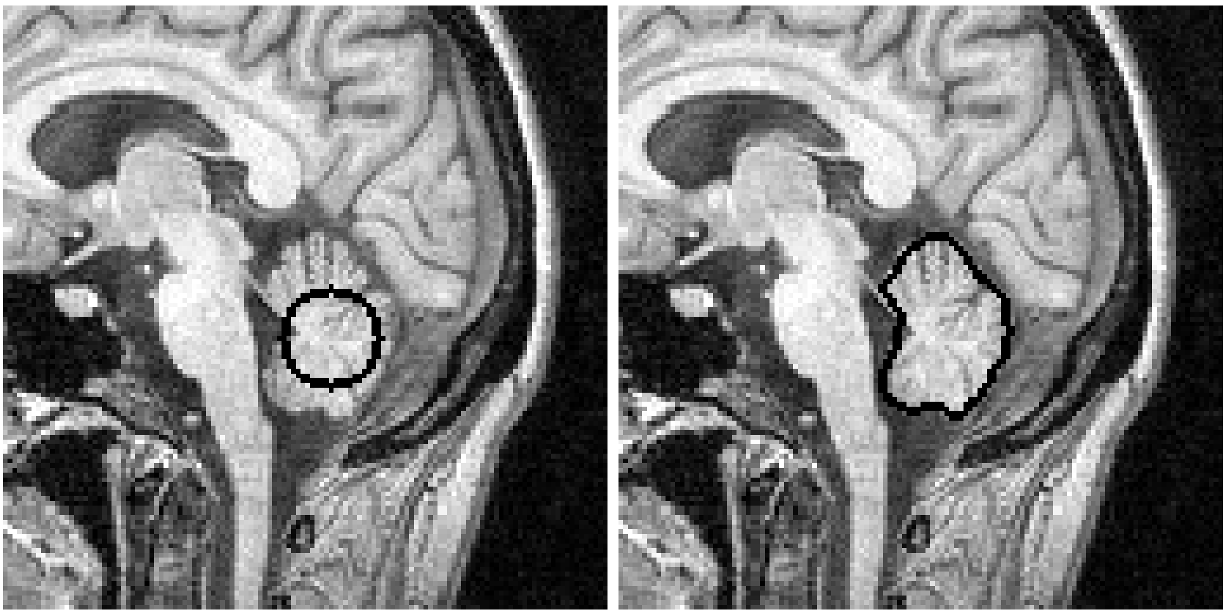
Geodesic Active Contours (7)



The image-induced metric is responsible for the fact that geodesic active contours may even expand. (a) **Left:** Initialisation of a geodesic active contour. (b) **Right:** Steady state.

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Region-Based Active Contours

(Chan/Vese 2001)

Motivation

- search for an alternative active contour model that avoids some of the shortcomings of geodesic active contours
- using region-based information instead of edge-based information allows to
  - be more robust under noise
  - have a higher globality in the convergence behaviour
  - include other features that can be measured within the region, e.g. texture features

The Chan–Vese Model

- Chan and Vese considered a region-based active contour model where the image is segmented into two phases.
- Its energy functional is a function of a contour $\mathbf{c}$:

$$E_{\mathrm{CV}}(\mathbf{c}, u_{\mathrm{in}}, u_{\mathrm{out}}) \;=\; \int\limits_{\text{inside } \mathbf{c}} (f(\mathbf{x}) - u_{\mathrm{in}})^2 \, d\mathbf{x} \;+\; \int\limits_{\text{outside } \mathbf{c}} (f(\mathbf{x}) - u_{\mathrm{out}})^2 \, d\mathbf{x} \;+\; \lambda \, l(\mathbf{c})$$

  where u_{in} and u_{out} are the arithmetic means of $f(x)$ inside resp. outside $\mathbf{c}$, and $l(\mathbf{c})$ is the contour length.

- The user specifies some initial contour $\mathbf{c}_0$ close to the segmentation one wants to find.
- Minimising $E_{\mathrm{CV}}(\mathbf{c}, u_{\mathrm{in}}, u_{\mathrm{out}})$ drives the curve towards the segment boundaries.

Region-Based Active Contours (3)

Comparison with the Mumford–Shah Model
(cf. Lecture 13)

- Chan–Vese model:

$$E_{\mathrm{CV}}(\mathbf{c}, u_{\mathrm{in}}, u_{\mathrm{out}}) \;=\; \int_{\text{inside } \mathbf{c}} (f(\mathbf{x}) - u_{\mathrm{in}})^2 \, d\mathbf{x} \;+\; \int_{\text{outside } \mathbf{c}} (f(\mathbf{x}) - u_{\mathrm{out}})^2 \, d\mathbf{x} \;+\; \lambda \, l(\mathbf{c})$$

- Mumford–Shah model (Lecture 13):

$$E_{\mathrm{MS}}(K, u) \;=\; \int_{\Omega} (f - u)^2 \, d\mathbf{x} \;+\; \beta \int_{\Omega \setminus K} |\nabla u|^2 \, d\mathbf{x} \;+\; \mu \, |K|$$

- Equivalent for piecewise constant approximations u (i.e. $\beta = 0$) and edge sets K that separate Ω only into **two** regions.

Region-Based Active Contours (4)

Numerical Realisation With Level Sets

- Level set formulation (image evolution instead of curve evolution):
  Embed the curve $\mathbf{c}$ as level line into some image v, and minimise

$$E_I(v) \;=\; \int_{\Omega} (f - u_{\mathrm{in}})^2 \, H(v) \, d\mathbf{x} + \int_{\Omega} (f - u_{\mathrm{out}})^2 \, (1 - H(v)) \, d\mathbf{x} \;+\; \lambda \int_{\Omega} |\nabla H(v)| \, d\mathbf{x}$$

  with the *Heaviside function* $H(z) := \begin{cases} 1 & (z \geq 0) \\ 0 & (z < 0) \end{cases}$.

- In order to render H differentiable, we approximate it by a smooth function H_ε, e.g.

$$H_\varepsilon(z) \;:=\; \frac{1}{2}\left(1 + \frac{2}{\pi} \arctan\left(\frac{z}{\varepsilon}\right)\right).$$

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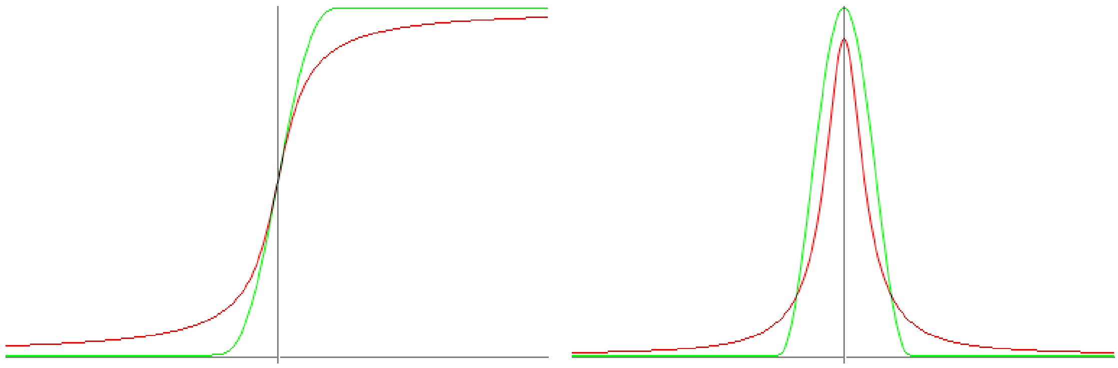
Region-Based Active Contours (5)



Left: Two regularised Heaviside functions proposed by Chan–Vese. **Right:** Their derivatives.

- Thanks to the regularisation of H, gradient descent can be applied:

$$\partial_t v \;=\; H'_\varepsilon(v)\left((f-u_{\text{out}})^2 - (f-u_{\text{in}})^2 + \lambda\,\text{div}\left(\frac{\nabla v}{|\nabla v|}\right)\right).$$

- Evolve alternatingly for v and update u_{out} and u_{in} (as the mean outside/inside the level line 0.5 of v).

- The final contour is the zero level line of the steady state solution for v.

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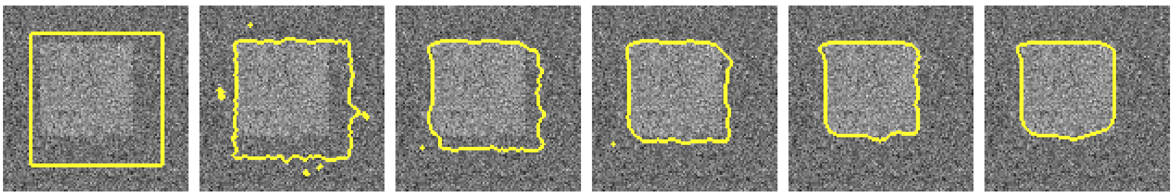
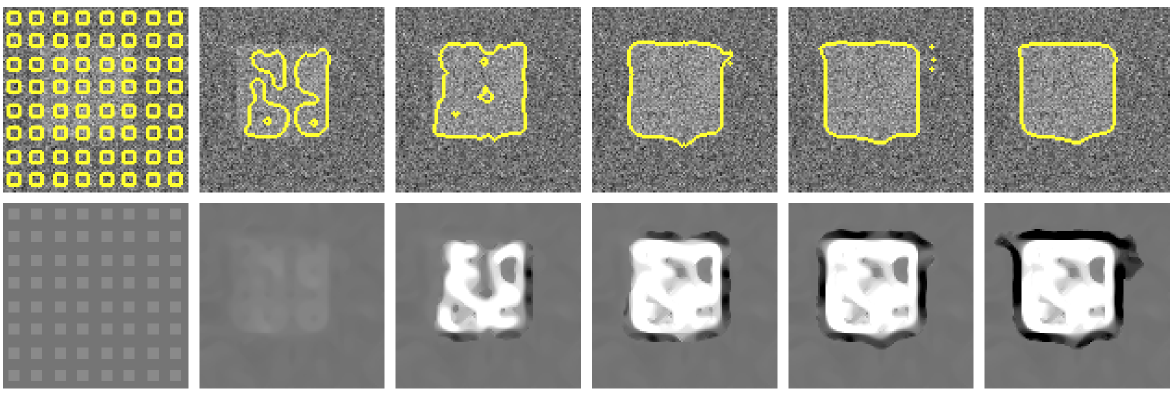
Region-Based Active Contours (6)



Curve evolution with the Chan–Vese model. Curve at times $t = 0, 1, 2, 3, 4, 10$. Authors: T. Chan, L. Vese (2001).



Top: Curve evolution with another initialization. **Bottom:** Corresponding level set function. Authors: T. Chan, L. Vese (2001).

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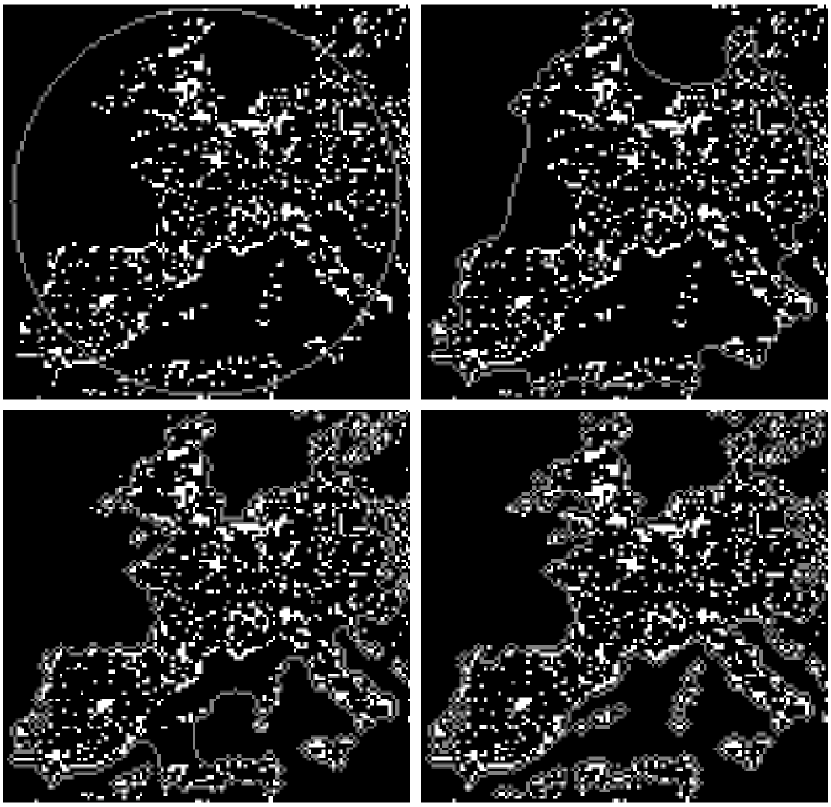
Region-Based Active Contours (7)



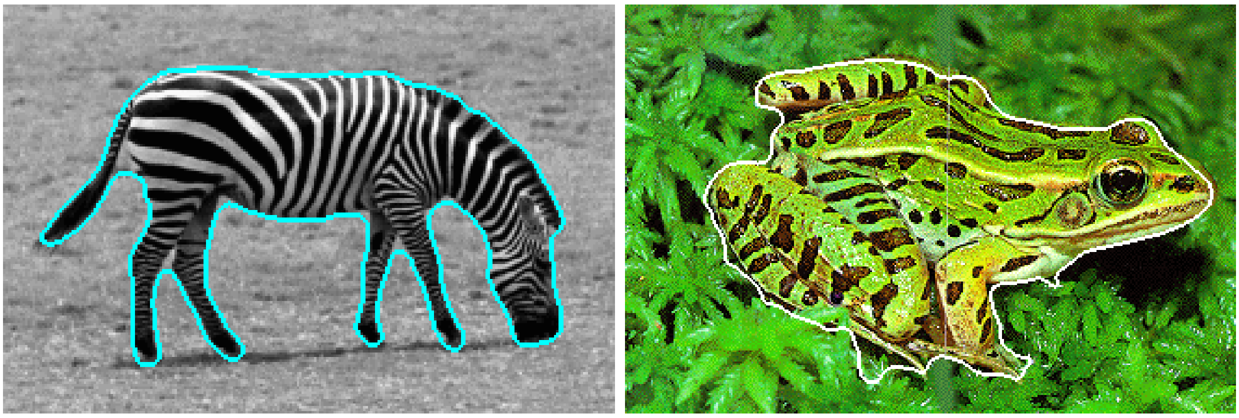
Top left to bottom right: Evolution of an active contour under the Chan-Vese model. The contour splits and creates a fairly good segmentation of Europe at night. Authors: T. Chan, L. Vese (2001).

Region-Based Active Contours (8)

Extensions of the Chan–Vese Model

- recent modifications of the Chan–Vese model include
  - more sophisticated features than the grey value (colour, texture, optic flow, ...)
  - additional statistical characterisations of a region (not only mean, but also standard deviation, ...)
  - multiphase models that allow more than two types of segments
  - sometimes even a-priori knowledge using a statistical characterisation of the shapes to be expected
- yield state-of-the-art results in segmentation

Region-Based Active Contours (9)



(a) Left: Segmented image using grey values and texture as features. **(b) Right:** Segmentation with colour and texture as feature channels. Author: T. Brox (2003).

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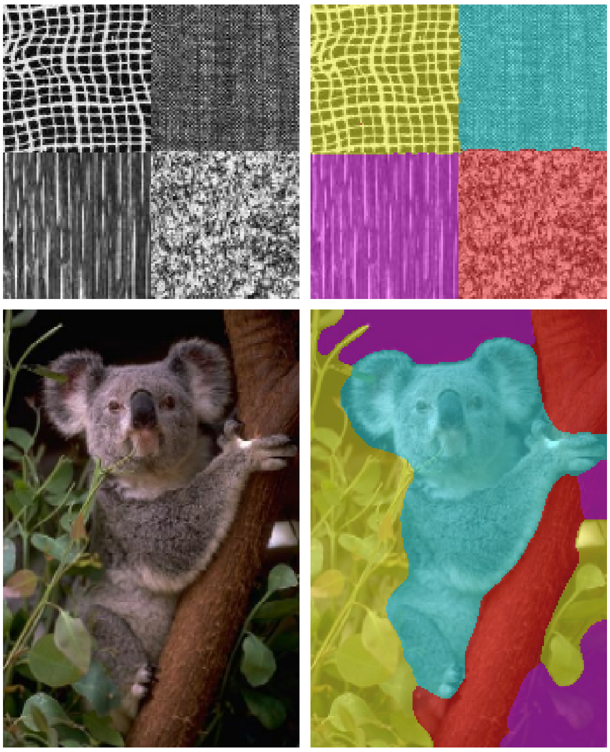
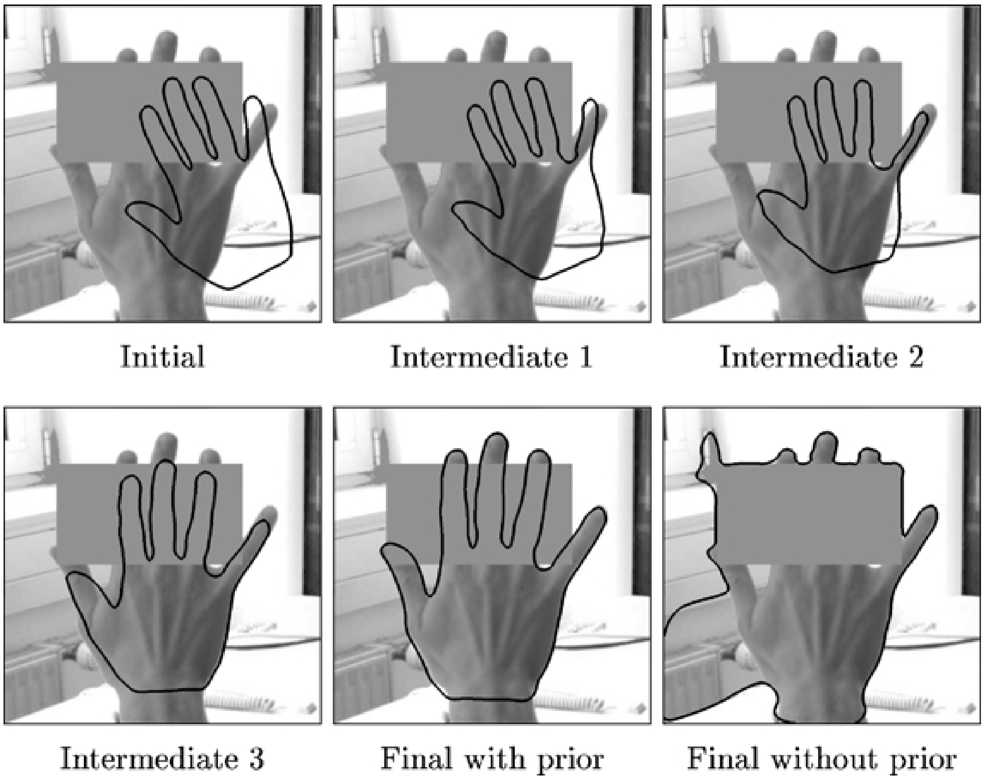
Region-Based Active Contours (10)



Left: Input image. **Right:** Segmented regions using a multiphase extension of the Chan-Vese model. Author: T. Brox (2004).



Segmentation of an occluded hand with and without a priori knowledge. Authors: Cremers et al. (2002).

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Summary

- Self-snakes:
  - edge-enhancing modifications of MCM
  - contain edge-stopping function within divergence expression
- Geodesic active contours:
  - level set based interactive segmentation tool
  - An initial contour is drawn and embedded into a level set evolution.
  - This evolution is steered by an edge stopping function within a modified MCM.
  - The level set of the steady state gives the segmentation result.
- Region-based active contours:
  - active contour evolutions that are fairly stable under noise
  - allow many useful modifications

References

- G. Sapiro: Vector (self) snakes: a geometric framework for color, texture and multiscale image segmentation. *Proc. 1996 IEEE International Conference on Image Processing*, (Lausanne, Switzerland, Sept. 1996), Vol. 1, pp. 817–820.
  (self-snakes)
- V. Caselles, R. Kimmel, G. Sapiro: Geodesic active contours. *International Journal of Computer Vision*, Vol. 22, 61–79, 1997.
  (one of the two journal papers where geodesic active contours have been introduced)
- S. Kichenassamy, A. Kumar, P. Olver, A. Tannenbaum, A. Yezzi, Conformal curvature flows: from phase transitions to active vision, *Archive for Rational Mechanics and Analysis*, Vol. 134, 275–301, 1996.
  (the other journal paper on geodesic active contours)
- T. F. Chan, L. A. Vese: Active contours without edges. *IEEE Transactions on Image Processing*, Vol. 10, No. 2, pp. 266–277, Febr. 2001.
  (region-based active contours)

Assignment P6 – Programming

You can download the file `Ex06.tar` from the web page

`http://www.mia.uni-saarland.de/Teaching/dic08.shtml`

To unpack these data, use `tar xvf Ex06.tar`.

1. The ANSI C programme `dilation.c` performs continuous-scale dilation with a disc-shaped structuring element. (7 points)
   - Have a look at it and supplement the corresponding programme `erosion.c` with the missing code.
   - Apply `erosion` to `noise.pgm` and stop it at $t = 9$.
     Apply `dilation` to the result and stop at $t = 9$ as well.
     Visualize the result with `xv` and normalize the grey level range to $[0, 255]$: Click on the image with the right mouse button. This opens a window with many options. Press the buttons for `windows`, then `color editor` and afterwards `Norm`.
   - Apply `erosion` to `head.pgm` (1 iteration with $\tau = 0.5$), and do the same with `dilation`. Take the modulus of the difference by applying `difference` to the result. This edge detector (*"morphological gradient"*) can be scaled by varying the evolution time of both underlying processes.

Assignment P6 (2)

2. The ANSI C programme `mcm.c` performs mean curvature motion. (2 points)
   - Use it for shape simplification of the image `tyre.pgm`. Useful parameters: $\tau = 0.2$, 100/500/2000 iterations.
   - Why is `mcm.c` not useful for fingerprint processing?
   - The images `check1.pgm` and `check2.pgm` are visually very similar. How do they evolve under MCM? Is this in contradiction to the theory?

3. The programme `corner_detect.c` uses the ideas of Alvarez and Morales to investigate the corner evolution with the affine morphological scale-space. (3 points)
   - Apply the programme to the images `corner01.pgm` and `corner02.pgm` with time step size $\tau = 0.01$ and maximal stopping time $t = 100$. The exact corner angles are $\alpha = 90^\circ$ for `corner01.pgm` and $\alpha = 53.13^\circ$ for `corner02.pgm`. How good are the results with respect to different stopping times?
   - To check the robustness of the method under noise, also try the images `corner01_50.pgm` and `corner02_50.pgm`. How is the result influenced by noise?

Assignment P6 (2)

For assessment: Use `tar -cvzf P6_`*yourname*`.tgz` *file1 file2 ...* to pack into an archive:

- the output files obtained by dilation, erosion and the difference computation with the given parameters,
- the `mcm`-filtered versions of `tyre.pgm` with the given parameters,
- representative filtering results from `check{1|2}.pgm`,
- the coordinates of the evolving corner point in Task 3 and the estimate of the corner,
- a `README` file listing the files and corresponding parameters.
  (Comment in this file also on the evolution of `check{1|2}.pgm` under MCM.)

Deadline for electronic submission: Friday, July 11, 10 am.