Lecture 13:
Image Enhancement IV:
Morphological Filters

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What are Morphological Filters?

Mathematical Morphology

- analyses the shape of objects in an image
- was founded by Jean Serra and George Matheron around 1965 at the Ecole Normale Superiéure des Mines in Fontainebleau near Paris
- one of the most successful classes of image analysis methods
- numerous applications:
  cell biology, medical image analysis, geostatistics, remote sensing, ...
- provides a nonlinear alternative to the linear system theory from Lecture 11

What are Morphological Filters? (2)

The building in Fontainebleau where mathematical morphology was born is now called Centre de Morphologie Mathématique (CMM). Source: `http://cmm.ensmp.fr/presentation.html`.

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What are Morphological Filters? (3)

Properties of Morphological Filters

- invariant under monotone increasing greyscale transformations (cf. Lecture 10: affine greyscale transformations, gamma corrections, histogram equalisations, ...):

  $$MGf = GMf$$

  for any image f, any morphological filter M and any monotoneously increasing greyscale transformation G.

- Hence, sensor characteristics and brightness of illumination do not matter. This implies, however, that image contrast does not matter as well.

- Morphological methods do only take into account the *level sets* $L_i(f) := \{(x,y) \mid f(x,y) \geq i\}$ of an image f.

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The Building Blocks: Dilation and Erosion

- *Dilation (in German: Dilatation)* replaces the grey value of a continuous image $f(x, y)$ by its supremum within a mask B:

  $$(f \oplus B)\,(x, y) \;:=\; \sup\,\{f(x{-}x',\, y{-}y') \mid (x', y'){\in}B\}.$$

  Erosion (in German: Erosion) uses the infimum instead:

  $$(f \ominus B)\,(x, y) \;:=\; \inf\,\{f(x{+}x',\, y{+}y') \mid (x', y'){\in}B\}.$$

  For discrete images: maximum / minimum instead of supremum / infimum.

- The mask B is called *structuring element (Strukturelement)*. Often convex structuring elements are used: disks or squares with reference point in their centre.

- For binary images with bright object and dark background, dilation propagates the object contour in outer normal direction, while erosion moves it in inner normal direction.

- Analogies:
  – flame propagation in a prairie fire
  – Huygens principle for wave propagation

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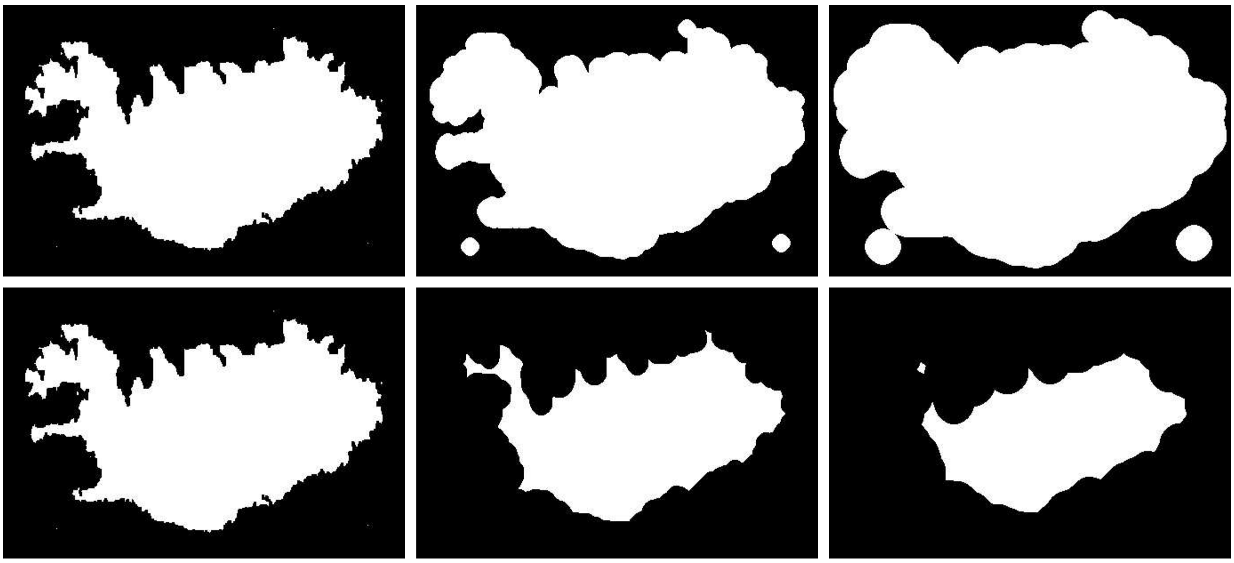
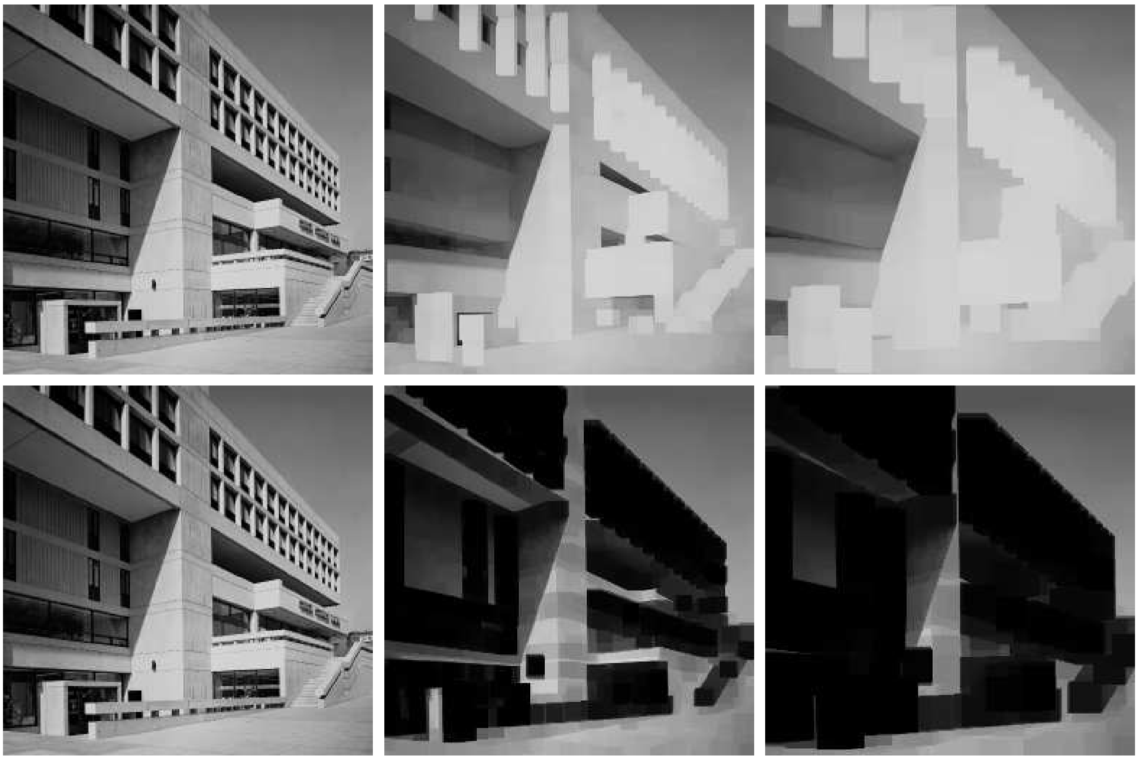


(a): Top row, from left to right: Dilation of a binary image of Iceland (452×305 pixels) with a disc with radius 10 and 20. **(b): Bottom row, from left to right:** Corresponding erosion. Author: J. Weickert (2002).

The Building Blocks: Dilation and Erosion (3)



(a) Top row, from left to right: Dilation of a greyscale image (256×256 pixels) with a square of length 11 and 21. **(b) Bottom row, from left to right:** Corresponding erosion. Author: J. Weickert (2002).

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The Building Blocks: Dilation and Erosion (4)

Algorithms for Convex Structuring Elements

- Convex structuring elements are scalable: Dilation / erosion with structuring element nB is equivalent to n dilations / erosions with structuring element B.
- Scalability is important for implementations in hardware:
  large structuring elements from a cascade of small ones

Algorithms for Rectangular Structuring Elements

- Rectangular structuring elements are separable:
  It is sufficient to find efficient 1-D algorithms.
- The van Herk algorithm (next page) allows to compute 1-D dilations / erosions with lines of arbitrary length as structuring elements with only 3 max-/min comparisons per pixel.
- Thus, 2-D dilations / erosions with rectangular structuring elements can be done in linear complexity with a factor that is independent of the size of the structuring element!

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The Building Blocks: Dilation and Erosion (5)

The van Herk Algorithm

Example: Dilation with structuring element of length 6 centred in the leftmost pixel.

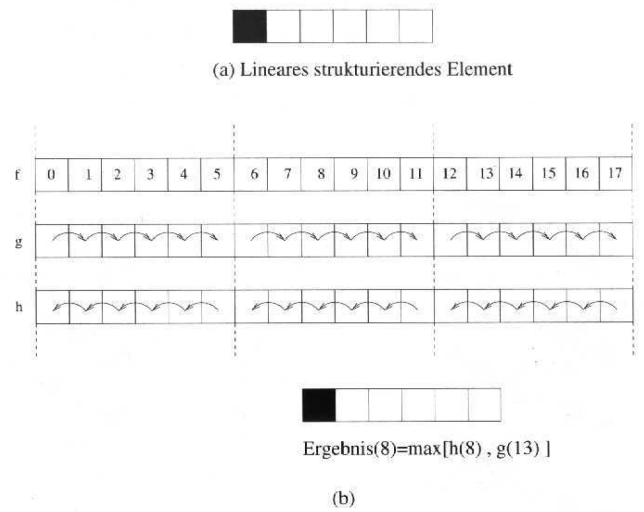


Abbildung 3.27. Iterativer van Herk-Algorithmus. f stellt die zu verarbeitende Bildzeile oder -spalte dar. Die Maximalwerte pflanzen sich von links nach rechts (Puffer g) und von rechts nach links (Puffer h) fort, wieder beginnend mit dem Eingangsbildwert für jeden Index gleich einem Vielfachen der Länge des SE.

The same algorithm has been re-published by Gil and Werman in 1993.

Morphological Lowpass Filters: Opening and Closing (1)

Morphological Lowpass Filters: Opening and Closing

Closing (Schließung):

- Goal: Simplify the image structure while avoiding the expansion effects of dilation
- perform an erosion after the dilation:

$$f \bullet B \;:=\; (f \oplus B) \ominus B$$

- closes gaps in bright structures by removing dark details

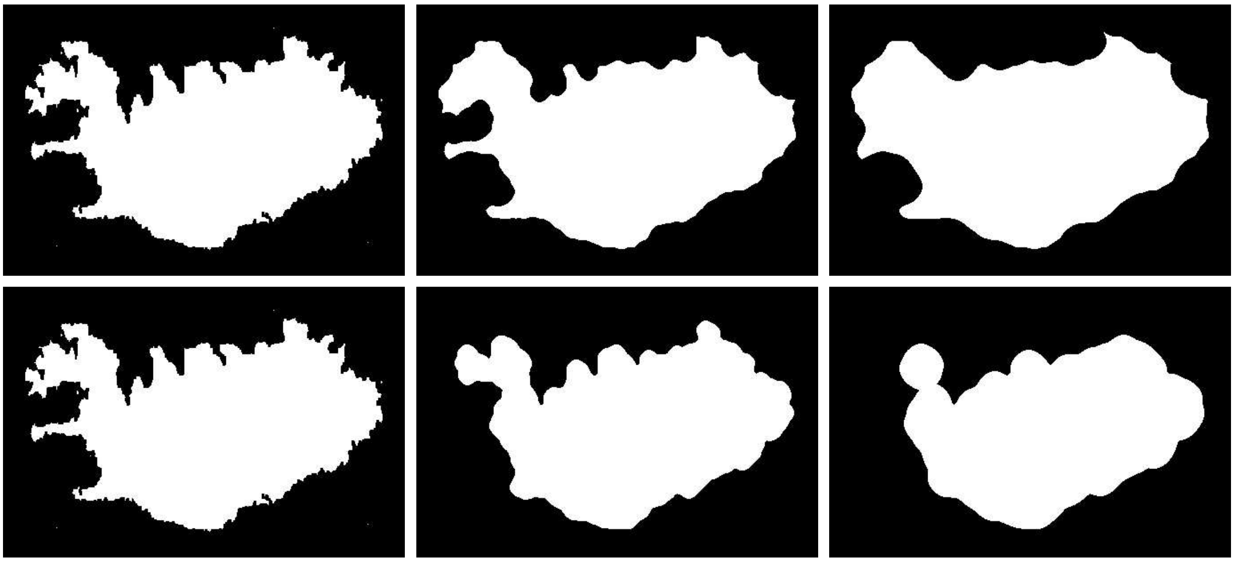
Opening (Öffnung):

- Goal: Simplify the image structure while avoiding the shrinkage effects of erosion
- perform a dilation after the erosion:

$$f \circ B \;:=\; (f \ominus B) \oplus B$$

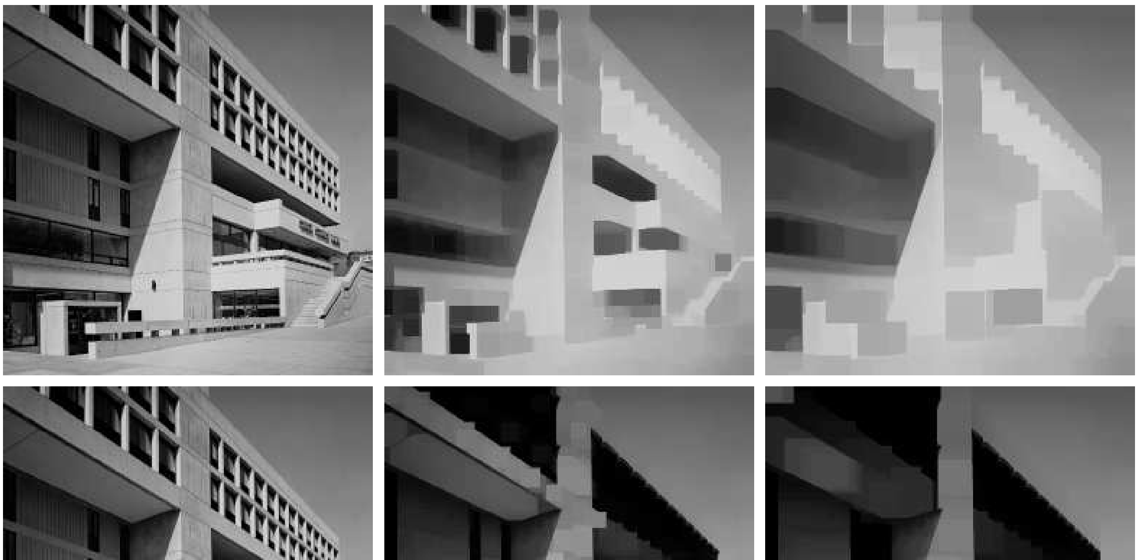
- closes gaps in dark structures by removing bright details

Morphological Lowpass Filters: Opening and Closing (2)



(a): Top row, from left to right: Closing of a binary image of Iceland (452×305 pixels) with a disc with radius 10 and 20. **(b): Bottom row, from left to right:** Corresponding opening. Author: J. Weickert (2002).

Morphological Lowpass Filters: Opening and Closing (3)



(a) Top row, from left to right: Closing of a greyscale image (256×256 pixels) with a square of length 11 and 21. **(b) Bottom row, from left to right:** Corresponding opening. Author: J. Weickert (2002).

Application: Granulometries (Granulometrien)

- Multiple openings or closings with the same structuring element do not alter the image any more *(idempotency)*:

$$(f \circ B) \circ B \;=\; f \circ B$$
$$(f \bullet B) \bullet B \;=\; f \bullet B$$

- Opening and closing are so-called *sieve operations (Sieboperationen):* Structures that pass the sieve (the filter) once also pass it a second time.
- Using structuring elements of increasing size, however, one can remove small-, middle- and coarse-scale structures step by step.
- Such a morphological image decomposition into structures of different size is called *granulometry*.
- Because of their simplifying nature, opening and closing play the role of morphological lowpass filters.
Granulometries play the role of morphological bandpass filters.

Morphological Highpass Filters: Top Hats

Top hats (Zylinderhüte) extract fine-scale details.

- *White Top Hat, Top Hat by Opening:*

$$\mathrm{WTH}(f) \;:=\; f \;-\; (f \circ B)$$

extracts small bright structures

- *Black Top Hat, Top Hat by Closing:*

$$\mathrm{BTH}(f) \;:=\; (f \bullet B) \;-\; f$$

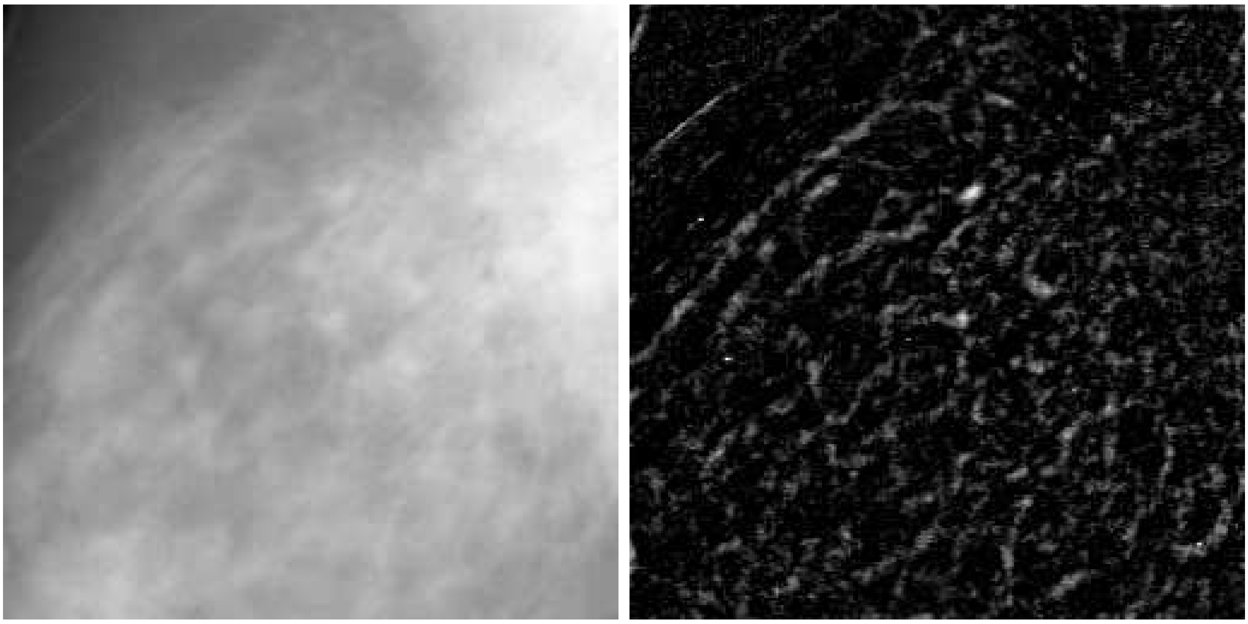
extracts small dark structures

- *Selfdual Top Hat:*

$$\rho(f) \;=\; \mathrm{WTH}(f) \;+\; \mathrm{BTH}(f) \;=\; (f \bullet B) \;-\; (f \circ B)$$

extracts all small structures; invariant with respect to greyscale inversion

Morphological Highpass Filters: Top Hats (2)



(a) Left: Mammogram, 256×256 pixels. **(b) Right:** White top hat with a square of size 7×7 pixels as structuring element. Microcalcifications and star-shaped structures become better visible.

Morphological Highpass Filters: Top Hats (3)

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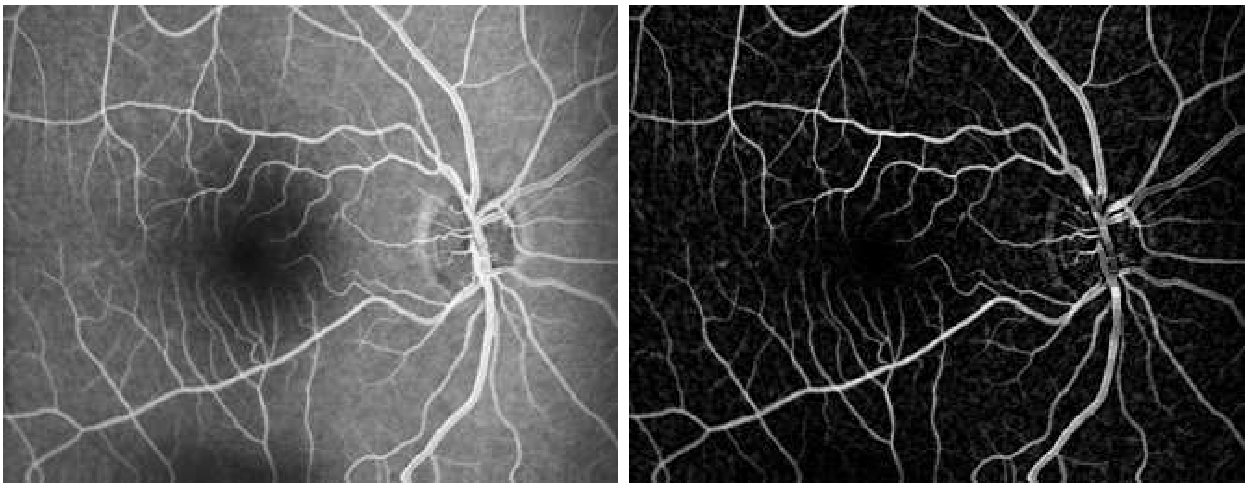

(a) Left: Image of the background of an eye. **(b) Right:** Vessel extraction with a white top hat. Compare the result with Lecture 11, Slide 23 where linear highpass filters cannot distinguish between bright and dark details.

Morphological System Theory: The Slope Transform

Motivation

The Fourier transform is essential for understanding *linear* systems:

- filter analysis and filter design in the Fourier domain
- (computationally expensive) convolution becomes (inexpensive) multiplication

Is there a similar transformation in morphology ? It should transform the basic morphological operation, the dilation, into a simple operation.

Generalisation of Dilation

- use *structuring function* $b(x, y) : \mathbb{R}^2 \to \mathbb{R}$ instead of structuring element B (renounces morphological invariance)
- We consider the *tangential dilation*

$$(f \check{\oplus} b)(x, y) \;:=\; \operatorname*{stat}_{x',y'} \Big(f(x-x', y-y') + b(x', y') \Big)$$

  where the stat operator denotes the stationary values, i.e. values with derivative 0 (maxima, minima, saddle points):

$$\operatorname*{stat}_{x',y'} g(x', y') := \{ g(x', y') \,|\, \nabla g(x', y') = 0 \}.$$

- When identifying a structuring element B with the "flat" structuring function

$$b(x, y) \;:=\; \begin{cases} 0 & ((x, y) \in B), \\ -\infty & ((x, y) \notin B) \end{cases}$$

  and considering strictly concave functions f, tangential dilation becomes classical dilation.

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The Slope Transform

(Dorst / van den Boomgaard 1993, Maragos 1993)

- The *slope transform (Slopetransformation)*

$$\mathcal{S}[f](u,v) := \operatorname*{stat}_{x,y} \Big(f(x,y) - (ux+vy) \Big)$$

  is the morphological analogue to the Fourier transform

$$\mathcal{F}[f](u,v) := \int\!\!\int f(x,y)\, e^{-i2\pi(ux+vy)}\, dx\, dy$$

- It transforms tangential dilation to addition,

$$\mathcal{S}[f \check{\oplus} b] \;=\; \mathcal{S}[f] \,+\, \mathcal{S}[b],$$

  while the Fourier transformation turns convolution into multiplication:

$$\mathcal{F}[f * b] \;=\; \mathcal{F}[f] \,\cdot\, \mathcal{F}[b]$$

- The *inverse slope transform* is given by

$$f(x,y) \;=\; \operatorname*{stat}_{u,v} \left(\mathcal{S}[f](u,v) + (ux+vy) \right)$$

  Inverse Fourier transform:

$$f(x,y) \;=\; \int\!\!\int \mathcal{F}[f](u,v)\, e^{i2\pi(ux+vy)}\, du\, dv$$

- Note the almost logarithmic connection between both worlds.
  There is an explanation for this connection (Burgeth / Weickert 2005):
  Morphological system theory is linear system theory in another algebra (max-plus algebra), where the $(+,\cdot)$ operations are replaced by $(\max,+)$.

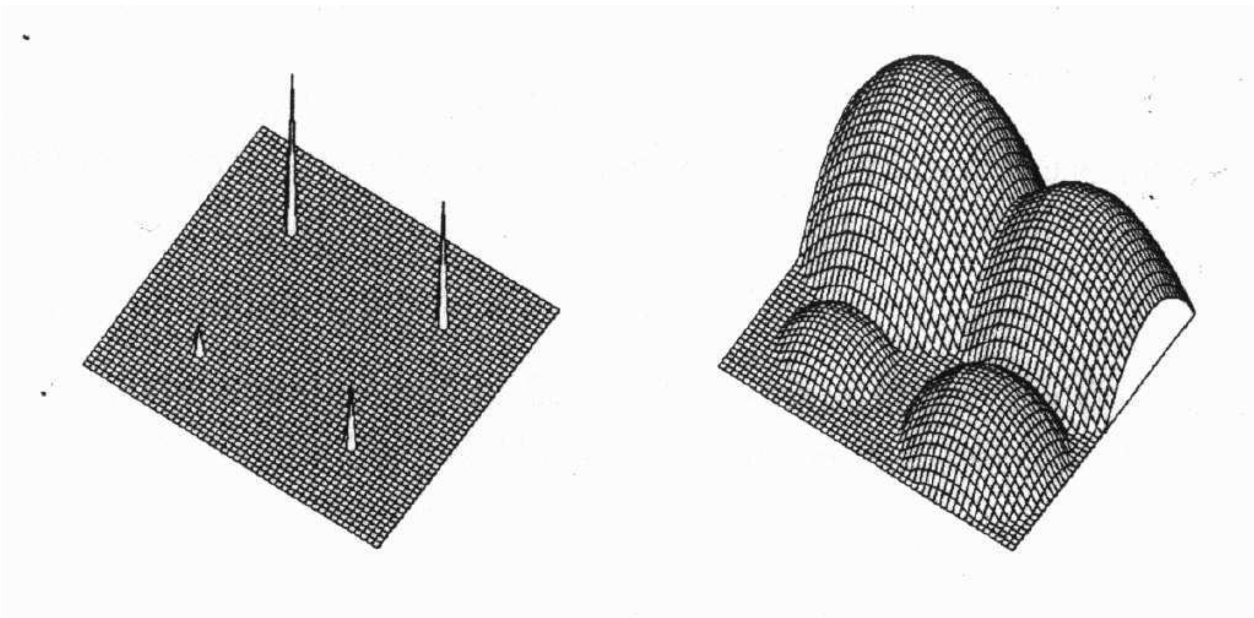
- In 1996, Lucet discovered a morphological analogue to the Fast Fourier Transform (FFT), the so-called *Fast Legendre Transform (FLT)*.

Morphological System Theory: The Slope Transform (5)

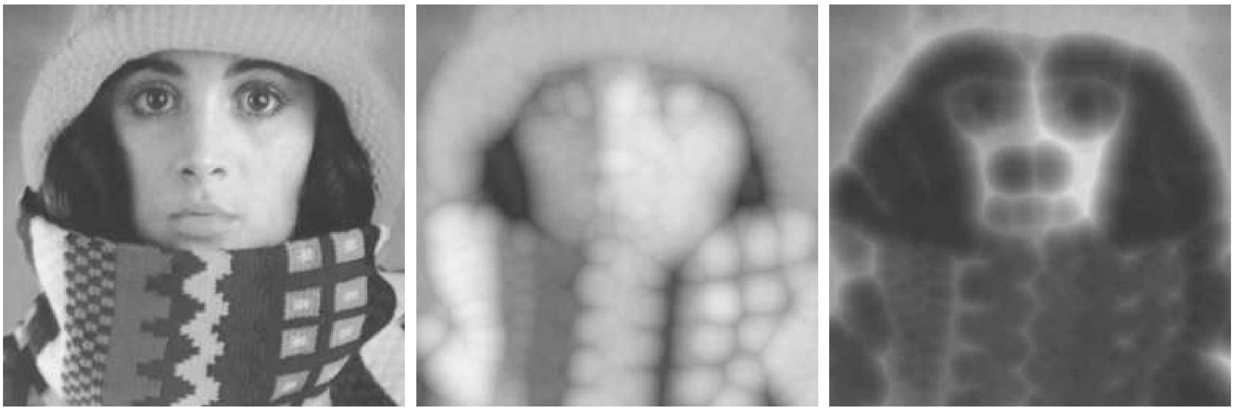
Das Morphological Analogue to Gaussian Convolution

- In linear system theory, Gaussian convolution plays a fundamental role:
  - Gaussians remain Gaussians under the Fourier transform.
  - Gaussians are the only separable and rotationally invariant convolution kernels.
- Analog results for morphology:
  - Paraboloids remain paraboloids under the slope transform.
  - Paraboloids are the only structuring functions that are separable and rotationally invariant.

Morphological System Theory: The Slope Transform (6)



A 2-D signal and its dilation with a paraboloid as structuring function. Author: R. van den Boomgaard (1992).



Paraboloids as structuring functions can create scary images. **Left:** Original image. **Middle:** Dilation. **Right:** Erosion. Author: J. Weickert (1999).

Application: Euclidean Distance Transformation

- given: binary image $f(x, y)$
- set foreground to 0 and background to ∞
- erode with a paraboloid $b(x, y) = -x^2 - y^2$ as structuring function
- gives squared distance transformation
- important e.g. in robotics (collision avoidance)
- tip for better visualisation:
  periodic greyscale repetition after a few pixels

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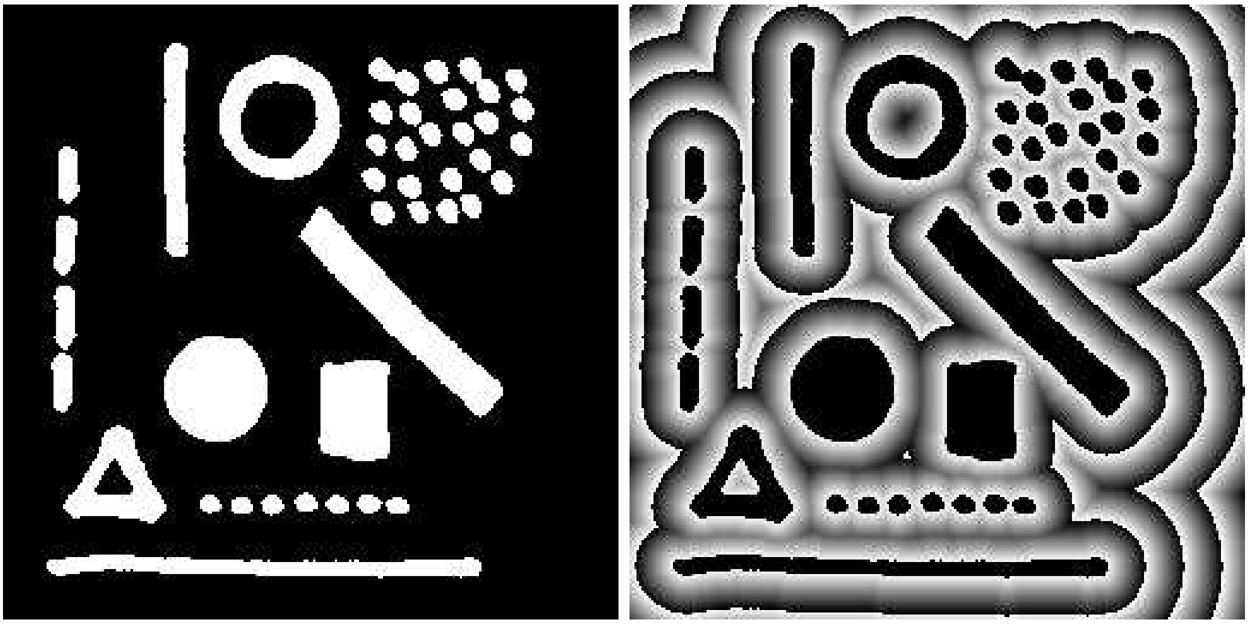
Morphological System Theory: The Slope Transform (9)



Left: Test image, 256×256 pixels. **Right:** Euclidean distance transformation. The greyscale is repeated periodically after 16 pixels. Author: J. Weickert (1999).

Summary (1)

Summary

- Morphological filters are invariant under monotoneously increasing greyscale transformations.
- By replacing a grey value by its maximum or minimum within a neighbourhood, dilation and erosion are obtained.
- Dilation and erosion are used for shape analysis.
- Sequential combinations of erosion und dilation create openings and closings. They act as morphological lowpass filters.
- Granulometries are examples for morphological bandpass filters.
- Top hats result from computing differences between closing, original image, and opening. They act as morphological highpass filters.
- There exists a morphological system theory that resembles linear system theory.
- The slope transform is the morphological analogue to the Fourier transform. It transforms (tangential) dilation into addition.
- Parabolas / paraboloids as structuring functions are the morphological analogues to Gaussians in linear system theory.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27

Literature

- P. Soille: *Morphological Image Analysis*. Second Edition. Springer, Berlin, 1999.
  (Well readable text book on morphology. Not much theory though.)
- M. van Herk: A fast algorithm for local minimum and maximum filters on rectangular and octogonal kernels. *Pattern Recognition Letters*, Vol. 13, 517–521, 1992.
  (describes the van Herk algorithm)
- R. van den Boomgaard: The morphological equivalent of the Gauss convolution. *Nieuw Archief Voor Wiskunde*, Vierde Serie, Deel 10, No. 3, pp. 219–236, 1992.
  (parabolic structuring elements as analogue to Gaussian convolution kernels)
- L. Dorst, R. van den Boomgaard: Morphological signal processing and the slope transform. *Signal Processing*, Vol. 38, pp. 79–98, 1994.
  (one of the earliest papers on the slope transform)
- B. Burgeth, J. Weickert: An explanation for the logarithmic connection between linear and morphological system theory. *International Journal of Computer Vision*, Vol. 64, No. 2/3, 157–169, Sept. 2005. (`http://www.mia.uni-saarland.de/publications.shtml`)
  (morphological system theory as linear system theory in max-plus algeba)
- Y. Lucet: A fast computational algorithm for the Legendre-Fenchel transform. *Computational Optimization and Applications*, Vol. 6, 27–57, 1996.
  (morphological analogue to the FFT)
- Morphology Digest: `http://ams.jrc.it/mdigest/`
  (latest news on mathematical morphology)