Lecture 18:

Image Sequence Analysis III: Large Displacements, High Accuracy Methods, and Illumination Changes

Contents

Large Displacements

- So far we have used linearised constancy assumptions.
- Examples:

(a) The brightness constancy assumption

$$f_{x_1} u_1 + f_{x_2} u_2 + f_{x_3} \;=\; 0$$

can be regarded as Taylor linearisation (around $\mathbf{x}$) of

$$f(\mathbf{x}+\mathbf{u}) - f(\mathbf{x}) \;=\; 0.$$

(b) The gradient constancy assumptions

$$\begin{aligned} f_{x_1x_1} u_1 + f_{x_1x_2} u_2 + f_{x_1x_3} &= 0, \\ f_{x_2x_1} u_1 + f_{x_2x_2} u_2 + f_{x_2x_3} &= 0 \end{aligned}$$

are linearisations of

$$\begin{aligned} f_{x_1}(\mathbf{x}+\mathbf{u}) - f_{x_1}(\mathbf{x}) &= 0, \\ f_{x_2}(\mathbf{x}+\mathbf{u}) - f_{x_2}(\mathbf{x}) &= 0. \end{aligned}$$

- Such linearisation are valid approximations if the optic flow field $\mathbf{u}$ involves only small displacements (in the order of one pixel).
- However, conventional video cameras often suffer from temporal undersampling, i.e. they produce displacements over several pixels.
- In this case we have to use constancy assumptions without linearisations.
- Example: Without linearisation, the Horn–Schunck model becomes

$$E(\mathbf{u}) \;=\; \int_{\Omega} \Big(\big(\mathbf{f}(\mathbf{x}+\mathbf{u}) - \mathbf{f}(\mathbf{x})\big)^{2} + \alpha(|\nabla \mathbf{u_1}|^{2} + |\nabla \mathbf{u_2}|^{2}) \Big)\, \mathbf{dx}.$$

- lead to nonconvex energy functionals with multiple local minima
- numerical problem: find a good local minimiser
- However, the quality gain may compensate for the additional problems.

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A Highly Accurate Model

(Brox et al. 2004)

Goals

- design a model that pushes the quality of optic flow estimation to the limit
- takes into account a number of successful concepts:
  - constancy assumptions on the greyvalue and its gradient
  - constancy assumptions without linearisation
  - robust penalisation with nonquadratic penalisers
  - flow-driven, spatiotemporal regularisation
- use a numerical strategy that finds a good local minimiser

A Highly Accurate Model (2)

Basic Assumptions

- greyvalue constancy $\quad f_{\mathbf{u}} := f(\mathbf{x}+\mathbf{u}) - f(\mathbf{x}) = 0$
- gradient constancy $\quad f_{x_1\mathbf{u}} := \partial_{x_1} f(\mathbf{x}+\mathbf{u}) - \partial_{x_1} f(\mathbf{x}) = 0$, $\quad f_{x_2\mathbf{u}} := \partial_{x_2} f(\mathbf{x}+\mathbf{u}) - \partial_{x_2} f(\mathbf{x}) = 0$
- spatiotemporal smoothness $\quad |\nabla_3 u_1|^2 + |\nabla_3 u_2|^2$ is small. $\nabla_3 = (\partial_{x_1}, \partial_{x_2}, \partial_{x_3})^\top$
- robustness $\quad \Psi\left(s^2\right) = \sqrt{s^2+\epsilon^2}$

Energy Functional

$$E(\mathbf{u}) = \int\limits_\Omega \Psi\left(f_{\mathbf{u}}^2 + \gamma \cdot \left(f_{x_1\mathbf{u}}^2 + f_{x_2\mathbf{u}}^2\right)\right) d\mathbf{x} + \alpha \int\limits_\Omega \Psi\left(|\nabla_3 u_1|^2 + |\nabla_3 u_2|^2\right) d\mathbf{x}$$

A Highly Accurate Model (3)

Euler-Lagrange Equations

$$\begin{aligned}
&\alpha \,\mathrm{div}\, \left(\Psi'(|\nabla_3 u_1|^2 + |\nabla_3 u_2|^2)\, \nabla_3 u_1\right) \\
&\quad = \Psi'\left(f_{\mathbf{u}}^2 + \gamma(f_{x_1\mathbf{u}}^2 + f_{x_2\mathbf{u}}^2)\right) \cdot \left(f_{x_1} f_{\mathbf{u}} + \gamma\left(f_{x_1x_1} f_{x_1\mathbf{u}} + f_{x_1x_2} f_{x_2\mathbf{u}}\right)\right) \\
&\alpha \,\mathrm{div}\, \left(\Psi'(|\nabla_3 u_1|^2 + |\nabla_3 u_2|^2)\, \nabla_3 u_2\right) \\
&\quad = \Psi'\left(f_{\mathbf{u}}^2 + \gamma(f_{x_1\mathbf{u}}^2 + f_{x_2\mathbf{u}}^2)\right) \cdot \left(f_{x_2} f_{\mathbf{u}} + \gamma\left(f_{x_1x_2} f_{x_2\mathbf{u}} + f_{x_2x_2} f_{x_2\mathbf{u}}\right)\right)
\end{aligned}$$

where the indices denote differences or partial derivatives:

$$\begin{array}{lll lll lll}
f_{\mathbf{u}} & := & f(\mathbf{x}+\mathbf{u}) - f(\mathbf{x}) & f_{x_1} & := & \partial_{x_1} f(\mathbf{x}+\mathbf{u}) & f_{x_2} & := & \partial_{x_2} f(\mathbf{x}+\mathbf{u}) \\
f_{x_1\mathbf{u}} & := & \partial_{x_1} f(\mathbf{x}+\mathbf{u}) - \partial_{x_1} f(\mathbf{x}) & f_{x_1x_1} & := & \partial_{x_1x_1} f(\mathbf{x}+\mathbf{u}) & f_{x_2x_2} & := & \partial_{x_2x_2} f(\mathbf{x}+\mathbf{u}) \\
f_{x_2\mathbf{u}} & := & \partial_{x_2} f(\mathbf{x}+\mathbf{u}) - \partial_{x_2} f(\mathbf{x}) & f_{x_1x_2} & := & \partial_{x_1x_2} f(\mathbf{x}+\mathbf{u}) & & &
\end{array}$$

This nonlinear elliptic system of PDEs has the structure

$$S(\mathbf{u}) = D(\mathbf{u})$$

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Minimisation with Warping

- widely used for optic flow computation with large displacements (e.g. Anandan 1989, Black/Anandan 1996, Mémin/Pérez 1998)
- downsample image data
- solve problem $S(\mathbf{u}) = D(\mathbf{u})$ at coarse scale
- use this flow field at next finer scale:
  warp image in order to compensate for this estimated motion
- solve modified problem (with other image data) at finer scale
- continue until finest scale reached
- sum up optic flow contributions from all scales

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Evaluation

We evaluate the quality of this method by

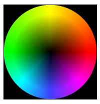
- quantitative comparisons with other methods:
  use synthetic sequences with ground truth
- investigating robustness under noise
- qualitative experiments with real-world data

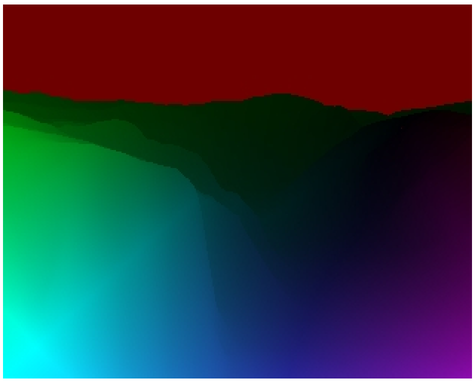
Evaluation (2)

Sequence

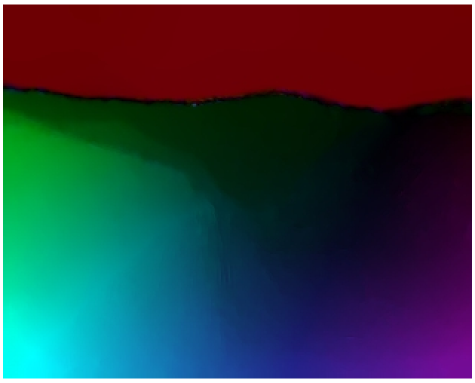
Yosemite Sequence

- synthetic sequence ($316 \times 252 \times 15$)
- known ground truth between frame 8 and frame 9





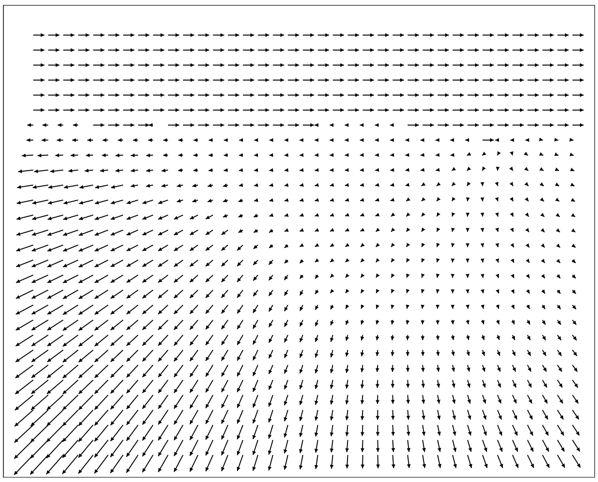
Ground Truth

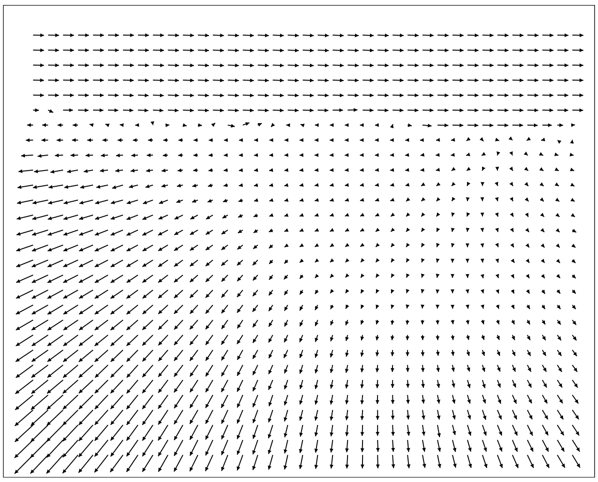


Computed Flow

Evaluation (3)

Qualitative Evaluation





Vector plot of the optic flow field for the Yosemite sequence **with** clouds. (a) **Left:** Ground truth. (b) **Right:** Computed flow.

Evaluation (4)

Quantitative Evaluation

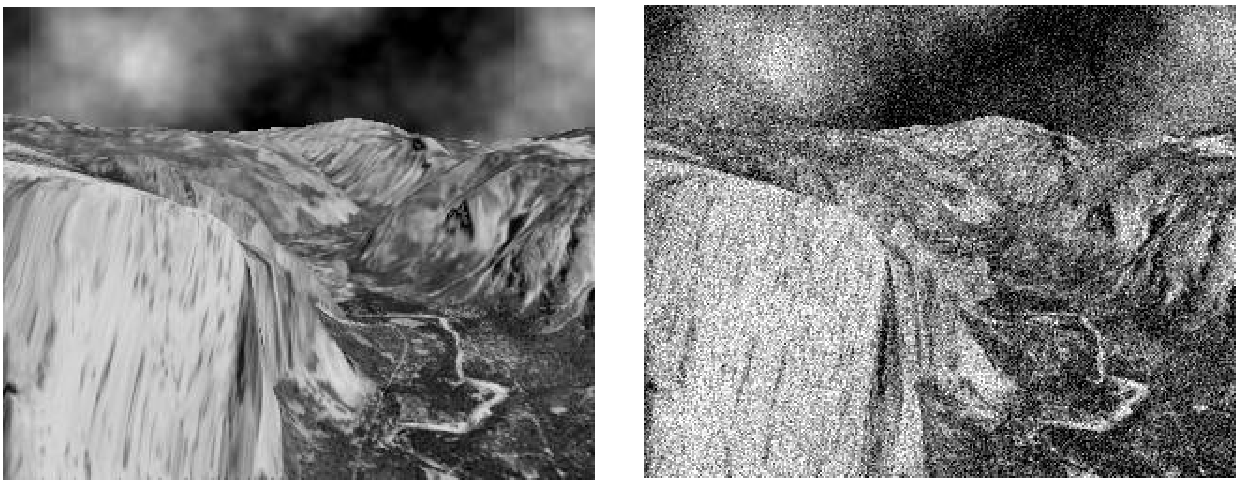
- Comparison to the best results from literature
- Average angular errors (AAE) for the Yosemite sequence **with** clouds (methods with 100 % density):

Yosemite with clouds	
Technique	AAE
Anandan 1989	13.36°
Nagel 1983	10.22°
Horn/Schunck, mod. 1981	9.78°
Uras *et al.* 1988	8.94°
Alvarez *et al.* 2000	5.53°
Bruhn *et al.* 2003	5.18°
Mémin/Pérez 1998	4.69°
our method 2004	1.94°

Evaluation (5)

Robustness under Noise

- Added Gaussian noise with zero mean and different standard deviations σ_n.



Frame 8 of the Yosemite sequence with clouds. (a) **Left:** Original. (b) **Right:** Gaussian noise with standard deviation $\sigma_n = 40$ added.

Evaluation (6)

- Even with $\sigma_n = 40$ the method outperforms all previous ones without noise:

Technique	AAE
Anandan 1989	13.36°
Nagel 1983	10.22°
Horn/Schunck, mod. 1981	9.78°
Uras *et al.* 1988	8.94°
Alvarez *et al.* 2000	5.53°
Weickert *et al.* 2003	5.18°
Mémin/Pérez 1998	4.69°
our method $(\sigma_n = 40)$	4.37°
our method $(\sigma_n = 30)$	3.77°
our method $(\sigma_n = 20)$	3.12°
our method $(\sigma_n = 10)$	2.50°
our method $(\sigma_n = 0)$	1.94°

Evaluation (8)

Robustness under Parameter Variations

- Three intuitive parameters:
  σ: Gaussian presmoothing of the input data
  α: weight of smoothness term
  γ: weight of gradient constancy term
- Parameter variation for the Yosemite sequence with clouds:

σ	α	γ	AAE
0.8	80	100	$1.94°$
0.4	"	"	$2.10°$
1.6	"	"	$2.04°$
0.8	80	100	$1.94°$
"	40	"	$2.67°$
"	160	"	$2.21°$
0.8	80	100	$1.94°$
"	"	50	$2.07°$
"	"	200	$2.03°$

- Deviations from the optimum by a factor 2 hardly influence the result.

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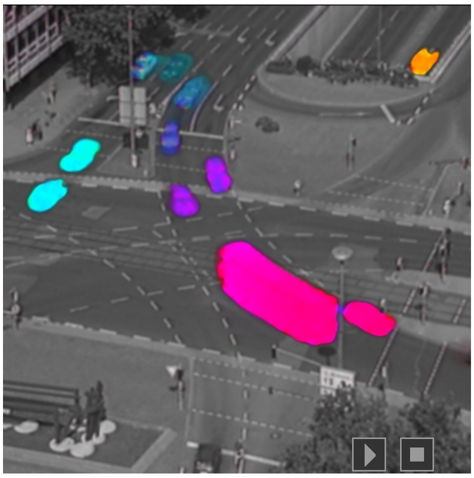
Real-World Data

- Real-world image sequence "Ettlinger Tor" by Nagel ($512 \times 512 \times 50$)

Sequence



Computed Flow

Illumination Changes

Problem

- Many OF methods are not robust under realistic illumination changes such as globally varying illumination, shadow/shading, highlights, specular reflections
- can create severe perturbations for driver assistance systems, robot navigation, ...

Classification of Illumination Changes

- global multiplicative changes
- local multiplicative changes: shadow, shading
- local additive changes: highlights, specular reflections

One can construct photometric invariants that are not influenced by these changes. In particular colour images offer a number of possibilities.

Illumination Changes (2)

Examples of Photometric Invariants

- **Log-Derivative Transform:**

$$(R, G, B)^\top \mapsto \big((\ln R)_x, (\ln R)_y, (\ln G)_x, (\ln G)_y, (\ln B)_x, (\ln B)_y\big)^\top$$

Invariant under global multiplicative illumination changes.

- **Chromaticity Space:**

$$(R, G, B)^\top \mapsto \left(\frac{R}{N}, \frac{G}{N}, \frac{B}{N}\right)^\top$$

with normalisation $N := \frac{1}{3}(R + G + B)$.
Invariant under global and local multiplicative illumination changes.

Illumination Changes (3)

- **HSV Colour Space:**
  - The HSV colour space (hue, saturation, value) is given by

$$(R, G, B)^\top \mapsto \left\{ \begin{array}{lcl} H & = & \left\{ \begin{array}{ll} \frac{G-B}{M-m} \times 60^\circ, & R \geq G, B, \\ (2 + \frac{B-R}{M-m}) \times 60^\circ, & G \geq R, B, \\ (4 + \frac{R-G}{M-m}) \times 60^\circ, & B \geq R, G, \end{array} \right. \quad (\bmod 360^\circ)\,, \\ S & = & \frac{M-m}{M} \\ V & = & M\,. \end{array} \right.$$

  where $M := \max(R, G, B)$ and $m := \min(R, G, B)$.
  - hue H denotes pure colour (angle)
  - saturation S is the achromatic component (radius)
  - value V denotes actual brightness (height)
  - The hue component H is invariant under global and local illumination changes as well as local additive changes.

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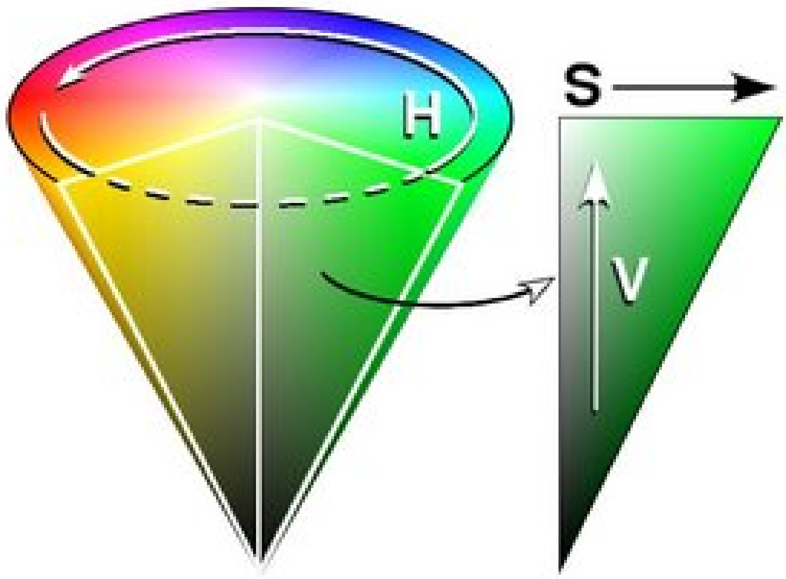
Illumination Changes (4)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26



HSV colour space. From Wikipedia.

Illumination Changes (5)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Incorporation of Photometric Invariants

- Let $f_1,...,f_N$ be photometric invariants. Then one chooses the data term

$$E_D(\mathbf{u}) \;:=\; \int_\Omega \psi_D\Big(\sum_{i=1}^{N} \gamma_i \left(f_i(\mathbf{x}+\mathbf{u}) - f_i(\mathbf{x})\right)^2\Big)\, d\mathbf{x}$$

  with positive weights $\gamma_1,...,\gamma_N$.

- no changes in the smoothness term

- yields models that offer advantages in case of illumination changes, if the photometric invariants do not discard too much information

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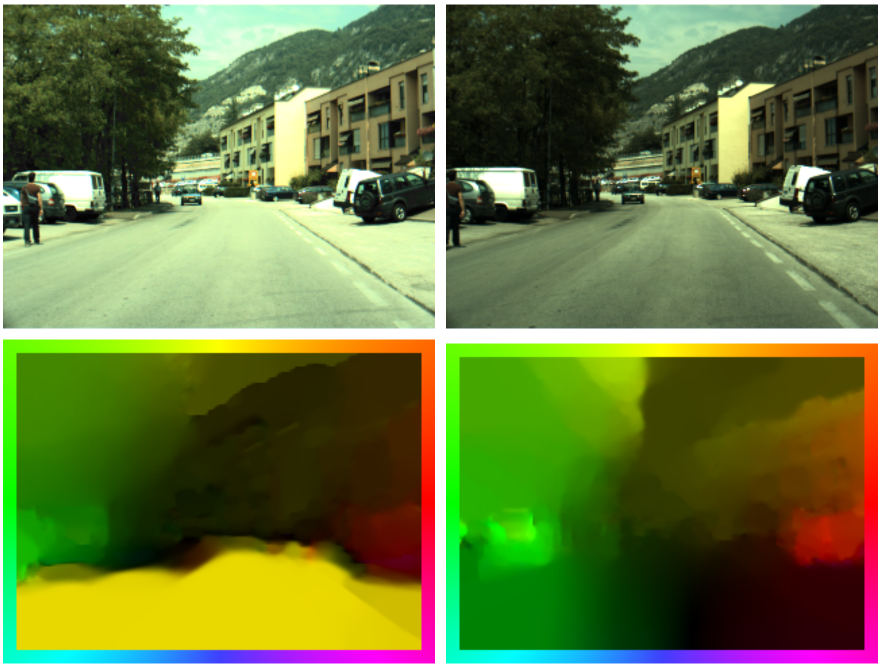

Results under real illumination conditions. **(a) Top left:** Left frame 205 of the *Road* stereo sequence of the DIPLODOC project (size 320×240). **(b) Top right:** Left frame 207. **(c) Bottom left:** Flow with RGB constancy assumption. At the road area, incorrect motion estimation takes place. **(d) Bottom right:** A hue constancy assumption performs better. From Mileva et al. (2007).

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Summary

- For large displacements, one should use constancy assumptions that refrain from linearisations.
- It is possible to design highly accurate variational methods when successful ingredients are used:
  - constancy assumptions on the greyvalue and its gradient
  - constancy assumptions without linearisation
  - robust penalisation with nonquadratic penalisers
  - flow-driven, spatiotemporal regularisation
- Illumination changes can be addressed by incorporating photometric invariants in the data term.

References

- T. Brox, A. Bruhn, N. Papenberg, J. Weickert: High accuracy optical flow estimation based on a theory for warping. In T. Pajdla, J. Matas (Eds.): *Computer Vision - ECCV 2004.* Lecture Notes in Computer Science, Vol. 3024, Springer, Berlin, 25–36, 2004.
  (`http://www.mia.uni-saarland.de/publications.shtml`)
  (discribes the highly accurate approach)
- N. Papenberg, A. Bruhn, T. Brox, S. Didas, J. Weickert: Highly accurate optic flow computation with theoretically justified warping. *International Journal of Computer Vision*, Vol. 67, No. 2, 141–158, April 2006.
  (`http://www.mia.uni-saarland.de/publications.shtml`)
  (more detailed journal version)
- Y. Mileva, A. Bruhn, J. Weickert: Illumination-invariant variational optical flow with photometric invariants. In F. Hamprecht, B. Jähne, C. Schnörr (Eds.): *Pattern Recognition.* Springer LNCS Vol. 4713, 152–162, 2007.
  (incorporation of photometric invariants)

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Assignment T5 – Theoretical Home Work

Problem 1 (Eigenvalues and Eigenvectors of Nagel's Method) (1+2 points)

(a) Let $\mathbf{v} \in \mathbb{R}^n$ be a vector. Then $\mathbf{v}\mathbf{v}^T \in \mathbb{R}^{n \times n}$ is a quadratic matrix. Which rank has this matrix? Determine its eigenvectors and eigenvalues.

(b) Nagel's optic flow method uses the regulariser

$$S_{AI}(\nabla f, \nabla \mathbf{u}) \;:=\; \sum_{i=1}^{2} \nabla u_i^\top \, D(\nabla f)\, \nabla u_i$$

where $D(\nabla f)$ is a regularised projection matrix on $\nabla f^\perp := (f_{x_2}, -f_{x_1})^\top$:

$$D(\nabla f) \;:=\; \frac{1}{|\nabla f|^2 + 2\lambda^2} \left(\nabla f^\perp \nabla f^{\perp\top} + \lambda^2 I \right).$$

Show that D has the eigenvectors $\mathbf{v}_1 = \nabla f$ and $\mathbf{v}_2 = \nabla f^\perp$ with corresponding eigenvalues

$$\lambda_1(|\nabla f|) \;=\; \frac{\lambda^2}{|\nabla f|^2 + 2\lambda^2}, \qquad \lambda_2(|\nabla f|) \;=\; \frac{|\nabla f|^2 + \lambda^2}{|\nabla f|^2 + 2\lambda^2}.$$

Assignment T5 (2)

Problem 2 (Motion Tensors) (1+1 points)

Derive the motion tensors for the following constancy assumptions:

(a) constancy of the Laplacian,

(b) constancy of the spatial image gradient magnitude of an RGB image.

Problem 3 (Least Squares in Motion Estimation) (2+1+1 points)

Consider constancy assumptions on some image features p_1,...,p_n and a data term of type

$$M \;=\; \sum_{i=1}^{n} \gamma_i (\mathbf{u}^\top \nabla_3 p_i)^2 \;=\; \mathbf{u}^\top J \mathbf{u}$$

with a motion tensor

$$J := \sum_{i=1}^{n} \gamma_i \nabla_3 p_i \nabla_3 p_i^\top .$$

(a) Interpret the minimisation of M as a least squares estimation problem $\min_{\tilde{\mathbf{u}}} |A\tilde{\mathbf{u}} - \mathbf{b}|^2$ with $\tilde{\mathbf{u}} := (u_1, u_2)^\top$. Specify A and $\mathbf{b}$ in terms of the entries of the motion tensor.

(b) Derive a corresponding linear system of equations that has to be satisfied in this minimum.

(c) What happens if the corresponding system matrix has rank 2, 1, or 0?

Assignment T5 (3)

Problem 4 (Design of Global Optic Flow Methods) (3 points)

Design an energy functional based on spatio-temporal regularisation that allows an accurate computation of the optic flow for *rotational motion* under *additive illumination changes*. Pay attention to specify correctly the domain of integration as well as differential operators and integration variables. How do the energy functional and the Euler–Lagrange equations look like?

Deadline for submission: Friday, June 20, 10 am (before the lecture).