

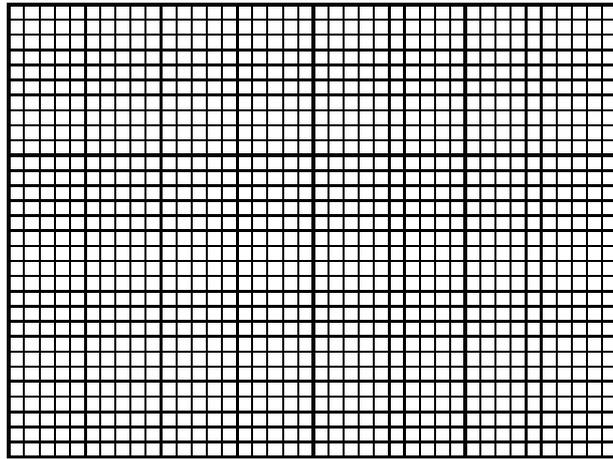
**Differential Equations in
Image Processing and Computer Vision
Self-Test Problems**

This problem sheet contains 6 problems which are intended to be similar in style and difficulty to a 180-minute written exam on DIC.

Problem 1 (Nonlinear Isotropic Diffusion)

Suppose we are given the diffusivity $g(s^2) = \exp\left(-\frac{cs^4}{\lambda^4}\right)$ with a constant c in a nonlinear isotropic diffusion process for an evolving image u .

- a) What is the associated flux function?
- b) Sketch this function on the rectangular grid given below.
- c) Determine the constant c such that the contrast parameter λ coincides with the boundary between forward and backward diffusion.



Problem 2 (Image Regularisation)

Consider the following energy functional for image regularisation:

$$E(u) = \int_{\Omega} (\beta \Psi((u - f)^2) + (1 - \beta) u_x^2) dx ,$$

where $\beta \in [0, \dots, 1]$.

- a) Derive a discrete version of this energy functional that uses a vector $\mathbf{u} = (\mathbf{u}_1, \dots, \mathbf{u}_n)^T \in \mathbb{R}^n$ instead of an function u as an argument. Take the spatial grid size to be $h > 0$ and assume homogeneous Neumann boundary conditions.
 - b) For which task this discrete energy functional could also be used if β is not set fixed but allowed to vary dependent on the location x ?
 - c) State the necessary conditions a vector $\mathbf{u} \in \mathbb{R}^n$ has to satisfy in order to minimize the discrete energy functional of part a).
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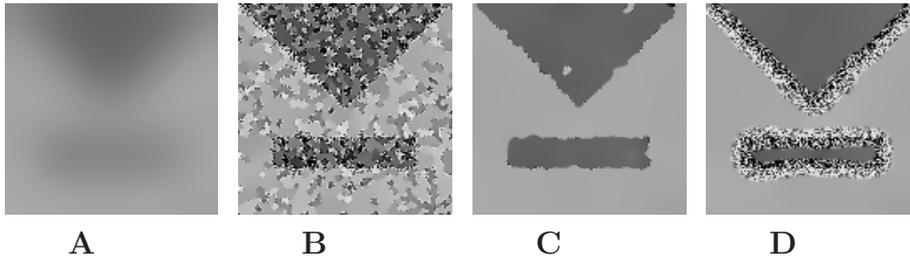
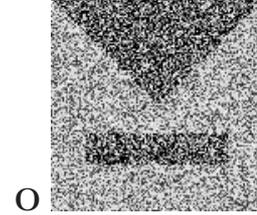
Problem 3 (Motion Analysis)

Let a sufficiently often continuously differentiable 2-D image sequence $f(x, y, z)$ be given, where $z \geq 0$ denotes time. The goal is to formulate a model for estimating the 2-D optical flow $u(x, y, z)$ and $v(x, y, z)$ which allows an accurate estimation for *translational motion* under *varying illumination* and which is *robust against outliers* in the data term at the same time. Use a spatial smoothness term of your choice.

- (a) Write down a suitable energy functional.
 - (b) Derive the differential equations which must necessarily be satisfied by a solution $u(x, y, z)$ and $v(x, y, z)$.
(Boundary conditions need *not* to be specified.)
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Problem 4 (Visual Analysis)

The images **A–D** show results of regularised nonlinear isotropic diffusion filtering applied to the original image **O**.



The parameter values are – not necessarily in the same order –

- | | |
|--------------------------------------|-------------------------------------|
| (i) $\lambda = 3.0, \sigma = 1.3,$ | (ii) $\lambda = 1.0, \sigma = 5.0,$ |
| (iii) $\lambda = 7.0, \sigma = 1.3,$ | (iv) $\lambda = 7.0, \sigma = 5.0$ |

where σ is the smoothing parameter, and λ the threshold parameter from the diffusivity function $g(|\nabla u_\sigma|^2) = 1 - \exp(-3.31488/(|\nabla u_\sigma|/\lambda)^8)$. In all images, 500 iterations of time step size 0.2 were performed.

Indicate for each of the parameter settings (i)–(iv) which of the filtered images A–D corresponds.

Problem 5 (Connection between Perona-Malik and Self-Snakes)

Let the isotropic nonlinear diffusion process of Perona-Malik

$$u_t = \operatorname{div} (g(|\nabla u|^2) \nabla u)$$

and the self-snake process

$$v_t = |\nabla v| \operatorname{div} \left(g(|\nabla v|^2) \frac{\nabla v}{|\nabla v|} \right)$$

be given. Show that the following relation holds:

$$u_t - v_t = g(|\nabla u|^2) u_{\eta\eta} .$$

Problem 6 (Multiple Choice)

Answer the following questions by writing Y (yes) or N (no) in the corresponding box to the right.

- a) The AOS scheme is invariant under rotations.
 - b) Pure backward diffusion processes are ill-posed problems.
 - c) An optic flow data term based on constancy assumptions on two different constancy assumptions is sufficient to overcome the aperture problem locally (without the help of the smoothness term).
 - d) A dilation step followed by an erosion step erases local maxima.
 - e) The number of extrema does not decrease during a 1-D linear diffusion process.
 - f) Anisotropic diffusion cannot be extended to matrix-valued images, since the diffusion tensor is a matrix itself.
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