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Lecture 20:

Feature Extraction III:
Contour Representations and the Hough Transform

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Chain Code (Kettencode)

Problem:

- Zero-crossings of the Laplacian (Lecture 18) yield closed contours as edges.
- How can such a contour be coded efficiently in a digital image?

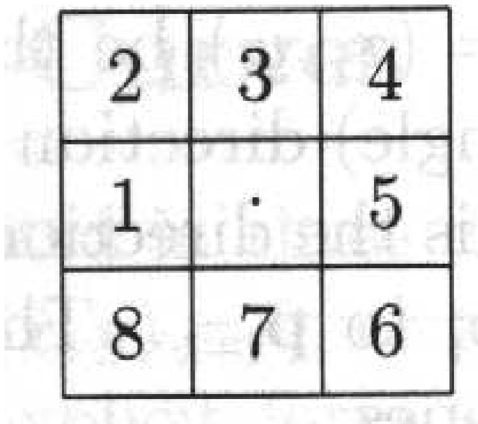
Chain Code:

1. Start with an arbitrary contour point and store its coordinates.
2. Walk clockwise along the contour, and store the location of the next 8-neighbour of the countour (can be represented by a number between 1 and 8, i.e. by 3 bits).

Advantages:

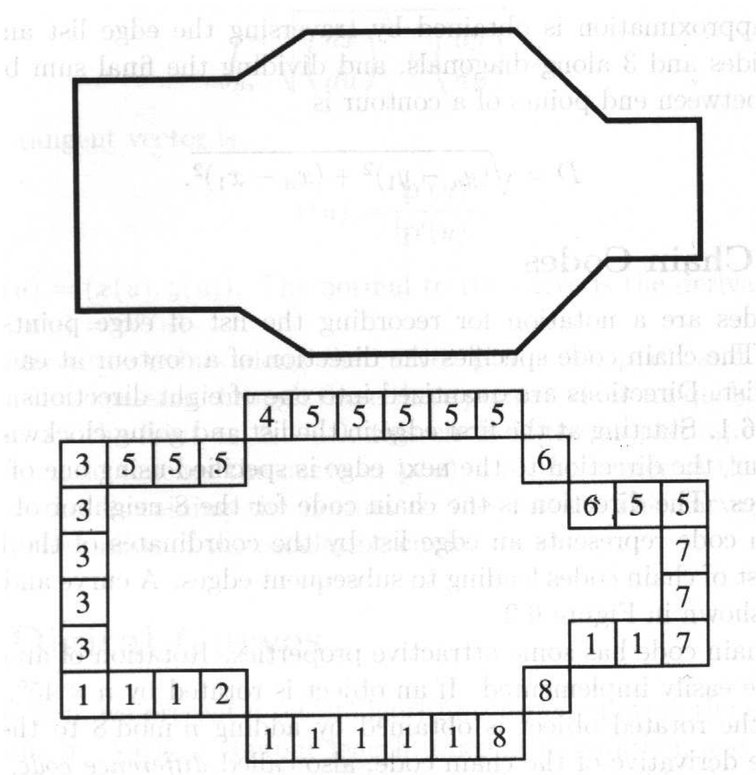
- Rotations by $n \cdot 45$ degrees add n mod 8 to the original code.
- The derivative of the chain code *(difference code)* is invariant under rotations.

Chain Code (2)



Chain code numbering. Authors: R. Jain, R. Kasturi and B. G. Schunck (1995).

Chain Code (3)



A curve and its chain code. Authors: R. Jain, R. Kasturi and B. G. Schunck (1995).

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Fourier Descriptors (Fourierdeskriptoren)

(Granlund 1972, Zahn/Roskies 1972)

Goal:

- contour description that is well-suited for filtering and compression

Solution:

- The coordinate (x_n, y_n) of the n-th contour point is represented by a complex number $u_n = x_n + iy_n$.
- The discrete Fourier transform of this complex 1-D signal $(u_0,...,u_{N-1})$ is given by (cf. Lecture 4):

$$\hat{u}_p \;=\; \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u_n \exp\left(-\frac{i2\pi pn}{N}\right) \qquad (p = 0, ..., N-1).$$

- The Fourier coefficients $\hat{u}_0,...,\hat{u}_{N-1}$ are called *Fourier descriptors*.

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Advantages of Fourier Descriptors

- allow a more compact (but lossy) contour representation if high-frequent components are eliminated: This lowpass filtering eliminates noise, but also corners.
- well-suited for curve matching, since typical operations like translation, rotation and scaling create simple transformations of the Fourier descriptors

Behaviour of Fourier Descriptors Under Transformations

- A translation by a vector (x, y) can be regarded as addition of a constant complex number $c = x + iy$.
  It only affects the zeroth Fourier descriptor:

$$v_n = u_n + c \quad \forall n \qquad \Longrightarrow \qquad \hat{v}_p = \begin{cases} \hat{u}_p + \sqrt{N}\,c & (p = 0) \\ \hat{u}_p & (p \neq 0) \end{cases}$$

Fourier Descriptors (3)

- Setting the zeroth Fourier descriptor to 0 translates the centre of gravity $\sum_{i=0}^{N-1}(x_i, y_i)^\top$ of the contour into the origin.

- The linearity of the DFT implies that scaling also scales the Fourier descriptors:

$$v_n = \alpha\, u_n \quad \forall\, n \qquad \Longrightarrow \qquad \hat{v}_p = \alpha\, \hat{u}_p \quad \forall\, p$$

- This also holds for rotations by some angle θ.
  They lead to multiplications with the complex number $\exp(i\theta)$:

$$v_n = u_n \exp(i\theta) \quad \forall\, n \qquad \Longrightarrow \qquad \hat{v}_p = \hat{u}_p \exp(i\theta) \quad \forall\, p$$

- A different starting point gives a cyclic phase shift of the Fourier descriptors:

$$v_n = u_{(n-n_0) \bmod N} \quad \forall\, n \qquad \Longrightarrow \qquad \hat{v}_p = \hat{u}_p \exp\left(-\frac{i2\pi n_o p}{N}\right) \quad \forall\, p$$

Fourier Descriptors (4)

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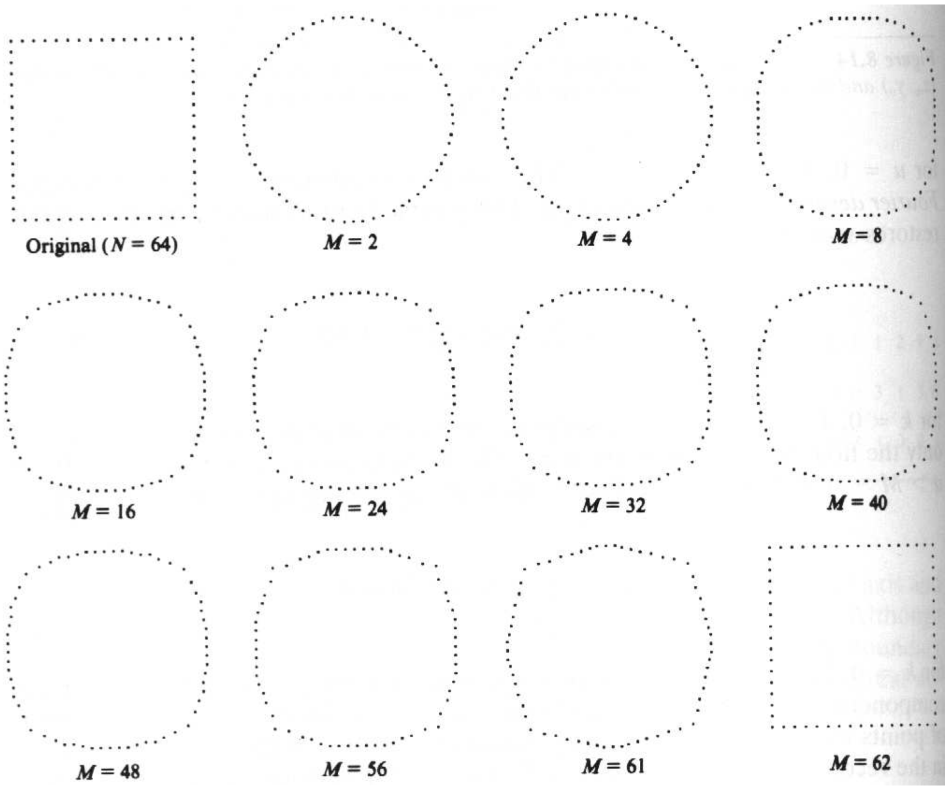
Disadvantages of Fourier descriptors:

- The preceding relations are only satisfied in a precise way if the contour has been sampled in an equidistant way.

- This is the case for a hexagonal grid (that is hardly used).

- On a rectangular grid this is only satisfied, if a 4-neighbourhood is used.

Fourier Descriptors (5)



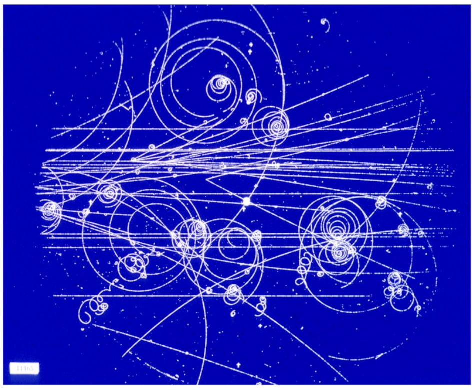
Example of a contour representation using a varying number of Fourier descriptors, where the highest frequencies have been removed. Authors: R. C. Gonzalez and R. E. Woods (1992).

Hough Transform (1)

The Hough Transform

Historical Origin:

- apparatus for line detection, patented by Hough in 1962
- application: automised analysis of bubble chamber images from high energy physics



Particle tracks in a hydrogen bubble chamber. A pion particle has interacted with a proton which created a number of new particles. Their motion in the magnetic field depends on their energy and their charge. Source: `http://www.particlephysics.ac.uk`

Hough Transform (2)

Basic Idea

- Goal: Method for detecting simple geometric objects that can be represented by a small set of parameters (e.g. lines, circles)
- Assumption: Object boundaries have large gradient magnitude: $|\nabla f| \geq T$
- Object boundary satisfies $g(x, y, p_1, ..., p_m) = 0$.
  Here p_1,...,p_m denote the parameters we want to determine.
- Example 1: A line can be represented by a normal vector $n = (\cos\phi, \sin\phi)^\top$ and the distance d to the origin: $x\cos\phi + y\sin\phi - d = 0$.
  Two parameters: ϕ, d.
- Example 2: Circle with centre (a, b) and radius r: $|x-a|^2 + |y-b|^2 - r^2 = 0$.
  Three parameters: a, b, r.
- Voting metheod:
  Every significant contour point (where $|\nabla f| \geq T$) votes for all parameters p_1,...,p_m that allow it to satisfy the equation $g(x, y, p_1, ..., p_m) = 0$.
  The majority wins: The parameters of the relevant contour get the most votes.

Hough Transform (3)

The Hough Transform for Lines

- Discretise the parameter space for (ϕ, d).
- For every pixel with $|\nabla f| \geq T$, increment the counter of all compatible parameter cells (ϕ, d) by 1.
- In this way a point (x_i, y_i) creates a trigonometric curve

$$d(\phi) = x_i\cos\phi + y_i\sin\phi.$$

  in the parameter space for (ϕ, d).
- For points belonging to the same line, the trigonometric curves intersect at the corresponding parameters of the line.

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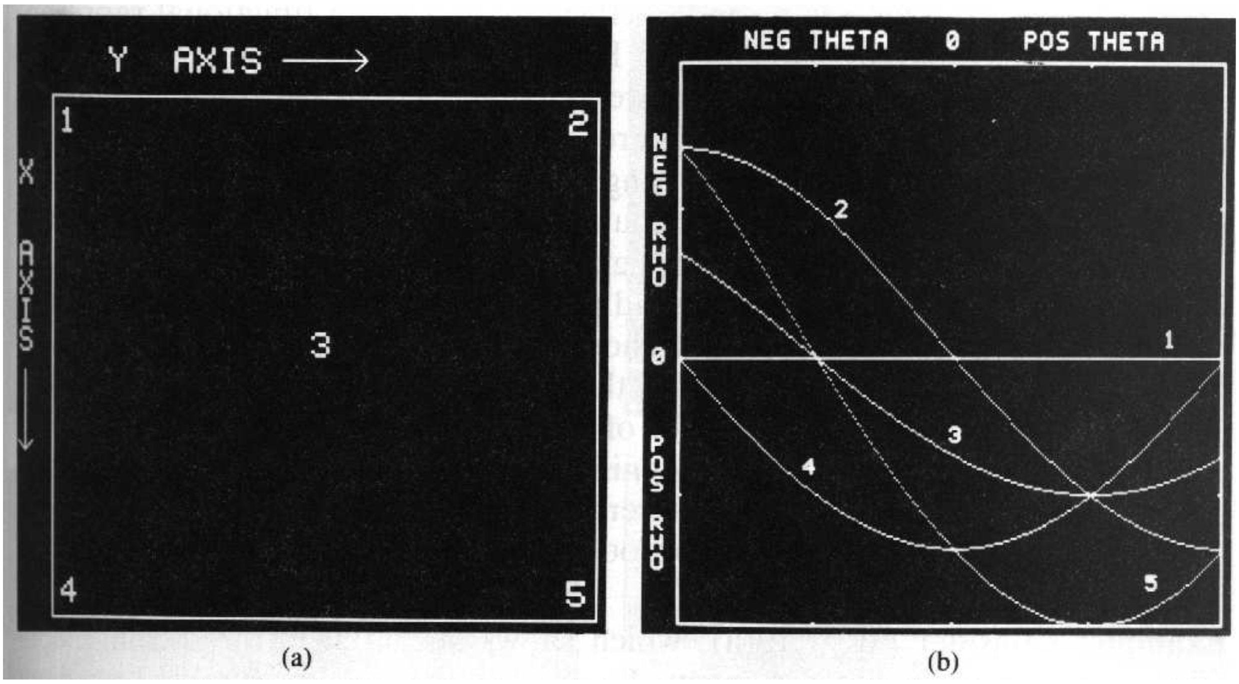
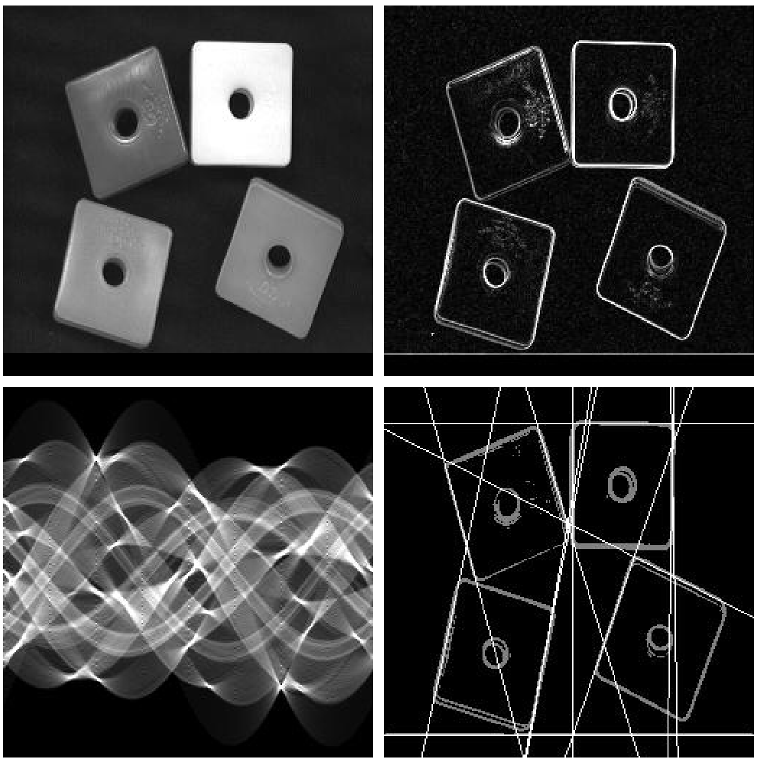
Hough Transform (4)



(a) Five points. (b) Curves in the parameter space (ϕ, d). The fact that the curves to the points 2, 3 and 4 intersect in one point shows that these three points belong to the same line. Authors: R. C. Gonzalez and R. E. Woods (1992).

Hough Transform (5)



(a) Top left: Original image, 256×256 pixels. **(b) Top right:** Gradient magnitude. **(c) Bottom left:** Representation in (ϕ, d)-space. **(d) Bottom right:** Most significant lines. Source: `http://infocom.cheonan.ac.kr/~nykwak/kuim/hough.html`.

Hough Transform (6)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

The Hough Transform for Circles

- Discretise the parameter space for a, b and r.
- For every pixel with $|\nabla f| \geq T$, increment the counter of all compatible cells (a, b, r) by 1.
- The cells with the most votes give the desired parameters for the circles.

Hough Transform (7)

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Advantages of the Hough Transform

- no need for explicit connection of lines
- object contours may even be interrupted or partially occluded
- The voting method is extremely robust:
  A simple majority of votes is sufficient, and everything is fully automised.

Main Disadvantage of the Hough Transform

- Memory requirements and computational effort increase rapidly with the number of parameters.
- Coarse-to-fine pyramid-like approaches can help addressing this problem.

Summary

- The chain code gives a compact contour description.
- Fourier descriptors allow a contour simplification and may be useful for matching curves.
- The Hough transform is useful for detecting simple geometric objects such as lines or circles.
- Every significant point votes for the compatible parameter sets.
- The parameter sets with the most votes specify the desired objects.

Literature

- R. Jain, R. Kasturi, B. G. Schunck: *Machine Vision*, Mc Graw-Hill, New York, 1995.
  (for chain code)
- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Third Edition, Pearson, Upper Saddle River, 2008.
  (Subsection 11.2.3 for Fourier descriptors, Subsection 10.2.7 for the Hough transform)
- E. R. Davies: *Machine Vision*. Academic Press, San Diego, 1997.
  (contains an extended chapter on the Hough transform; other chapters are less recommendable)
- G. H. Granlund: Fourier preprocessing for hand print character recognition. *IEEE Transactions on Computers*, Vol. 21, pp. 195–201, Febr. 1972.
  (for Fourier descriptors)
- C. T. Zahn, R. Z. Roskies: Fourier descriptors for plane closed curves. *IEEE Transactions on Computers*, Vol. 21, pp. 269–281, March 1972.
  (for Fourier descriptors)
- P. V. C. Hough: *Methods and means for recognizing complex pattens.* U.S. Patent 3069654, Dec. 1962.
  (original patent that gave rise to the Hough transform)

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Assignment C5 – Classroom Work

Problem 1 (Structure Tensor Analysis)

(a) Let $J \in \mathbb{R}^{n \times n}$ be a symmetric $n \times n$ matrix with real-valued entries. We consider its corresponding quadratic form

$$E : \mathbb{R}^n \rightarrow \mathbb{R}, \quad E(\mathbf{v}) = \mathbf{v}^\top J\, \mathbf{v}\ .$$

Show that among all vectors $\mathbf{v} \in \mathbb{R}$ with Euclidean norm $||\mathbf{v}||_2 = 1$, the function value $E(\mathbf{v})$ is minimal for the eigenvalue of J corresponding to its smallest eigenvalue. What can we say about E if J is positive definite ?

(b) Let J_ρ denote the structure tensor as defined in Lecture 19, page 14. If we set the outer scale $\rho = 0$, can the tensor still be used for edge and corner detection. Please explain your answer.

(*This problem shows how to solve total least squares problems that are usually formulated in terms of the minimisation of quadratic forms. This will also be useful in the context of motion estimation.*)

Assignment T5 – Theoretical Homework

Problem 1 (Derivative Filters) (5+3+1 points)

Let (f_i) be a discrete 1-D signal that has been sampled from a sufficiently smooth continuous function $f(x)$ with pixel distance h.

(a) Write down the linear equation system that has to be solved if one is interested in approximating the first derivative $f'(x)$ in pixel i by means of the 5 points $f_{i-2},\ f_{i-1},\ f_i,\ f_{i+1},\ f_{i+2}$.

(b) The result of the preceding linear system of equations is given by the filter mask $\frac{1}{12h}(1, -8, 0, 8, -1)$. Derive the order of consistency order of this approximation.

(c) Is there, in general, any lower bound of the order of consistency, if a derivative of order d is approximated with n points ?

(*This problem shows you how to design finite difference approximations to derivative expressions.*)

Problem 2 (Hough Transform) (3+3 points)

(a) Consider the point set $\{(-1, 1), (2, 0), (3, 1), (1, 3), (0, 2)\}$. Using the Hough transform, determine which 3 points belong to the same line.

(b) Determine also with the Hough transform which 3 points belong to the same circle with radius 2.

(*This problem shows how to apply the Hough transform for detecting of simple geometric primitives* .)

Problem 3 (Cooccurrence Matrices) (5 points)

Compute the cooccurrence matrix of the 7×6 image

3	1	0	2	2	2	3
0	3	1	3	3	2	2
1	0	0	3	3	2	1
1	2	1	1	1	2	3
1	1	0	1	1	0	2
3	2	1	2	0	3	2

with $d = (-1, -1)^\top$. Assume that the x axis points to the right and the y-axis points downwards.

(*This problem shows how to extract local contextual information for texture analysis* .)

Deadline for submission: Tuesday, January 22, 10 am (before the lecture).