Lecture 17:
Image Enhancement VIII:
Fourier Methods and Deconvolution

Contents

Motivation

- So far, we have mainly investigated filters for denoising tasks.
- However, there are also other perturbations in images, such as
  - periodic artifacts due to rasterisation
  - space-variant illumination
  - blur
- Removal of these artifacts requires specifically adapted filters.
- We will study some of the most popular methods for these problems.

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Removal of Structured Perturbations

- Periodic perturbations appear e.g. as a grid patterns in rastered newspaper images.
- Lined or chequered paper also creates periodic artifacts.
- Periodic artifacts are well-localised in the Fourier domain where it is easy to remove them. (cf. Lecture 4, Assignment P1, Problem 3(a)).

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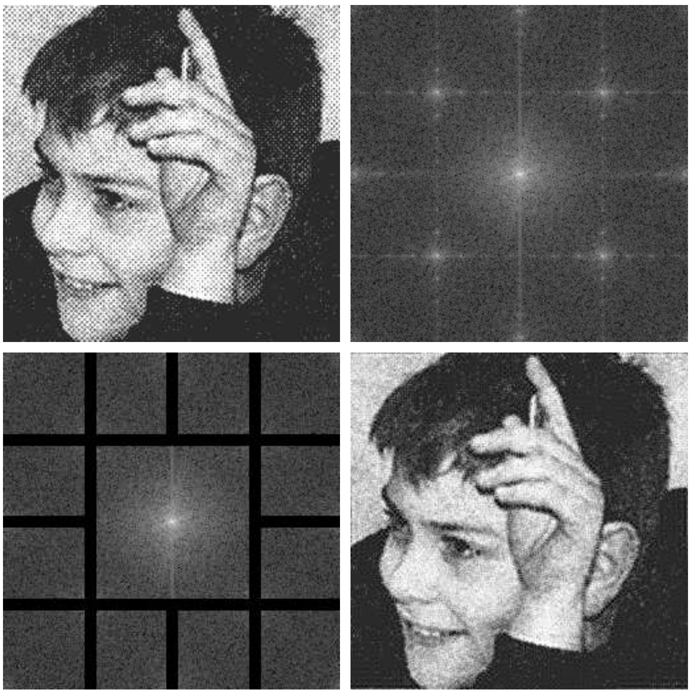
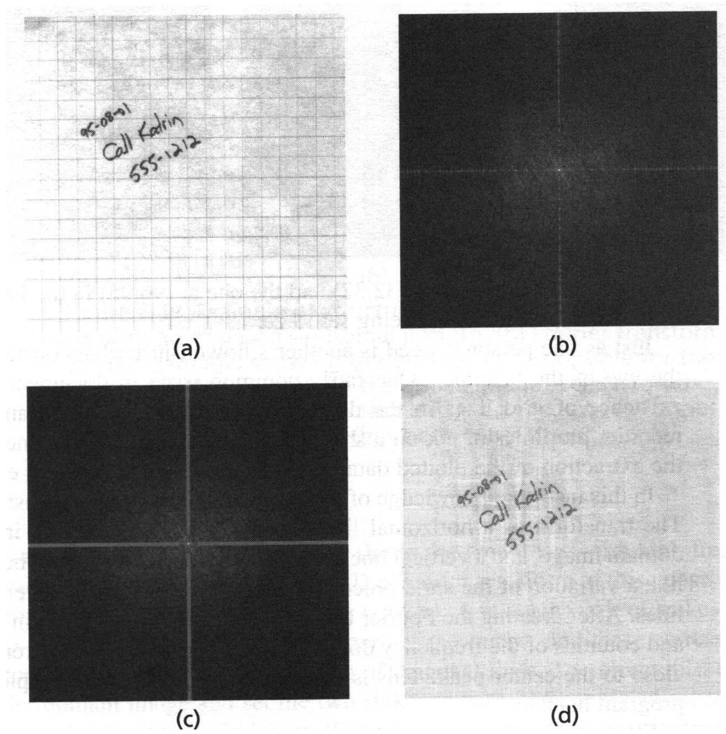


(a) Top left: Newspaper image with rastered structures, 256×256 pixels. **(b) Top right:** Fourier spectrum (logarithmically scaled). **(c) Bottom left:** Removal of the rastered structures in the Fourier domain. **(d) Bottom right:** Backtransformation of the filtered Fourier image. Author: J. Weickert (2002).



Removal of grid lines. **(a) Top left:** Original image. **(b) Top right:** Fourier spectrum. **(c) Bottom left:** Removal of chequered structures in the Fourier spectrum. **(d) Bottom right:** Backtransformation. Author: J. R. Parker (1997).

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Homomorphic Filtering (Homomorphe Filterung)

Goal:

- restore an image that has been recorded under space-variant illumination conditions

Grey Values:

- product of illumination intensity $I(x, y)$ of the light source and reflectance $R(x, y)$ of the illuminated object:

$$f(x, y) = I(x, y)\, R(x, y)$$

- Often the illumination intensity is only slowly varying in space.
  Thus, it contributes a low-frequent multiplicative perturbation.

- The reflectance can change rapidly, e.g. at edges.
  Thus, it contains many high-frequent components.

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Homomorphic Filtering (2)

Basic Idea Behind Homomorphic Filtering

- Compute the logarithm of all grey values:

$$\log f(x,y) \;=\; \log I(x,y) + \log R(x,y)$$

  Because of this superposition, one can now exploit the linearity of the Fourier transform. (Without taking the logarithm, the multiplicative perturbation gives a convolution in the Fourier domain.)

- Compute the Fourier transform of $\log f(x,y)$.
- Multiply in the Fourier domain with a function that attenuates low frequencies and boosts high frequencies.
- Compute the inverse Fourier transform.
- Compute the exponential function of the result.

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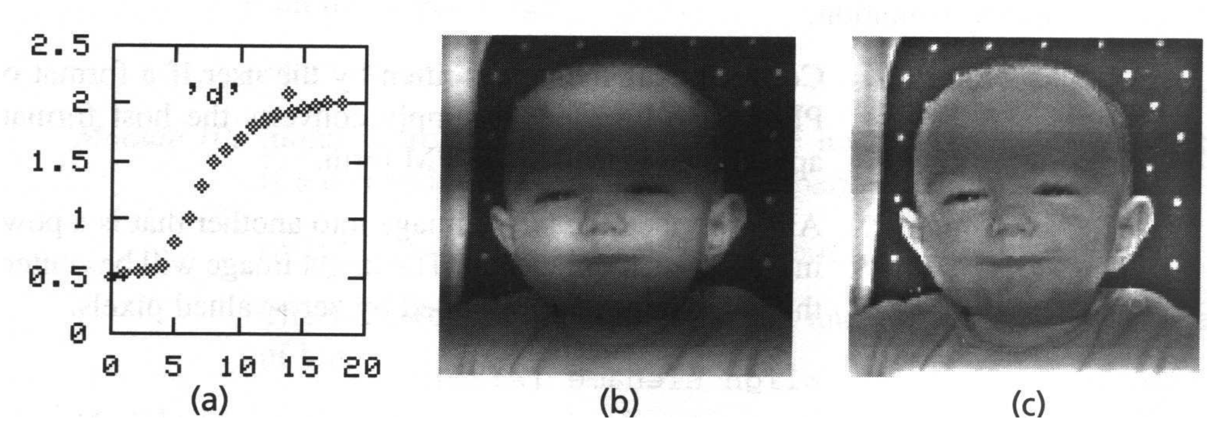
Homomorphic Filtering (3)



Homomorphic filtering. **(a) Left:** High pass filter that is used in the Fourier domain. **(b) Middle:** Original image with space-variant illumination. **(c) Right:** Reduction of illumination differences by homomorphic filtering. Author: J. R. Parker (1997).

Inverse Filtering

Image Degradation Model (cf. Lecture 2):

$$f(x,y) \;=\; (h * u)(x,y) + n(x,y)$$

- f: degraded image
- u: (unknown) ideal image
- h: shift-invariant (known) convolution kernel
- n: noise (caused by the sensor, transmission, quantisation, ...).

The convolution kernel h models e.g.

- a defocused optical system (cylinder-like shape)
- motion blur (oriented box function)
- atmospheric perturbations in telescopes (almost Gaussian)
- flub during the production of the Hubble telescope (Gaussian)

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How Can u be Restored ?

- If (!) noise is negligible, one obtains in the Fourier domain:

$$\hat{f} = \hat{h} \cdot \hat{u}.$$

- If (!) $\hat{h}$ does not vanish, u can be reconstructed from

$$\hat{u} \;=\; \frac{\hat{f}}{\hat{h}}$$

  and applying the inverse Fourier transform.

- This process is called *inverse filtering*.

Inverse Filtering (3)

Problem:

- h is usually a smoothing kernel (lowpass filter).
- Thus, for high frequencies, $\hat{h}$ is close to 0.
- In this case inverse filtering massivley amplifies even tiny high-frequent noise (e.g. quantisation noise).
- Without additional stabilisation, inverse filtering is often of no use.
- simplest stabilisation *(pseudoinverse filtering)*:

$$\hat{u} = \begin{cases} \frac{\hat{f}}{\hat{h}} & \text{if } |\hat{h}| > \varepsilon, \\ 0 & \text{else.} \end{cases}$$

Inverse Filtering (4)

(a) Left: Original image, 256×256 pixels. **(b) Middle:** Perturbed by simulated motion blur of 31 pixels. **(c) Right:** Deblurring with pseudoinverse filtering. Source: `www.owlnet.rice.edu/~elec431/projects95/lords/wolf.html`.

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Wiener Filtering

- stabilised variant of inverse filtering
- approximates $\hat{u}$ by
$$\hat{u} \;\approx\; \left(\frac{1}{\hat{h}}\,\frac{|\hat{h}|^2}{|\hat{h}|^2+K}\right)\,\hat{f}$$
with a positive parameter K.
- cannot diverge for $\hat{h} \to 0$:
$$\left(\frac{1}{\hat{h}}\,\frac{|\hat{h}|^2}{|\hat{h}|^2+K}\right)\,\hat{f} \to 0 \quad \text{for} \quad \hat{h} \to 0.$$
- acts like a bandpass filter:
  - highpass properties of inverse filtering
  - lowpass properties due to K

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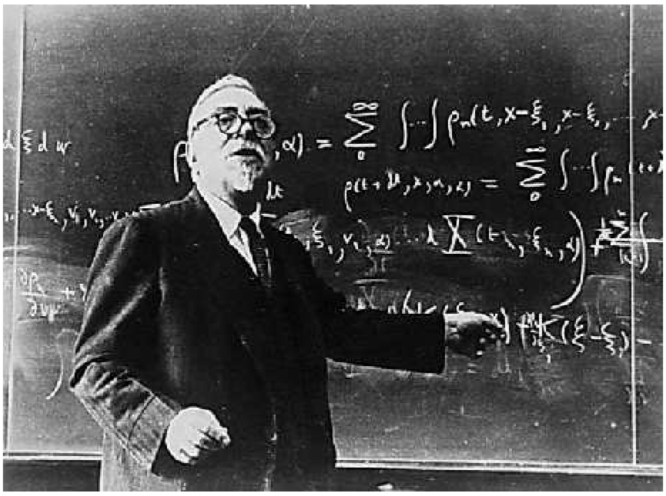
- significantly more robust with respect to noise than simple inverse filtering
- is regarded as one of the best linear methods for deconvolution problems with additive Gaussian noise
- K should depend on the estimated noise variance σ^2:
$$K := 2\sigma^2$$

Wiener Filtering (3)



The American mathematician Norbert Wiener (1894–1964) studied problems of control theory and communication. He is regarded as the founder of cybernetics and a co-founder of information theory. Source: `http://perso.wanadoo.fr/metasystems/Cybernetics.html`.

"He drove 150 miles to a math conference at Yale University. When the conference was over, he forgot he came by car, so he returned home by bus. The next morning, he went out to his garage to get his car, discovered it was missing, and complained the police that while he was away, someone stole his car."

Wiener Filtering (4)

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(a) Top left: Original image (U2, Joshua Tree). **(b) Top right:** With simulated motion blur and noise. **(c) Bottom left:** Deblurring with Wiener filtering. Source: `www.owlnet.rice.edu/~elec431/projects95/lords/wiener.html`.

Variational Deconvolution Methods

- variational image restoration studied so far (Lecture 14):

$$E_f(u) := \frac{1}{2} \int_\Omega \left((u-f)^2 + \alpha\, |\nabla u|^2 \right) \, dx \, dy$$

- modification by introducing a shift-invariant symmetric blurring kernel h:

$$E_f(u) := \frac{1}{2} \int_\Omega \left((h*u-f)^2 + \alpha\, |\nabla u|^2 \right) \, dx \, dy$$

- resulting Euler–Lagrange equation:

$$0 = h*(h*u-f) - \alpha\, \Delta u.$$

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- A discrete model like in Lecture 15 creates a linear system of equations. However, due to the convolution it is no longer sparse in most cases. It can be solved efficiently in the Fourier domain if the kernel h is not space-variant.

- If discontinuity-preserving deconvolution is required, one can use a nonquadratic regulariser $\Psi(|\nabla u|^2)$ (cf. Lecture 16). It leads to a nonlinear system of equations.

- The method can also be modified to treat nonsymmetric kernels and colour images.

Variational Deconvolution Methods (3)

From left to right: (a) Original colour image, 240×320 pixels. **(b)** Blurring with a nonsymmetric kernel. **(c)** Blurring kernel, 61×52 pixels. **(d)** Result of a variational deconvolution approach with nonquadratic penalisation. Authors: M. Welk, D. Theis, T. Brox, J. Weickert (2005).

Blind Deconvolution (1)

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Blind Deconvolution

Problem:

- Often the blurring kernel is unknown.
- Is is possible to estimate a shift-invariant convolution kernel h and the unknown image u at the same time ?
- This task is called *blind deconvolution*.

Blind Deconvolution (2)

Variational Blind Deconvolution:

- One minimises an energy with regularisers for both u and h:

$$E(u, h) \;:=\; \frac{1}{2} \int_{\Omega} \left((h * u - f)^2 + \alpha\, |\nabla u|^2 + \beta\, |\nabla h|^2 \right) dx\, dy$$

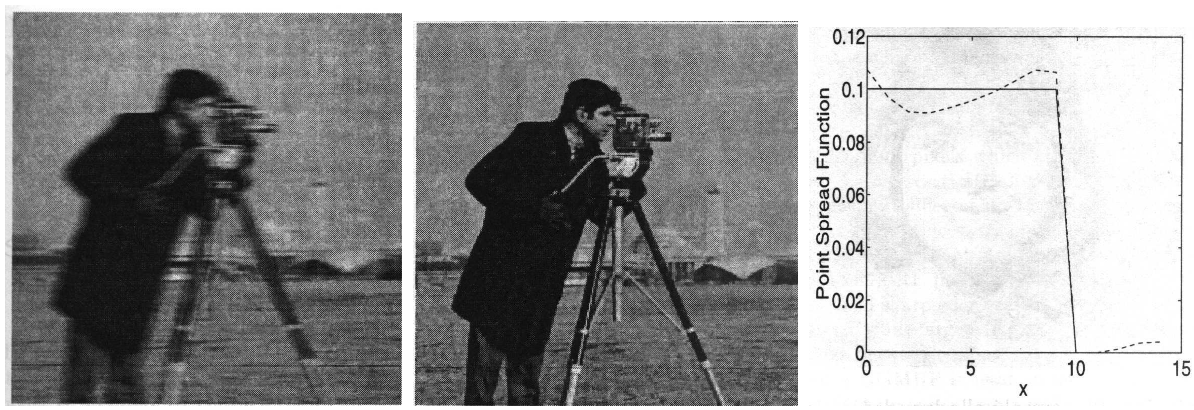
- creates two coupled Euler–Lagrange equations (cf. Lecture 14):

$$\begin{aligned} 0 &= h * (h * u - f) - \alpha\, \Delta u, \\ 0 &= u * (h * u - f) - \beta\, \Delta h \end{aligned}$$

- Its discretisations can be solved in an iterative and alternating manner.

- Also here one can use nonquadratic regularisers.

Blind Deconvolution (3)



(a) Left: Original image with noise and motion blur. **(b) Middle:** Blind deconvolution with nonquadratic regularisers. **(c) Right:** Estimated kernel (dotted line) compared with the correct kernel. Authors: Y.-L. You, M. Kaveh (1996).

Summary

- Fourier methods allow to remove periodically structured perturbations.
- Homomorphic filtering reduces illumination inhomogeneities by a logarithmic transform with subsequent highpass filtering.
- For a known shift-invariant blurring kernel, deconvolution can be performed in the Fourier domain.
- A naive inverse filtering is usually unstable. It is better to use pseudoinverse filtering.
- For deconvolution problems with additive Gaussian noise Wiener filtering is among the best linear methods.
- Alternatively one can use variational methods. For nonquadratic penalisers, they come down to nonlinear systems of equations.
- Blind deconvolution simultaneously estimates also the unknown convolution kernel.

Literature

- F. M. Wahl: *Digitale Bildsignalverarbeitung.* Springer, Berlin, 1984.
  (Homomorphic filtering is treated in Section 3.3.3, and Chapter 4 gives a good description of image deconvolution methods in the Fourier domain.)
- J. M. Parker: *Algorithms for Image Processing and Computer Vision.* Wiley, New York, 1997.
  (usually not a recommendable book, but the image restoration part is okay)
- K. R. Castleman: *Digital Image Processing.* Prentice Hall, Upper Saddle River, 1996.
  (for deconvolution in the Fourier domain)
- M. Petrou, P. Bosdogianni: Image Processing: The Fundamentals. Wiley, Chichester, 1999.
  (Chapter 6 provides you with many mathematical details about deconvolution in the Fourier domain.)
- M. Welk, D. Theis, T. Brox, J. Weickert: PDE-based deconvolution with forward-backward diffusivities and diffusion tensors. In R. Kimmel, N. Sochen, J. Weickert (Eds.): *Scale-Space and PDE Methods in Computer Vision.* Lecture Notes in Computer Science, Vol. 3459, Springer, Berlin, pp. 585–597, 2005 (`http://www.mia.uni-saarland.de/Publications/welk-tbw-ss05.pdf`).
  (variational deconvolution)
- Y.-L. You, M. Kaveh: Anisotropic blind image restoration. *Proc. IEEE International Conference on Image Processing* (ICIP–96, Lausanne, Sept. 16–19, 1996), Vol. 2, pp. 461–464, 1996.
  (variational blind deconvolution)

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