Lecture 16:

Image Sequence Analysis I: Models for the Smoothness Term

Contents

Introduction

The Optic Flow Problem

- *given:* image sequence $f(x_1, x_2, x_3)$ where (x_1, x_2) is location and x_3 time
  f can be Gaussian-smoothed: $f = K_\sigma * f_0$
- *wanted:* displacement field *(optic flow, OF)* between subsequent frames

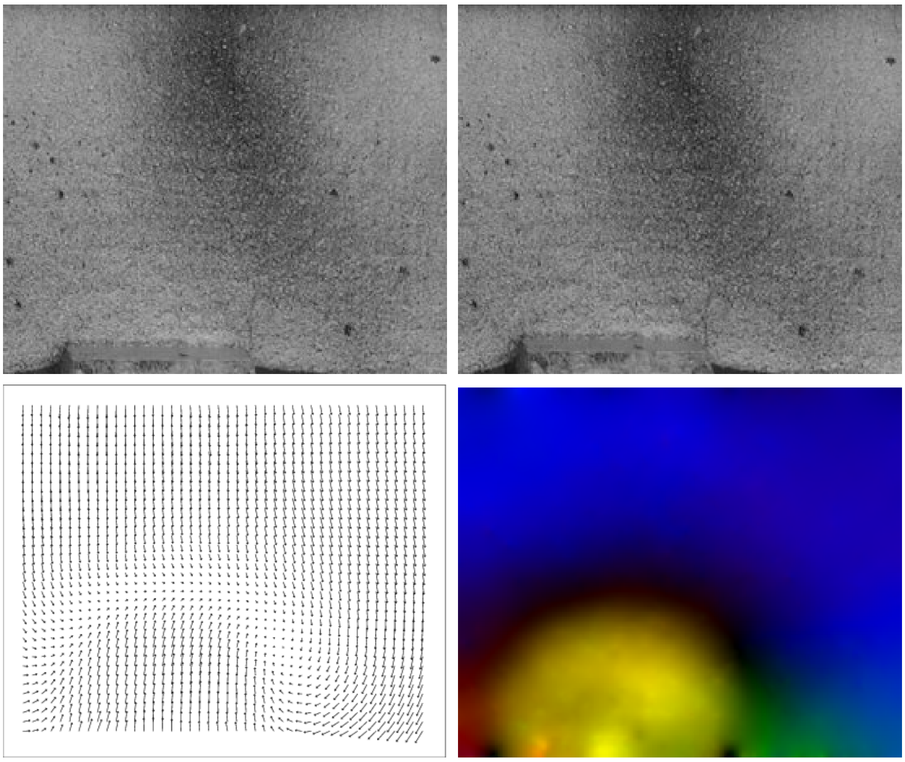
$$\mathbf{u} = \begin{pmatrix} u_1(x_1, x_2, x_3) \\ u_2(x_1, x_2, x_3) \\ 1 \end{pmatrix}$$

What is Optic Flow Good for?

- extracting motion information e.g. in robotics
- compact coding of image sequences
- related correspondence problems in computer vision:
  e.g. stereo reconstruction and medical image registration

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Introduction (2)



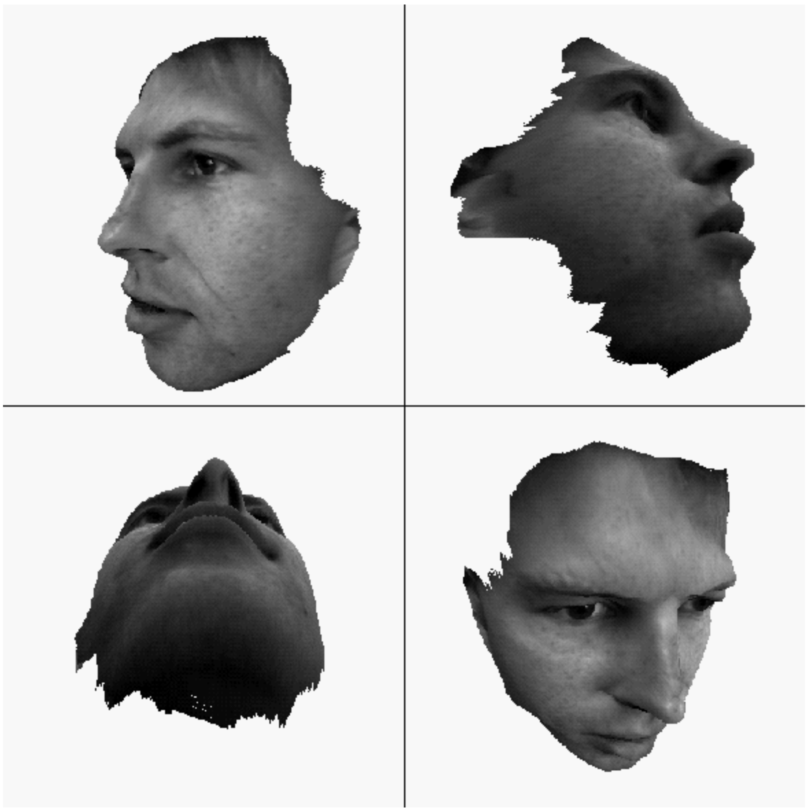
Deformation analysis of plastic foam using an optic flow method. (a) **Top left:** Frame 1 of a deformation sequence. (b) **Top right:** Frame 2. (c) **Bottom left:** Vector plot of the displacement field. (d) **Bottom right:** Colour-coded displacement field.

Introduction (3)

Pair of stereo images.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Introduction (4)



Four views of a stereo reconstruction algorithm based on optic flow ideas. **Authors:** Alvarez/Deriche/Sánchez/W. (2002)

Introduction (5)

Hairstyle simulation using an optic flow based registration approach. (a) **Left:** Customer face. (b) **Middle:** Model face with desired hairstyle. (c) **Right:** Optic flow computes a registration from the model face to the customer face. By adapting the hairstyle in the same way, a convincing registration to the customer face is obtained. **Author:** O. Demetz (2006).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Variational Optic Flow Methods

- optic flow as minimiser of a suitable energy functional:
  data constraints plus smoothness constraints
- first model due to Horn and Schunck (1981),
  but many improvements in the meantime:
  - modified data and smoothness constraints
  - theoretical foundation
  - efficient numerical algorithms
- competitive performance

M I A

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

General Structure

Variational OF methods recover the optic flow field $\mathbf{u}(x_1, x_2, x_3)$ as minimiser of

$$E(\mathbf{u}) = \int_\Omega \Big(\underbrace{M(D^k f, \mathbf{u})}_{\text{data term}} + \alpha \underbrace{S(\nabla f, \nabla \mathbf{u})}_{\text{regulariser}} \Big)\, d\mathbf{x}$$

- Ω is either a spatial or a spatiotemporal domain
- spatial case: $\mathbf{x} := (x_1, x_2)^\top$ and $\nabla := \nabla_2 := (\partial_{x_1}, \partial_{x_2})^\top$
  spatiotemporal case: $\mathbf{x} := (x_1, x_2, x_3)^\top$ and $\nabla := \nabla_3 := (\partial_{x_1}, \partial_{x_2}, \partial_{x_3})^\top$
- $D^k f$: spatiotemporal partial derivatives of order k
- data term $M(D^k f, \mathbf{u})$ penalises deviations from constancy assumptions
- smoothness term $S(\nabla f, \nabla \mathbf{u})$ penalises deviations from (piecewise) smoothness ($\nabla \mathbf{u} := (\nabla u_1, \nabla u_2)$)
- regularisation parameter $\alpha > 0$ determines smoothness

Specific Features of Variational OF Models

- transparent formalism without hidden model assumptions
- continuous energy functional allows rotationally invariant modelling
- can be arbitrarily well approximated by consistent numerical methods
- strictly convex functionals allow unique minimiser and well-posedness
- minimiser can be found by relatively simple globally convergent algorithms (such as gradient descent)
- global method giving *filling-in effect* for small data terms: dense flow fields without subsequent interpolation steps

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

The Data Term

Brightness Constancy Assumption

- most frequently used optic flow assumption
- The brightness f is supposed not to change along the path $(x_1(x_3), x_2(x_3))$ of a moving structure:

$$\begin{aligned} 0 &= \frac{df(x_1(x_3), x_2(x_3), x_3)}{dx_3} \\ &= f_{x_1}\partial_{x_3}x_1 + f_{x_2}\partial_{x_3}x_2 + f_{x_3} \\ &= f_{x_1}u_1 + f_{x_2}u_2 + f_{x_3} \end{aligned}$$

  where $f_{x_i} := \partial_{x_i} f$, and the OF component $u_i = \partial_{x_3} x_i$ gives the velocity in direction x_i.

- Deviations from the brightness constancy assumption can be penalised by the data term (Horn/Schunck 1981)

$$M(D^1 f, \mathbf{u}) := (f_{x_1}u_1 + f_{x_2}u_2 + f_{x_3})^2$$

The Smoothness Term

Diffusion Processes Induced by the Smoothness Constraint

Minimising

$$E(\mathbf{u}) = \int_\Omega \Big(M(D^k f, \mathbf{u}) \; + \; \alpha \, S(\nabla f, \nabla \mathbf{u}) \Big) \, d\mathbf{x}$$

by means of steepest descent gives the diffusion–reaction system

$$\begin{aligned} \partial_t u_1 &= \partial_{x_1} S_{u_{1,x_1}} + \partial_{x_2} S_{u_{1,x_2}} - \frac{1}{\alpha} \partial_{u_1} M, \\ \partial_t u_2 &= \partial_{x_1} S_{u_{2,x_1}} + \partial_{x_2} S_{u_{2,x_2}} - \frac{1}{\alpha} \partial_{u_2} M \end{aligned}$$

where $S_{u_{i,x_j}}$ denotes the partial derivative of S with respect to $\partial_{x_j} u_i$.

- diffusion processes caused by smoothness term $S(\nabla f, \nabla \mathbf{u})$, reaction processes caused by data term $M(D^k f, \mathbf{u})$
- steady state for $t \to \infty$ gives minimising flow field $u(x_1, x_2, x_3)$.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Diffusion Filtering of Multichannel Images

For filtering some multichannel image

$$\mathbf{f} = (f_1(\mathbf{x}), ..., f_m(\mathbf{x}))^\top$$

one of the following diffusion processes can be used (cf. also Lecture 14):

(a)	*Homogeneous diffusion:*	$\partial_t u_i = \Delta u_i$
(b)	*Linear isotropic diffusion:*	$\partial_t u_i = \text{div}\,(g(\nabla \mathbf{f})\, \nabla u_i)$
(c)	*Linear anisotropic diffusion:*	$\partial_t u_i = \text{div}\,(D(\nabla \mathbf{f})\, \nabla u_i)$
(d)	*Nonlinear isotropic diffusion:*	$\partial_t u_i = \text{div}\,(g(\nabla \mathbf{u})\, \nabla u_i)$
(e)	*Nonlinear anisotropic diffusion:*	$\partial_t u_i = \text{div}\,(D(\nabla \mathbf{u})\, \nabla u_i)$

with $\mathbf{f}$ as initial condition:

$$u_i(\mathbf{x}, 0) = f_i(\mathbf{x}) \qquad\qquad (i = 1, ..., m).$$

The Smoothness Term (3)

Diffusion filtering of a colour image. (a) **Top Left:** Noisy original image. (b) **Top Middle:** Homogeneous diffusion. (c) **Top Right:** Linear isotropic diffusion. (d) **Bottom Left:** Linear anisotropic diffusion. (e) **Bottom Middle:** Nonlinear isotropic diffusion. (f) **Bottom Right:** Nonlinear anisotropic diffusion.

The Smoothness Term (4)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Table illustrating the one-to-one correspondence between vector-valued diffusion filtering and optic flow regularisation.

optic flow regulariser $S(\nabla f, \nabla \mathbf{u})$	vector-valued diffusion process $\partial_t u_i = \partial_{x_1} S_{u_{i,x_1}} + \partial_{x_2} S_{u_{i,x_2}}$
homogeneous (Horn/Schunck 1981) $\sum_i \lvert\nabla u_i\rvert^2$	homogeneous (Iijima 1959) $\partial_t u_i = \Delta u_i$
image-driven, isotr. (Alvarez et al. 1999) $g(\lvert\nabla f\rvert^2) \sum_i \lvert\nabla u_i\rvert^2$	linear isotropic (Fritsch 1992) $\partial_t u_i = \mathrm{div}\ \left(g(\lvert\nabla \mathbf{f}\rvert^2)\ \nabla u_i\right)$
image-driven, anisotropic (Nagel 1983) $\sum_i \nabla u_i^\top D(\nabla f)\, \nabla u_i$	linear anisotropic (Iijima 1962) $\partial_t u_i = \mathrm{div}\ (D(\nabla \mathbf{f})\ \nabla u_i)$
flow-driven, isotropic (Schnörr 1994) $\Psi\Big(\sum_i \lvert\nabla u_i\rvert^2\Big)$	nonlinear isotr. (Gerig et al. 1992) $\partial_t u_i = \mathrm{div}\ (\Psi'(\sum_k \lvert\nabla u_k\rvert^2)\ \nabla u_i)$
flow-driven, anisotr. (W./Schnörr 2001) $\mathrm{tr}\,\Psi\Big(\sum_i \nabla u_i \nabla u_i^\top\Big)$	nonlinear anisotropic (W. 1994) $\partial_t u_i = \mathrm{div}\ \Big(\Psi'(\sum_k \nabla u_k \nabla u_k^\top)\ \nabla u_i\Big)$

The Smoothness Term (5)

Homogeneous Regularisation (Horn/Schunck 1981)

- uses the Whittaker-Tikhonov regulariser

$$S_H(\nabla f, \nabla \mathbf{u}) \;:=\; |\nabla u_1|^2 + |\nabla u_2|^2.$$

- corresponds to homogeneous diffusion

$$\partial_t u_i \;=\; \Delta u_i$$

- Problem: blurs across motion boundaries

The Smoothness Term (6)

Isotropic Image-Driven Regularisation (Alvarez et al. 1999)

- many flow boundaries coincide with image boundaries
- reduce smoothing at *image* boundaries:

$$S_{II}(\nabla f, \nabla \mathbf{u}) \;:=\; g(|\nabla f|^2)\,\left(|\nabla u_1|^2 + |\nabla u_2|^2\right)$$

with some decreasing smooth function g, such as

$$g(s^2) \;:=\; \varepsilon + \frac{1-\varepsilon}{\sqrt{1+s^2/\lambda^2}}$$

- corresponds to linear isotropic diffusion

$$\partial_t u_i \;=\; \text{div}\,\left(g(|\nabla f|^2)\,\nabla u_i\right)$$

- Problems:
  - Smoothing along image edges without blurring across them would be useful. This requires anisotropic models.
  - Textured objects have more images boundaries than flow boundaries. This gives oversegmented flow fields.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

The Smoothness Term (7)

Anisotropic Image-Driven Regularisation (Nagel 1983)

- reduce smoothing *across* image boundaries, permit smoothing *along* them
- consider regulariser

$$S_{AI}(\nabla f, \nabla \mathbf{u}) \;:=\; \sum_{i=1}^{2} \nabla u_i^\top \, D(\nabla f) \, \nabla u_i$$

where $D(\nabla f)$ is a regularised projection matrix on $\nabla f^\perp := (f_{x_2}, -f_{x_1})^\top$:

$$D(\nabla f) \;:=\; \frac{1}{|\nabla f|^2 + 2\lambda^2} \left(\nabla f^\perp \nabla f^{\perp\top} + \lambda^2 I \right),$$

- corresponding diffusion is linear and anisotropic:

$$\partial_t u_i \;=\; \mathrm{div}\,\left(D(\nabla f)\, \nabla u_i \right)$$

where D has eigenvectors ∇f, $\nabla f^\perp$ with eigenvalues

$$\lambda_1(|\nabla f|) \;=\; \frac{\lambda^2}{|\nabla f|^2 + 2\lambda^2}, \qquad \lambda_2(|\nabla f|) \;=\; \frac{|\nabla f|^2 + \lambda^2}{|\nabla f|^2 + 2\lambda^2}.$$

The Smoothness Term (8)

- isotropic behaviour at flat image regions:

$$\lim_{|\nabla f| \to 0} \lambda_1(|\nabla f|) \;=\; \frac{1}{2},$$
$$\lim_{|\nabla f| \to 0} \lambda_2(|\nabla f|) \;=\; \frac{1}{2}.$$

- anisotropic behaviour at image edges:

$$\lim_{|\nabla f| \to \infty} \lambda_1(|\nabla f|) \;=\; 0,$$
$$\lim_{|\nabla f| \to \infty} \lambda_2(|\nabla f|) \;=\; 1.$$

- Problem: may still suffer from oversegmented flow fields when the scene is strongly textured

The Smoothness Term (9)

Isotropic Flow-Driven Regularisation (Schnörr 1994)

- goal: reduce smoothing at *flow* discontinuities
- use regularisers of type

$$S_{IF}(\nabla f, \nabla \mathbf{u}) \;:=\; \Psi\left(|\nabla u_1|^2 + |\nabla u_2|^2\right),$$

  where $\Psi(s^2)$ is increasing, differentiable, convex in s, and there exist $c_1, c_2 > 0$ such that $c_1 s^2 \leq \Psi(s^2) \leq c_2 s^2$.

- Example: Charbonnier penaliser

$$\Psi(s^2) \;=\; 2\lambda^2 \sqrt{1 + s^2/\lambda^2} - 2\lambda^2.$$

- S_{IF} corresponds to the isotropic nonlinear diffusion processes

$$\partial_t u_i \;=\; \operatorname{div}\left(\Psi'\left(|\nabla u_1|^2 + |\nabla u_2|^2\right)\, \nabla u_i\right)$$

- Problem: Smoothing along flow edges would be desirable.
  This requires anisotropic models.

The Smoothness Term (10)

Anisotropic Flow-Driven Regularisation (W./Schnörr 2001)

- reduce smoothing *across* flow discontinuities, but permit smoothing *along* them
- uses regulariser

$$S_{AF}(\nabla f, \nabla \mathbf{u}) \;:=\; \operatorname{tr} \Psi\left(\nabla u_1 \nabla u_1^\top + \nabla u_2 \nabla u_2^\top\right)$$

- creates the anisotropic nonlinear diffusion processes

$$\partial_t u_i \;=\; \operatorname{div}\left(D(\nabla \mathbf{u})\, \nabla u_i\right)$$

  with a diffusion tensor

$$D(\nabla \mathbf{u}) \;:=\; \Psi'\left(\nabla u_1 \nabla u_1^\top + \nabla u_2 \nabla u_2^\top\right).$$

- Let $\mathbf{v}_1$, $\mathbf{v}_2$ denote the eigenvectors of $\nabla u_1 \nabla u_1^\top + \nabla u_2 \nabla u_2^\top$, and μ_1, μ_2 the corresponding eigenvalues.
  Then $D(\nabla \mathbf{u})$ has eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ with eigenvalues $\Psi'(\mu_1)$ and $\Psi'(\mu_2)$.
  This is exactly the desired anisotropy.

The Smoothness Term (11)

- Note that isotropic and anisotropic regularisers result from a simple interchange of trace operator and penaliser:

$$S_{IF}(\nabla f, \nabla \mathbf{u}) \quad := \quad \Psi\Big(\mathrm{tr} \sum_i \nabla u_i \nabla u_i^\top\Big),$$

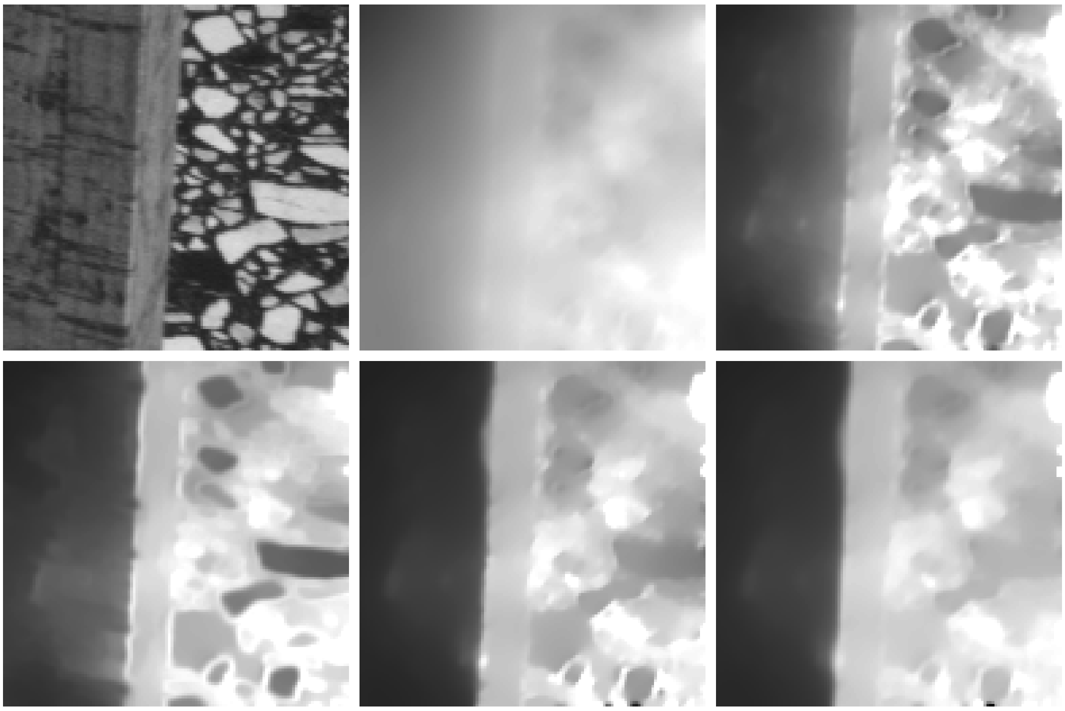
$$S_{AF}(\nabla f, \nabla \mathbf{u}) \quad := \quad \mathrm{tr}\, \Psi\Big(\sum_i \nabla u_i \nabla u_i^\top\Big).$$

The Smoothness Term (12)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

(a) **Top left:** Frame 16 of the marbled block sequence (512×512 pixels). (b) **Top middle:** Optic flow magnitude between Frame 16 and 17 for homogeneous regularisation. (c) **Top right:** Result for image-driven isotropic regularisation (d) **Bottom left:** Image-driven anisotropic regularisation. (e) **Bottom middle:** Flow-driven isotropic regularisation (f) **Bottom right:** Flow-driven anisotropic regularisation.

The Smoothness Term (13)



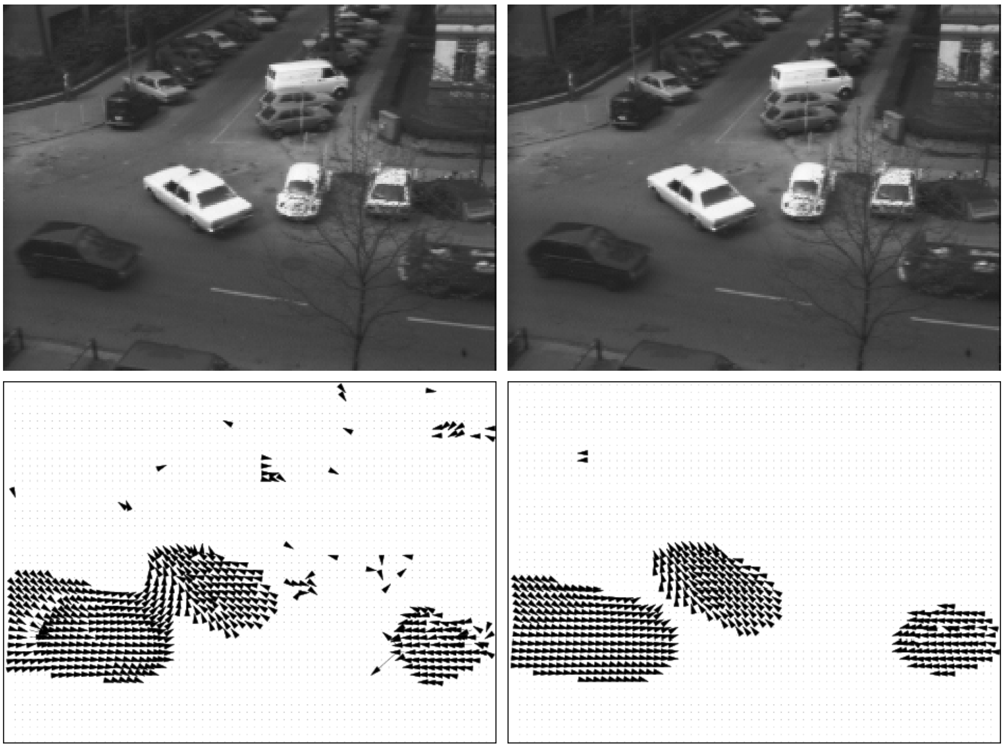
(a) **Top left:** Detail from the lower right part of Frame 16 (128×128 pixels). (b) **Top middle:** Optic flow magnitude for homogeneous regularisation. (c) **Top right:** Image-driven isotropic regularisation (d) **Bottom left:** Image-driven anisotropic regularisation. (e) **Bottom middle:** Flow-driven isotropic regularisation (f) **Bottom right:** Flow-driven anisotropic regularisation.

The Smoothness Term (14)

Spatial versus Spatiotemporal Regularisation

- Extending the spatial formulation into the spatiotemporal setting leads to 3-D diffusion–reaction instead of 2-D diffusion–reaction
- first proposed by Nagel (1990), but not implemented at that time
- computational cost not much higher than 2-D approach for an entire sequence
- memory requirements uncritical for typical test sequences
- better results than with pure spatial regularisers

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34



(a) **Top Left:** Frame 10 of the Hamburg taxi sequence. (b) **Top Right:** Frame 11. (c) **Bottom Left:** Spatial processing. (d) **Bottom Right:** Spatio-temporal processing.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Well-Posedness

Assumptions

- $\Psi(s^2)$ is differentiable and strictly convex in $s \in \mathbb{R}$.
- There exist $c_1, c_2 > 0$ such that $c_1 s^2 \leq \Psi(s^2) \leq c_2 s^2$ for all s.
- The initial data are sufficiently smooth: $f \in H^1(\Omega)$.
- f_{x_1} and f_{x_2} are linearly independent in $L^2(\Omega)$ and have finite $L^\infty(\Omega)$ norm.

Well-Posedness Result (W./Schnörr 2001)

The energy functional

$$E(\mathbf{u}) = \int_\Omega \Big((f_{x_1} u_1 + f_{x_2} u_2 + f_{x_3})^2 \; + \; \alpha \, S(\nabla f, \nabla \mathbf{u}) \Big) \, d\mathbf{x}$$

has a unique minimiser $(u_1, u_2) \in H^1(\Omega) \times H^1(\Omega)$.
It depends in a continuous way on the image sequence f.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Extensions

Design Principle for Anisotropic Regularisers

Assume that we are given some isotropic regulariser $\Psi(\sum_i |\nabla u_i|^2)$, and a decomposition of its argument

$$\sum_i |\nabla u_i|^2 = \sum_j \rho_j,$$

where the ρ_j are rotationally invariant expressions.
Then the regulariser $\sum_j \Psi(\rho_j)$ is rotationally invariant and anisotropic.

Remark

The transition from isotropic to anisotropic is just an exchange of the order of penalisation and summation:

- isotropic: $\Psi\left(\sum_j \rho_j\right)$
- anisotropic: $\sum_j \Psi(\rho_j)$

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Example 1

Previously used decomposition was trace identity

$$|\nabla u_1|^2 + |\nabla u_2|^2 \;=\; \mathrm{tr}\,(J) \;=\; \mu_1 + \mu_2$$

where μ_1, μ_2 are eigenvalues of

$$J = \nabla u_1 \nabla u_1^\top + \nabla u_2 \nabla u_2^\top .$$

Then the isotropic regulariser

$$\Psi(|\nabla u_1|^2 + |\nabla u_2|^2) = \Psi(\mu_1 + \mu_2)$$

induces the anisotropic regulariser

$$\Psi(\mu_1) + \Psi(\mu_2) \;=\; \mathrm{tr}\,\Psi(J) \;=\; \mathrm{tr}\,\Psi(\nabla u_1 \nabla u_1^\top + \nabla u_2 \nabla u_2^\top).$$

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Example 2

The identity

$$|\nabla u_1|^2 + |\nabla u_2|^2 = \frac{1}{2}\left(\mathrm{div}^2\mathbf{u} + \mathrm{curl}^2\mathbf{u} + \mathrm{sh}^2\mathbf{u}\right)$$

with

$$\begin{aligned} \mathrm{curl}\,\mathbf{u} &:= u_{2x_1} - u_{1x_2} \\ \mathrm{sh}\,\mathbf{u} &:= \sqrt{(u_{2x_2} - u_{1x_1})^2 + (u_{1x_2} + u_{2x_1})^2} \end{aligned}$$

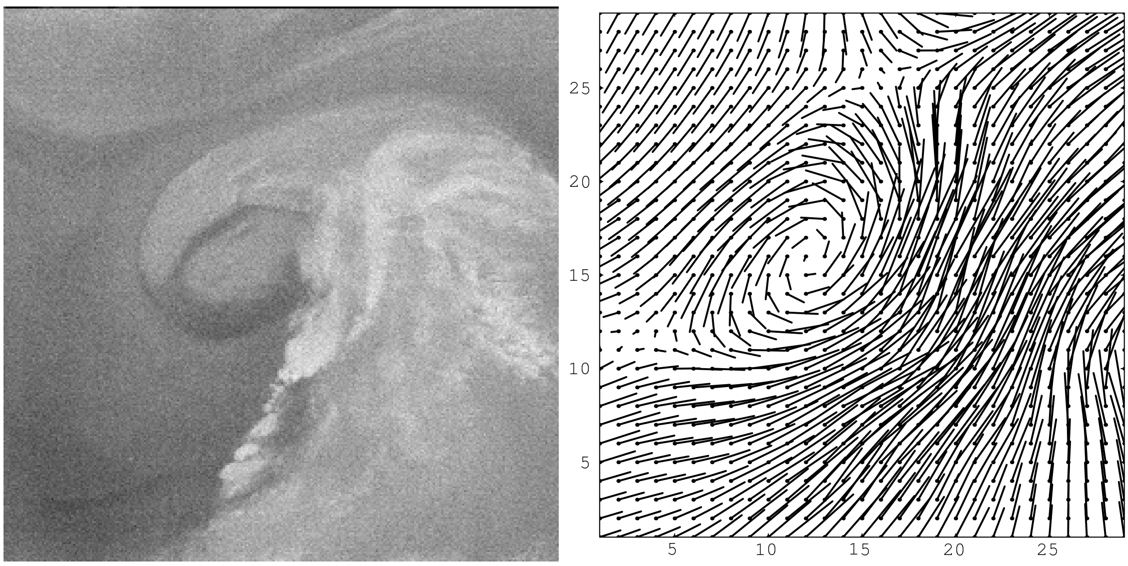
gives the anisotropic regulariser (Schnörr 1994)

$$V_{AFS}(\nabla f, \nabla \mathbf{u}) := \Psi\left(\mathrm{div}^2\mathbf{u}\right) + \Psi\left(\mathrm{curl}^2\mathbf{u}\right) + \Psi\left(\mathrm{sh}^2\mathbf{u}\right).$$

This creates the highly anisotropic processes

$$\begin{aligned} \partial_t u_1 &= \partial_{x_1}\left(\left(\Psi'(\mathrm{div}^2\mathbf{u}) + \Psi'(\mathrm{sh}^2\mathbf{u})\right) u_{1x_1} + \left(\Psi'(\mathrm{div}^2\mathbf{u}) - \Psi'(\mathrm{sh}^2\mathbf{u})\right) u_{2x_2}\right) \\ &\quad + \partial_{x_2}\left(\left(\Psi'(\mathrm{sh}^2\mathbf{u}) + \Psi'(\mathrm{curl}^2\mathbf{u})\right) u_{1x_2} + \left(\Psi'(\mathrm{sh}^2\mathbf{u}) - \Psi'(\mathrm{curl}^2\mathbf{u})\right) u_{2x_1}\right), \\ \partial_t u_2 &= \partial_{x_1}\left(\left(\Psi'(\mathrm{sh}^2\mathbf{u}) - \Psi'(\mathrm{curl}^2\mathbf{u})\right) u_{1x_2} + \left(\Psi'(\mathrm{sh}^2\mathbf{u}) + \Psi'(\mathrm{curl}^2\mathbf{u})\right) u_{2x_1}\right) \\ &\quad + \partial_{x_2}\left(\left(\Psi'(\mathrm{div}^2\mathbf{u}) - \Psi'(\mathrm{sh}^2\mathbf{u})\right) u_{1x_1} + \left(\Psi'(\mathrm{div}^2\mathbf{u}) + \Psi'(\mathrm{sh}^2\mathbf{u})\right) u_{2x_2}\right). \end{aligned}$$

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34



Left: Frame from a cyclone image sequence. **Right:** Optic flow field with the regulariser V_{AFS}.

Summary

- Rotationally invariant convex optic flow regularisers can be classified using a diffusion framework.
- This taxonomy includes the following regularisers:
  - isotropic and anisotropic ones
  - image-driven and flow-driven ones
  - spatial and spatio-temporal ones
- Well-posedness results exist for all these methods.
- There are interesting structural similarities between isotropic and anisotropic flow-driven models.
- There is a design principle for creating anisotropic regularisers from isotropic ones.

References

- B. Horn, B. Schunck: Determining optical flow. *Artificial Intelligence*, Vol. 17, 185–203, 1981. *(homogeneous regularisation)*
- L. Alvarez, J. Esclarín, M. Lefébure, J. Sánchez: A PDE model for computing the optical flow. *Proc. XVI Congreso de Ecuaciones Diferenciales y Aplicaciones* (Las Palmas de Gran Canaria, Spain, Sept. 1999), 1349–1356, 1999. *(image-driven isotropic regularisation)*
- H.-H. Nagel: Constraints for the estimation of displacement vector fields from image sequences. *Proc. Eighth International Joint Conference on Artificial Intelligence* (Karlsruhe, Germany, Aug. 1983), Vol. 2, 945–951, 1983. *(image-driven anisotropic regularisation)*
- C. Schnörr: Segmentation of visual motion by minimizing convex non-quadratic functionals. *Proc. Twelfth International Conference on Pattern Recognition* (Jerusalem, Israel, Oct. 1994), Vol. A, 661–663, 1994. *(flow-driven isotropic regularisation)*
- J. Weickert, C. Schnörr: A theoretical framework for convex regularizers in PDE-based computation of image motion. *International Journal of Computer Vision*, Vol. 45, No. 3, 245–264, Dec. 2001. *(flow-driven anisotropic regularisation, diffusion taxonomy, well-posedness)*
- J. Weickert, C. Schnörr: Variational optic flow computation with a spatio-temporal smoothness constraint. *Journal of Mathematical Imaging and Vision*, Vol. 14, No. 3, 245–255, May 2001. *(spatio-temporal regularisation)*

Assignment P4 – Programming

You can download the file Ex04.tar from the web page

`http://www.mia.uni-saarland.de/Teaching/dic08.shtml`

To unpack these data, use `tar xvf Ex04.tar`.

1. The object code `inp.o` has all routines on board in order to become an explicit scheme for edge-enhancing-diffusion (EED) based inpainting, except for routines for calculating the diffusion tensor and for computing the impainting from the results of an explicit EED step.
   - Supplement the file `inpainting.c` with the missing code. You may use the included routines for principal axis transformation and backtransformation. Compile the programme with `gcc -O2 -o inp inp.o inpainting.c -lm`.
   - Use the obtained programme `inp` to inpaint the images `trui.pgm` and `neworleans.pgm`. The required inpainting masks are given by `trui_mask.pgm` and `neworleans_mask.pgm`. Try to find good parameters for λ and σ.
   - In order to determine the quality of your inpainted results compute the average Euclidean error (AEE) and the average absolute error (AAE) compared to the original (complete) image `trui_orig.pgm`. To this end, supplement the file `errors.c` with the missing code.

M I A

For assessment:
Use `tar -cvzf P4_`*yourname*`.tgz` *file1 file2 …* to pack the following files into an archive:

- the supplemented file `inpainting.c`
- two results for variational image inpainting for each of the two test images
- the supplemented file `errors.c`
- the corresponding error measures for all your results

Include a short `README` file stating the parameters used in filtering all of the submitted images. Send the archive by e-mail to the address specifiec by your tutor.

Deadline for electronic submission: Friday, June 13, 10 am.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34