Lecture 29: Object Recognition I: Invariants

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Introduction

What's the Problem?

- We want to check if a binary 2-D image object is identical to a binary 2-D object in another image.
- The second object may have undergone several transformations such as translations, scalings and rotations.
- Potential applications include
  - optical character recognition (OCR)
  - recognition of workpieces (Werkzeugteile)
- Are there certain expressions that are invariant under such transformations?
- Such invariances would be very useful for object recognition.

Moments (Momente)

- For a continuous image $f : \mathbb{R}^2 \to \mathbb{R}$ one defines the *moment of order $p+q$* by

$$m_{p,q} := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p \, y^q \, f(x,y) \, dx \, dy \qquad (p,q = 0,1,2,...).$$

- *Uniqueness Theorem for Moments (Papoulis 1991):*
If $f(x,y)$ is piecewise continuous and is nonvanishing only on a finite subdomain of $\mathbb{R}^2$, then all moments do exist. The sequence of moments $(m_{p,q})$ is not only uniquely determined by $f(x,y)$, but even the reverse holds: If the moments $(m_{p,q})$ satisfy a certain growth condition, then they uniquely characterise the image $f(x,y)$.

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- For a discrete image $(f_{i,j})$ with $i \in \{1,...,N\}$ and $j \in \{1,...,M\}$ one defines the moments in a similar way:

$$m_{p,q} := \sum_{i=1}^{N} \sum_{j=1}^{M} i^p \, j^q \, f_{i,j}$$

with $p = 0,...,N-1$ and $q = 0,...,M-1$.

- These NM moments $(m_{p,q})$ are sufficient to uniquely determine the discrete image $(f_{i,j})$ with NM pixels.

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Incorporating Translation Invariance

- Goal: Modify the moments such that they are invariant under translations.
- The *central moments* of a continuous image $f(x,y)$ are defined as

$$\mu_{p,q} := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\bar{x})^p \, (y-\bar{y})^q \, f(x,y) \, dx \, dy$$

  where $\bar{x} := \frac{m_{1,0}}{m_{0,0}}$ and $\bar{y} := \frac{m_{0,1}}{m_{0,0}}$ denote the x and y coordinates of the centre of gravity (also called centroid).
- These expressions are shift invariant.
- For a discrete image $(f_{i,j})$ one defines the central moments as

$$\mu_{p,q} := \sum_{i=1}^{N} \sum_{j=1}^{M} (i-\bar{i})^p \, (j-\bar{j})^q \, f_{i,j}$$

  where $\bar{i} := \frac{m_{1,0}}{m_{0,0}}$, $\bar{j} := \frac{m_{0,1}}{m_{0,0}}$, and $m_{0,0}$, $m_{1,0}$ and $m_{0,1}$ denote discrete moments.

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From Moments to Central Moments

- It is not difficult to show that the central moments can be computed directly from the moments:

$$\begin{aligned}
\mu_{0,0} &= m_{0,0}, \\
\mu_{1,0} &= 0, \\
\mu_{0,1} &= 0, \\
\mu_{2,0} &= m_{2,0} - \bar{i} m_{1,0}, \\
\mu_{1,1} &= m_{1,1} - \bar{j} m_{1,0}, \\
\mu_{0,2} &= m_{0,2} - \bar{j} m_{0,1}, \\
\mu_{3,0} &= m_{3,0} - 3\bar{i} m_{2,0} + 2\bar{j}^2 m_{1,0}, \\
\mu_{1,2} &= m_{1,2} - 2\bar{j} m_{1,1} - \bar{i} m_{0,2} + 2\bar{j}^2 m_{1,0}, \\
\mu_{2,1} &= m_{2,1} - 2\bar{i} m_{1,1} - \bar{j} m_{2,0} + 2\bar{i}^2 m_{0,1}, \\
\mu_{0,3} &= m_{0,3} - 3\bar{j} m_{0,2} + 2\bar{j}^2 m_{0,1}.
\end{aligned}$$

- *Note the difference between the central moments is this lecture (they refer to the image) and the ones in Lecture 21 that refer to the histogram.*

Incorporating Scale Invariance

- Goal: Modify the central moments such that they are not only invariant under translations, but also under (spatial) rescalings.
- The *normalised central moments* are defined as

$$\eta_{p,q} := \frac{\mu_{p,q}}{\mu_{0,0}^{\gamma}}$$

with $\gamma := \frac{p+q}{2} + 1$ and $p + q = 2,\, 3,\, \ldots$

- One can show that they are invariant under translations and scalings.

What about rotations?

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Incorporating Rotation Invariance

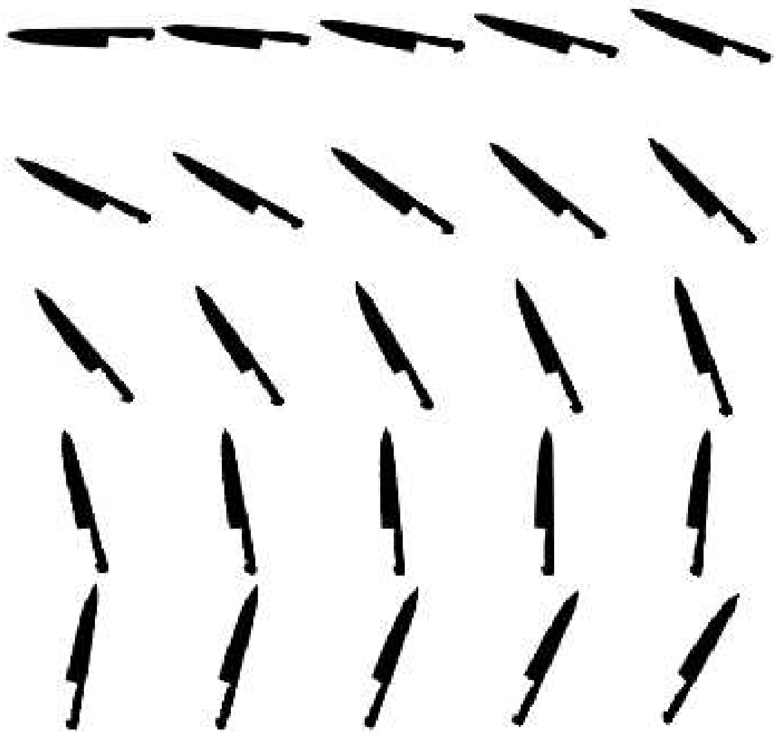
- The *seven moment invariants of Hu* (1962) have been derived as combinations of normalised central moments such that they are also invariant under rotations:

$$\begin{aligned}
\phi_1 &= \eta_{2,0} + \eta_{0,2},\\
\phi_2 &= (\eta_{2,0} - \eta_{0,2})^2 + 4\eta_{1,1}^2,\\
\phi_3 &= (\eta_{3,0} - 3\eta_{1,2})^2 + (3\eta_{2,1} - \eta_{0,3})^2,\\
\phi_4 &= (\eta_{3,0} + \eta_{1,2})^2 + (\eta_{2,1} + \eta_{0,3})^2,\\
\phi_5 &= (\eta_{3,0} - 3\eta_{1,2})(\eta_{3,0} + \eta_{1,2})[(\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2] +\\
&\quad (3\eta_{2,1} - \eta_{0,3})(\eta_{2,1} + \eta_{0,3})[3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2],\\
\phi_6 &= (\eta_{2,0} - \eta_{0,2})[(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2] +\\
&\quad 4\eta_{1,1}(\eta_{3,0} + \eta_{1,2})(\eta_{2,1} + \eta_{0,3}),\\
\phi_7 &= (3\eta_{2,1} - \eta_{0,3})(\eta_{3,0} + \eta_{1,2})[(\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2] +\\
&\quad (3\eta_{1,2} - \eta_{3,0})(\eta_{2,1} + \eta_{0,3})[3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2].
\end{aligned}$$

- Under mirroring, ϕ_1,...,ϕ_6 remain invariant, but ϕ_7 changes sign.

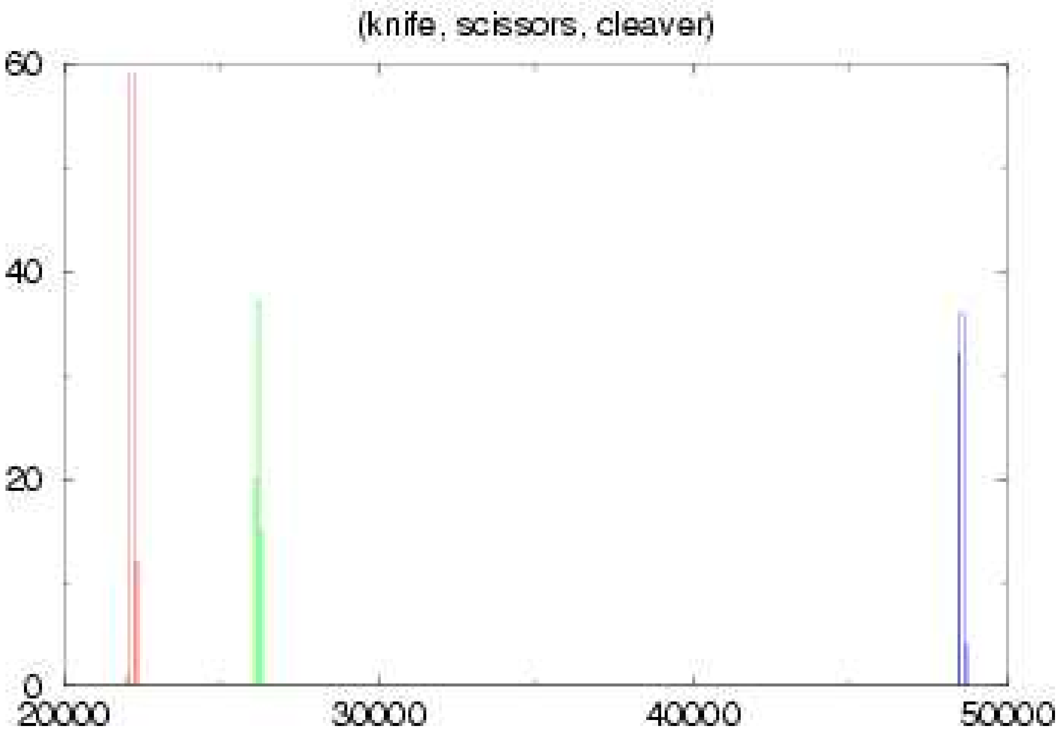
Application to Object Recognition

Set of 3 object classes: scissors, cleaver, and knife. Author: M. J. Andrews (1999).



Rotation of the knife by some angles between 0 and 120 degrees. Author: M. J. Andrews (1999).

Incorporating Rotation Invariance (4)



Already the first moment invariant ϕ_1 is sufficient for discriminating all three object classes. One observes that the rotation does hardly lead to a broadening of the clusters due to discretisation artifacts. Author: M. J. Andrews (1999).

Incorporating Rotation Invariance (5)

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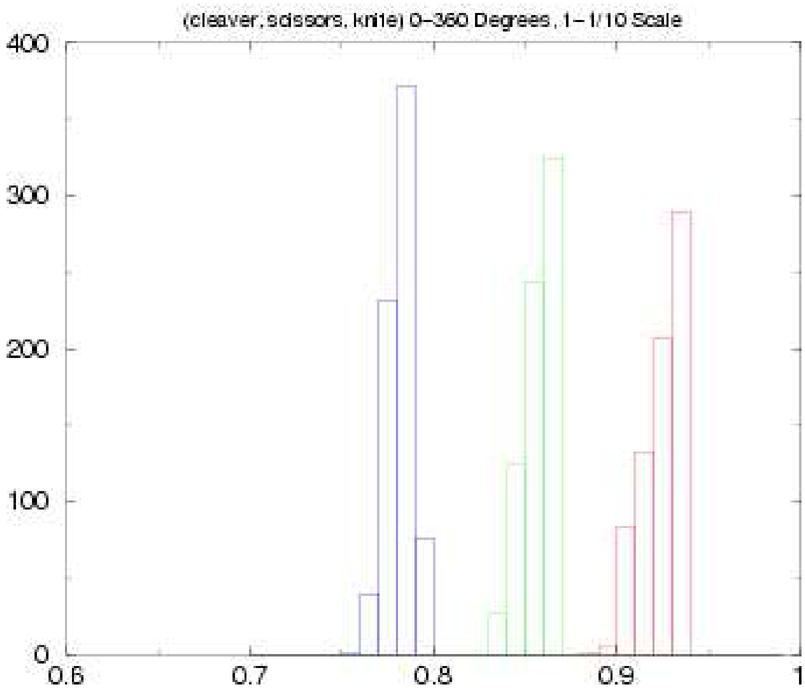

If one does not only rotate the object but also rescales them by factors between 0.1 and 1, then the moment invariant ϕ_2 does still allow to distinguish all classes. The broadening of the clusters results from discretisation effects that become particularly dangerous if the object is not very large compared to the pixel size. Author: M. J. Andrews (1999).

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Incorporating Affine Invariance

- In 1993, Flusser and Suk derived a set of four moment invariants that are even invariant under affine transformations (transformations of type $\mathbf{u} = A\mathbf{f} + \mathbf{b}$ with $\mathbf{u}$, $\mathbf{f}$, $\mathbf{b} \in \mathbb{R}^2$ and a nonsingular matrix $A \in \mathbb{R}^{2\times 2}$).
- In terms of central moments, they are given by

$$
\begin{aligned}
I_1 &= \frac{\mu_{2,0}\mu_{0,2} - \mu_{1,1}^2}{\mu_{0,0}^4}, \\
I_2 &= \frac{\mu_{3,0}^2\mu_{0,3}^2 - 6\mu_{3,0}\mu_{2,1}\mu_{1,2}\mu_{0,3} + 4\mu_{3,0}\mu_{1,2}^3 + 4\mu_{2,1}^3\mu_{0,3} - 3\mu_{2,1}^2\mu_{1,2}^2}{\mu_{0,0}^{10}}, \\
I_3 &= \frac{\mu_{2,0}(\mu_{2,1}\mu_{0,3} - \mu_{1,2}^2) - \mu_{1,1}(\mu_{3,0}\mu_{0,3} - \mu_{2,1}\mu_{1,2}) + \mu_{0,2}(\mu_{3,0}\mu_{1,2} - \mu_{2,1}^2)}{\mu_{0,0}^7}, \\
I_4 &= (\mu_{2,0}^3\mu_{0,3}^2 - 6\mu_{2,0}^2\mu_{1,1}\mu_{1,2}\mu_{0,3} - 6\mu_{2,0}^2\mu_{0,2}\mu_{2,1}\mu_{0,3} + 9\mu_{2,0}^2\mu_{0,2}\mu_{1,2}^2 \\
&\quad +12\mu_{2,0}\mu_{1,1}^2\mu_{2,1}\mu_{0,3} + 6\mu_{2,0}\mu_{1,1}\mu_{0,2}\mu_{3,0}\mu_{0,3} - 18\mu_{2,0}\mu_{1,1}\mu_{0,2}\mu_{2,1}\mu_{1,2} \\
&\quad -8\mu_{1,1}^3\mu_{3,0}\mu_{0,3} - 6\mu_{2,0}\mu_{0,2}^2\mu_{3,0}\mu_{1,2} + 9\mu_{2,0}\mu_{0,2}^2\mu_{2,1}^2 \\
&\quad +12\mu_{1,1}^2\mu_{0,2}\mu_{3,0}\mu_{1,2} - 6\mu_{1,1}\mu_{0,2}^2\mu_{3,0}\mu_{2,1} + \mu_{0,2}^3\mu_{3,0}^2) \;/\; \mu_{0,0}^{11}.
\end{aligned}
$$

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Incorporating Invariance under Motion Blur

- In 1996 Flusser, Suk and Saic have investigated how one can construct expressions that are invariant under translation and under linear motion blur.
- They have derived an explicit formula how one can compute invariants of arbitrary high order (and computed the first 30 invariants).
- Experiments show that these invariants can also be well approximated in a discrete setting and that they are also fairly robust under noise.
- In their original formulation, these moments are not invariant under rotations and scalings.
- However, there are possibilities how to incorporate rotation and scaling invariance.
- In this way Flusser et al. have shown that the shift, rotation and scaling invariant Hu moments ϕ_3, ϕ_4, ϕ_5 and ϕ_7 are even invariant under linear motion blur.

Incorporating Invariance under Motion Blur (2)

(a) Top left: Lena. **(b) Top middle:** Horizontal motion blur, 30 pixels. **(c) Top right:** Additional Gaussian noise with $\sigma = 10$. **(d) Bottom left:** Image (b) with Gaussian noise with $\sigma = 30$. **(e) Bottom middle:** Horizontal motion blur, 80 pixels. **(f) Bottom right:** Lisa. Authors: Flusser et al. (1996).

Incorporating Invariance under Motion Blur (3)

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	Lena orig.	Motion 30 pix.	Motion 30 STD = 10	Motion 30 STD = 30	Motion 80 pix.	Lisa
$M_0[10^6]$	6.4586	6.4589	6.4607	6.4674	6.4590	5.0860
$M_1[10^{10}]$	0.2560	0.2559	0.2553	0.2612	0.2560	−0.6036
$M_2[10^{10}]$	3.4740	3.4742	3.4750	3.4816	3.4743	2.3215
$M_3[10^{11}]$	−1.1022	−1.1024	−1.0968	−1.0616	−1.1017	1.4777
$M_4[10^{11}]$	1.0456	1.0462	1.0570	1.0639	1.0450	−2.9602
$M_5[10^{11}]$	2.2436	2.2434	2.2313	2.2541	2.2444	0.4081
$M_6[10^{11}]$	−5.3930	−5.3925	−5.3901	−5.4370	−5.3934	6.8344
$M_7[10^{14}]$	3.3918	3.3920	3.3918	3.4093	3.3921	2.0830
$M_8[10^{14}]$	0.1776	0.1775	0.1772	0.1811	0.1776	−0.5208
$M_9[10^{14}]$	−0.0513	−0.0513	−0.0507	−0.0544	−0.0513	0.0426
$M_{10}[10^{14}]$	−0.2293	−0.2292	−0.2293	−0.2277	−0.2292	0.5762
$M_{11}[10^{15}]$	−1.5087	−1.5088	−1.4996	−1.4809	−1.5081	2.7966
$M_{12}[10^{15}]$	4.7607	4.7606	4.7382	4.7451	4.7622	0.7743
$M_{13}[10^{15}]$	−3.1958	−3.1954	−3.1843	−3.2365	−3.1957	0.6806
$M_{14}[10^{15}]$	0.0734	0.0739	0.0985	0.0330	0.0732	−2.8659
$M_{15}[10^{16}]$	−0.2281	−0.2283	−0.2280	−0.2314	−0.2282	0.9275
$M_{16}[10^{16}]$	1.7525	1.7524	1.7540	1.7645	1.7528	−3.5346
$M_{17}[10^{18}]$	3.9858	3.9860	3.9817	4.0315	3.9863	2.3215
$M_{18}[10^{18}]$	0.1484	0.1484	0.1483	0.1505	0.1485	−0.5753
$M_{19}[10^{18}]$	−0.0520	−0.0520	−0.0503	−0.0552	−0.0519	0.0603
$M_{20}[10^{18}]$	−0.2045	−0.2044	−0.2058	−0.2041	−0.2044	0.5220
$M_{21}[10^{18}]$	0.1186	0.1186	0.1191	0.1229	0.1185	−0.1292
$M_{22}[10^{18}]$	0.7187	0.7186	0.7195	0.7164	0.7187	−2.3919

The values of the first 22 motion invariants computed for the images from the previous slide. One observes that all Lena images yield almost the same number, wheras the Lisa image differs quite strongly from these numbers. Authors: Flusser et al. (1996).

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Photometric Invariants

Problem

- Many computer vision methods are not robust under realistic illumination changes such as globally varying illumination, shadow/shading, highlights, specular reflections
- can create severe perturbations for real world applications

Classification of Illumination Changes

- global multiplicative changes
- local multiplicative changes: shadow, shading
- local additive changes: highlights, specular reflections

One can construct so-called *photometric invariants* that are not influenced by these changes. In particular colour images offer a number of possibilities.

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Examples of Photometric Invariants

- **Log-Derivative Transform:**

$$(R, G, B)^\top \mapsto \big((\ln R)_x, (\ln R)_y, (\ln G)_x, (\ln G)_y, (\ln B)_x, (\ln B)_y\big)^\top$$

  Invariant under global multiplicative illumination changes.

- **Chromaticity Space:**

$$(R, G, B)^\top \mapsto \left(\frac{R}{N}, \frac{G}{N}, \frac{B}{N}\right)^\top$$

  with normalisation $N := \frac{1}{3}(R + G + B)$.
  Invariant under global and local multiplicative illumination changes.

- **HSV Colour Space:**

  The hue component H (cf. Lecture 7) is invariant under global and local illumination changes as well as local additive changes.

Summary

- Invariants are a very important tool for object recognition under differing conditions.
- Often images that depict the same object have undergone transformations such as translations, rotations and scalings.
- With a sufficient number of moments one can characterise an image with arbitrary accuracy.
- The moment invariants of Hu are a set of seven expressions that are invariant under translations, scalings and rotations.
- It is possible to derive similar expressions if other invariances are needed such as invariance under affine transformations or under linear motion blur.
- Discretisation effects are the reason why these invariances are never fully invariant in practice.
- Photometric invariants can be used to make computer vision systems invariant under illumination changes.

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Literature

- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Addison-Wesley, Reading, Second Edition, 2002.
  (Section 11.3 briefly lists the important moment invariants.)
- M. Sonka, V. Hlaváč, R. Boyle: *Image Processing, Analysis, and Machine Vision*. Thomson Learning, London, 1999.
  (Section 6.3.2 includes affine invariant moments.)
- M. J. Andrews: *Moment representation of blobs in 2-D intensity images.*
  (`www.gweep.net/~rocko/Moment/`)
  (describes experiments)
- J. Shutler: *Statistical moments – An introduction.*
  (`homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/SHUTLER3/`)
  (good overview)
- M. K. Hu: Visual pattern recognition by moment invariants. *IRE Transactions on Information Theory*, Vol. 8, pp. 179–187, Feb. 1962.
  (classical paper on the seven Hu invariants)
- J. Flusser, T. Suk: Pattern recognition by affine moment invariants. *PatternRecognition*, Vol. 26, pp. 167–174, 1993. `http://library.utia.cas.cz/asep/FLUS_JAN.html`
  (introduced affine moment invariants)
- J. Flusser, T. Suk, S. Saic: Recognition of images degraded by linear motion blur without restoration. *Computing*, Supplement 11, pp. 37–51, 1996.
  `http://library.utia.cas.cz/asep/FLUS_JAN.html`
  (moment invariants for linear motion blur)

Summary (3)

- A. G. Mamistvalov: n-dimensional moment invariants and conceptual mathematical theory of recognition of n-dimensional data sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 20, No. 8, pp. 819–831, 1998.
  (generalises the Hu theory to arbitrary dimensions)
- K. Voss, H. Süße: *Adaptive Modelle und Invarianten für zweidimensionale Bilder*. Shaker, Aachen, 1995.
  (recommendable book on moment invariants)
- A. Papoulis, S. U. Pillai: *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, New York, 2002.
  (uniqueness theorem for moments)

Announcement

Announcement

Next semester:

- **Proseminar Simulation der Welt** (Joachim Weickert, Martin Welk)
  Anmeldung: e-Mail an `welk@mia.uni-saarland.de`.
  Vorbesprechung: Donnerstag, 20. Februar, 16:15 in E1.1, Raum 306
- **Seminar Processing of Matrix-Valued Images** (Stephan Didas)
  Registration: e-mail to `didas@mia.uni-saarland.de`.
  First meeting: Friday, February 21, 2:15 pm in E1.1, Room 306.

First come, first serve.

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