Lecture 10: Image Enhancement I: Point Operations

Contents

What are Point Operations?

Image processing operations transform an image into another one that is more useful for a human– or computer–based interpretation.

Point Operations:

- simplest transformations for image enhancement: global greyscale modification
- do not take into account grey value configuration in the neighbourhood
- for greyscale images, all point operations are of the structure

$$\phi : \quad f(x,y) \quad \longmapsto \quad g(x,y) = \phi(f(x,y))$$

- note that the location (x,y) does not matter for ϕ
- can be very useful in many applications and should be tried first

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Affine Greyscale Transformations

- have a simple structure: $g(x, y) \; = \; a \cdot f(x, y) + b$
- for $b = 0$ they are also called *linear* greyscale transformations
- allow numerous possibilities:

(1)	*identity:*	$a = 1,$	$b = 0$
(2)	*contrast enhancement:*	$a > 1$	
(3)	*contrast attenuation:*	$a < 1$	
(4)	*brightening:*	$a = 1,$	$b > 0$
(5)	*darkening:*	$a = 1,$	$b < 0$
(6)	*greyscale reversion:*	$a = -1,$	$b = g_{max}$

- frequent application:
  greyscale transformation to the interval $[0, 255]$
  (which is easy to display and to store)
- disadvantage:
  - does not take into account how often a grey value is present
  - one outlier can spoil the result
- remedy: histogram equalisation (later)

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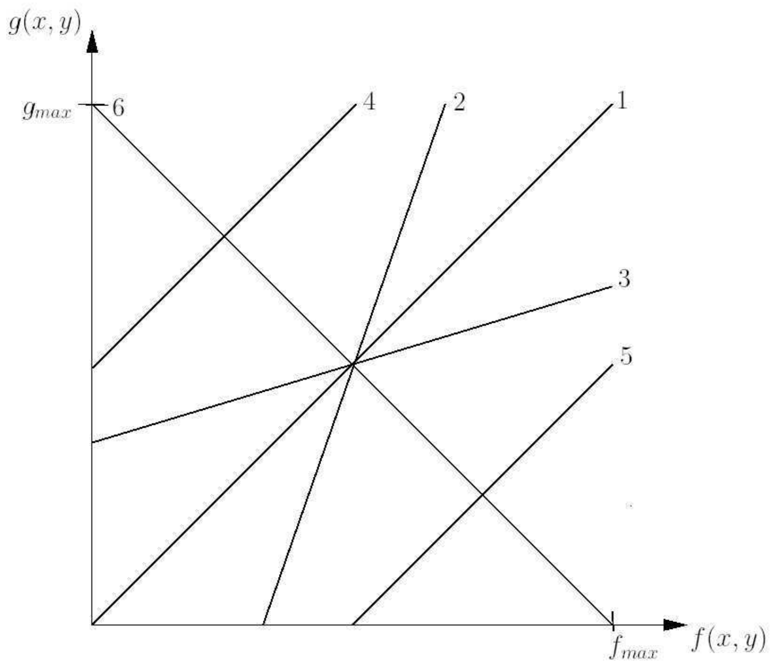

Affine greyscale transformations. **(1)** Identity. **(2)** Contrast enhancement. **(3)** Contrast attenuation. **(4)** Brightening. **(5)** Darkening. **(6)** Greyscale reversion. Author: S. Zimmer (2002).

(a) Left: Underexposed original image. **(b) Right:** After affine greyscale mapping to the interval $[0, 255]$. Author: J. Weickert (2000).

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Nonlinear Greyscale Transformations

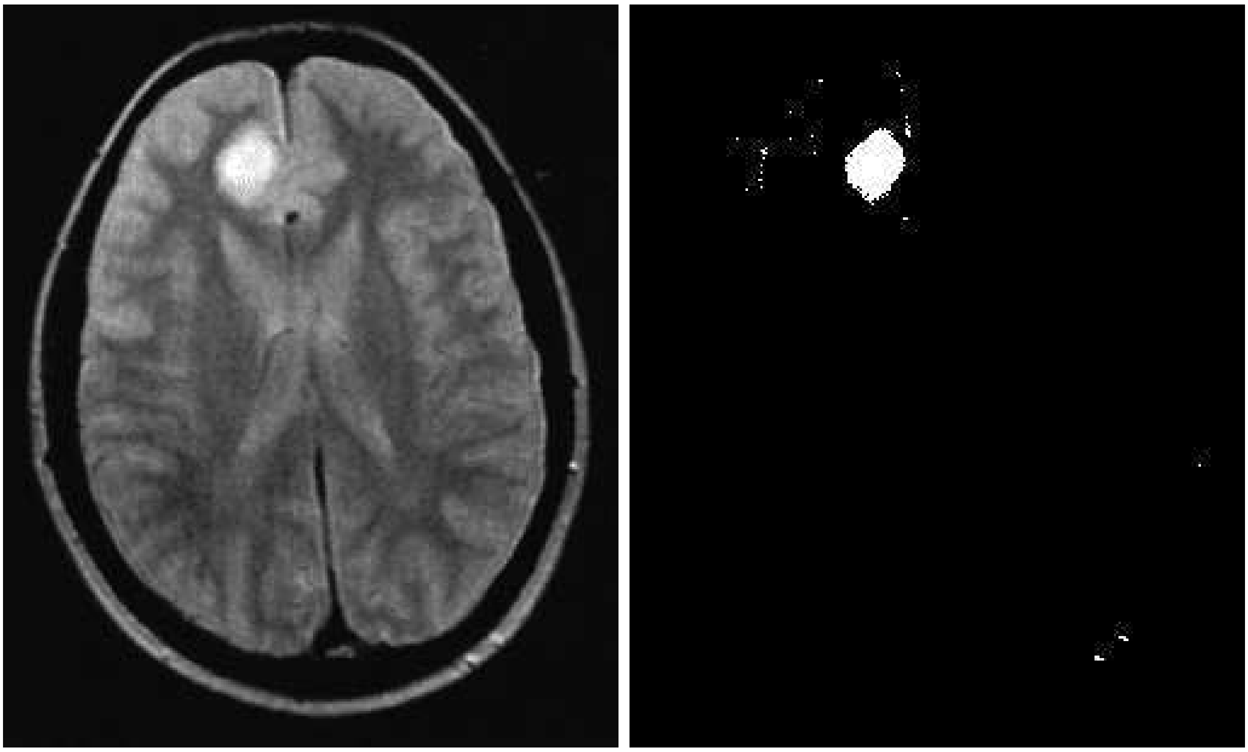
(a) Thresholding (Binarisation, Schwellwertbildung)

- can be described by the transformation

$$\phi(f(x,y)) \;:=\; \begin{cases} g_{max} & \text{for } f(x,y) \geq T \\ 0 & \text{else} \end{cases}$$

- simplest method for segmentation
- most difficult part: finding a good threshold parameter T

Nonlinear Greyscale Transformations (2)



Thresholding. **(a) Left:** MR image of a human head with a tumour. **(b) Right:** Thresholding allows to segment the tumour. Author: J. Weickert (2000).

Nonlinear Greyscale Transformations (3)

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(b) Logarithmic Dynamic Compression

- For $f(x,y) \geq 0$, one can compute the transformation

$$\phi(f(x,y)) \;:=\; c \,\log\left(1 + f(x,y)\right)$$

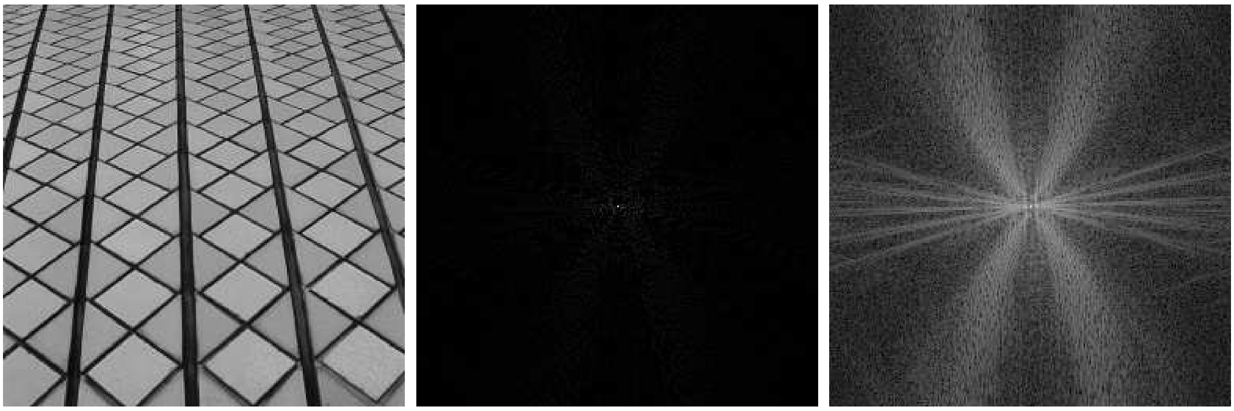
- useful if dynamic range is too large, e.g. for visualising the Fourier spectrum $|\hat{f}(u,v)|$ (cf. Lecture 4)

- Often c is chosen such that $\max\limits_{x,y} \phi(f(x,y)) = 255$:

$$c \;:=\; \frac{255}{\log\left(1 + \max\limits_{x,y} f(x,y)\right)}$$

  Then all transformed grey values are in $[0, 255]$.

Nonlinear Greyscale Transformations (4)



Logarithmic dynamic compression. **(a) Left:** Original image, 256×256 pixels. **(b) Middle:** Fourier spectrum without logarithmic dynamic compression. The white pixel in the centre corresponds to the sum of all grey values. It dominates over all other Fourier coefficients. **(c) Right:** After logarithmic dynamic compression, the entire Fourier spectrum is well visible. The constant c is chosen such that the range of the transformed spectrum coincides with the interval [0,255]. Author: J. Weickert (2002).

Nonlinear Greyscale Transformations (5)

(c) Gamma Correction (Gammakorrektur)

- Many video cameras transform a light intensity I into a grey value f that is proportional to I^γ. Often $\gamma \approx 0.4$.
- Similar nonlinearities can also be observed for computer monitors where the value for γ varies from brand to brand.
- Sometimes the γ value is even changed by software.
- As a result, an image may look unpleasant on a specific monitor.
- To compensate for these effects, a so-called *gamma correction* can be used. For an image with greyscale range $[0, f_{max}]$ it is given by

$$\phi(f(x,y)) \;:=\; f_{\max} \left(\frac{f(x,y)}{f_{max}} \right)^{1/\gamma} \qquad (\gamma > 0).$$

- Thus, the range $[0, f_{max}]$ is not affected.

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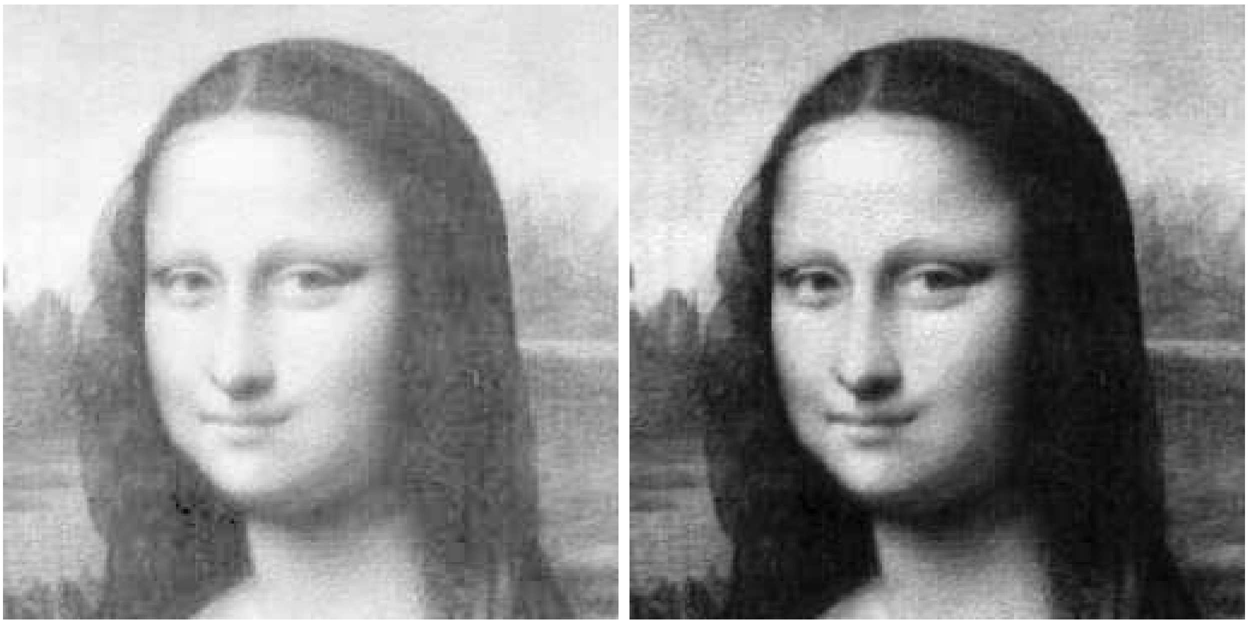
Nonlinear Greyscale Transformations (6)



Gamma correction. **(a) Left:** Although the entire greyscale range $[0, 255]$ is used, the Mona Lisa image appears pale and not very rich in contrast. **(b) Right:** A gamma correction with $\gamma = 0.4$ is a remedy. Authors: L. da Vinci (1503–1506), J. Weickert (2002).

Histogram Equalisation (1)

Histogram Equalisation (Histogrammegalisierung)

Basic Idea:

- another important nonlinear point operation
- function ϕ depends on *entire* grey level set $\{f(x,y) \,|\, (x,y) \in \Omega\}$
- goal: transformation such that all grey values occur equally frequent
- often dramatic improvements in the subjective image quality

Histogram:

- specifies relative frequency of occurence of a grey value within an image
- spatial context does not matter: any pixel permutation gives same histogram

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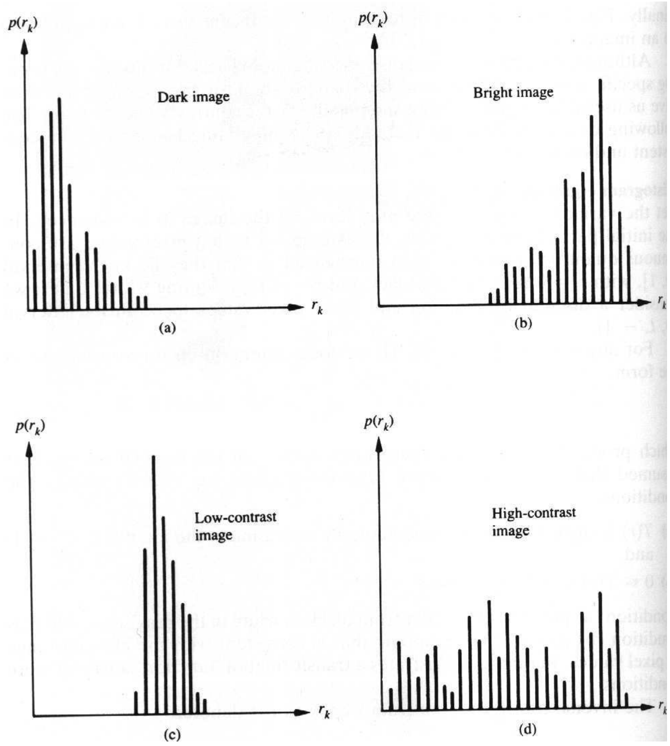
Histogram Equalisation (2)



Histograms of different types of images. **(a):** Dark image. **(b):** Bright image. **(c):** Low-contrast image. **(d):** High-contrast image. Authors: R. C. Gonzalez and R. E. Woods (1992).

Histogram Equalisation (3)

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Algorithm for Direct Histogram Equalisation of a Discrete Image

- Given: p_i: number of pixels of image f having grey value v_i $(i = 1, ..., m)$
  q_j: desired number of pixels of g with grey value w_j $(j = 1, ..., n)$
  (for N pixels and 256 grey scales: $q_j := \frac{N}{256}$)
- Set $k_0 := 0$.
- For $r = 1, ..., n$:
  Search the largest index $k_r \leq m$ with

$$\sum_{i=1}^{k_r} p_i \leq \sum_{j=1}^{r} q_j$$

  and map the grey values $v_{k_{r-1}+1}, ..., v_{k_r}$ to w_r.

Histogram Equalisation (4)

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Remarks

- Sums with upper index smaller than the lower index are set to 0.
  Nothing is mapped in this case.
- The algorithm does not only perform histogram equalisation, it can also transform a histogram in any other histogram *(histogram specification)*.
  All one has to do is to use other values for $q_1, \ldots, q_n$.
- For a general discrete image, histogram equalisation can only be approximated (see next page).

Histogram Equalisation (5)

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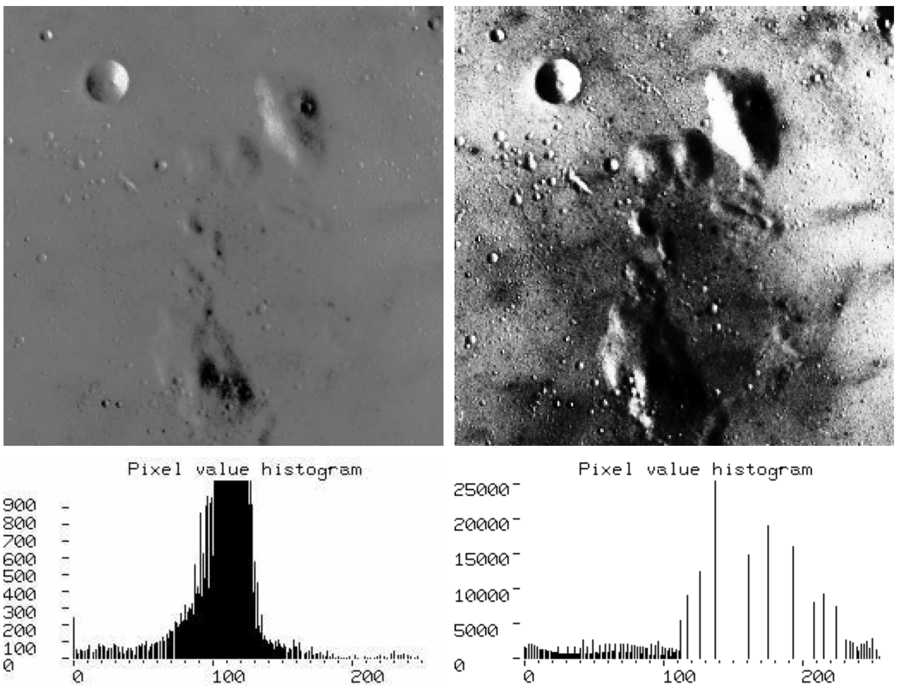

Histogram equalisation. **(a) Top left:** Original image of the surface of the moon. **(b) Top right:** After discrete histogram equalisation. **(c) Bottom left:** Histogram of the original image. **(d) Bottom right:** Histogram of the equalised image. Authors: R. Fisher, S. Perkins, A. Walker, E. Wolfa (2000), `www.dai.ed.ac.uk/HIPR2/histeq.htm`.

Pseudocolour Representation of Greyscale Images (1)

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Pseudocolour Representation of Greyscale Images

- Humans can distinguish only 40 greyscales, but $2,000,000$ colours.
- Thus, colouring grey values allows better visual discrimination.
- There are numerous possibilities to design mappings of type

$$f(x,y) \quad \longmapsto \quad \begin{pmatrix} \phi_r(f(x,y)) \\ \phi_g(f(x,y)) \\ \phi_b(f(x,y)) \end{pmatrix}$$

- used e.g. in X-ray scanners at airports

Pseudocolour Representation of Greyscale Images (2)

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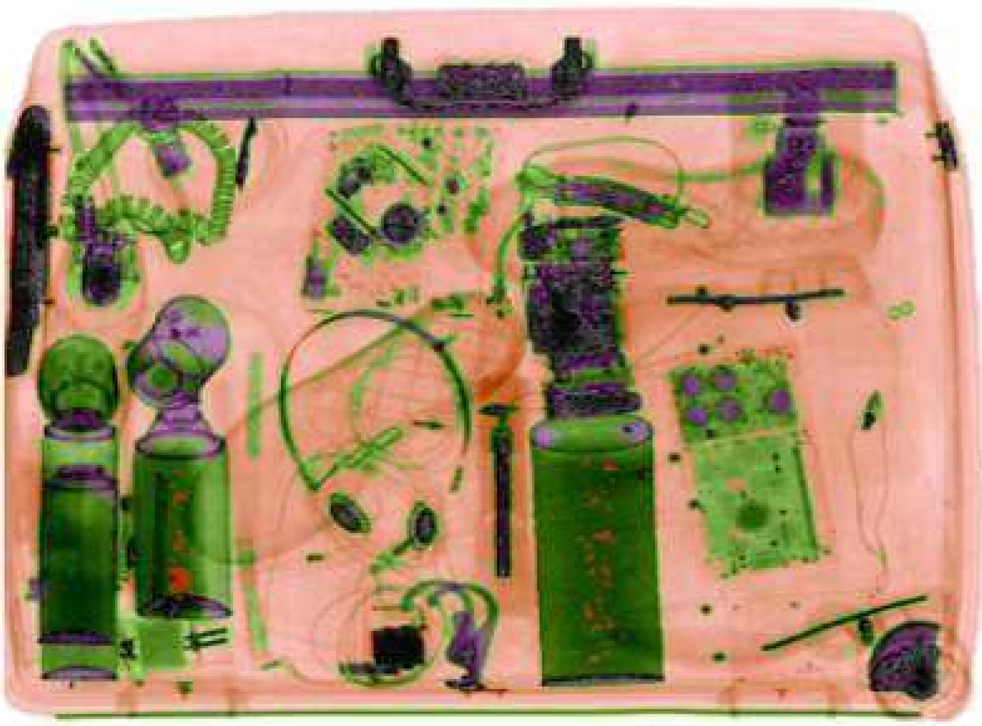
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30



Colouring X-ray images at airport security checks allows a human observer to distinguish objects in a better way. Source: `http://static.howstuffworks.com/gif/airport-security-xray2.jpg`.

False Colour Representation of Vectorial Images

- tranforms a multichannel image into a colour image
- often used in astronomical satellite imaging:
  frequencies outside the visible spectrum are mapped to colours

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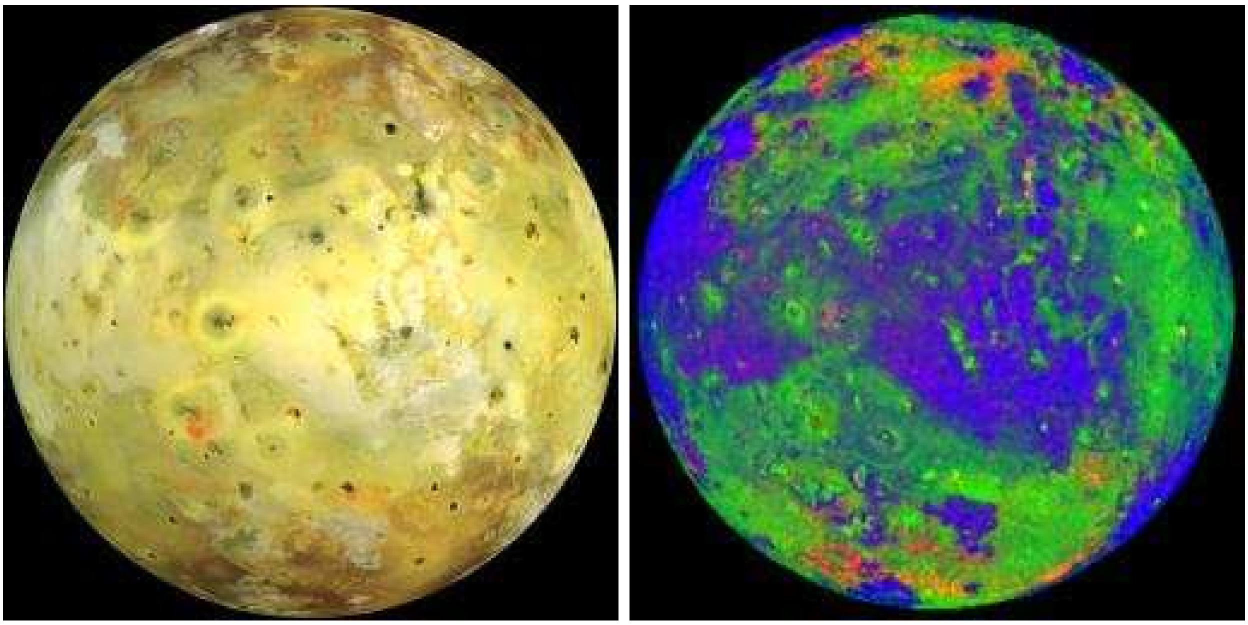

(a) Left: True colour representation of Jupiter's moon Io. **(b) Right:** False colour representation that combines informations from two visible and two infrared frequency bands. The depicted colours red, green and blue shows the fraction between two of the four channels. They allow a better interpretation of the surface structure: Red depicts hot vulcanoes, green presumably characterises regions with much sulphur, and blue indicates frozen sulphur dioxide. Source: NASA, `www.jpl.nasa.gov/galileo/images/io/iocolor.html`.

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Adding Images

Problem:

- Some imaging methods (e.g. electron microscopy) create very noisy images:

$$\underbrace{g(x,y)}_{\text{noisy}} = \underbrace{f(x,y)}_{\text{no noise}} + \underbrace{n(x,y)}_{\text{noise}}$$

Solution:

- If (!) the object can be recorded multiple times under the same conditions, one can average the images.
- For uncorrelated noise with mean 0, averaging of M images with noise variance

$$\sigma^2 \;=\; \frac{1}{|\Omega|} \int\limits_{\Omega} |n(x,y)|^2 \, dx \, dy$$

  creates a reduced noise variance $\bar{\sigma}^2 = \sigma^2/M$.
- If one wants to reduce the standard deviation $\bar{\sigma}$ of the averaged image to $1/10$ of its original value σ, one needs $10^2 = 100$ images !

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Denoising by averaging. **(a) Top Left:** Original image, 323×279 pixels, standard deviation: 52.29. **(b) Top Middle:** Gaussian noise with standard deviation 52.29 added. **(c) Top Right:** Averaged 4 times. **(d) Bottom Left:** Averaged 16 times. **(e) Bottom Middle:** Averaged 64 times. **(f) Bottom Right:** Averaged 256 times. Author: J. Weickert (2004).

MIA
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Subtracting Images

Example: Digital Subtraction Angiography (DSA)

- medical imaging method for visualising the blood flow through the vessels
- X ray images are taken before and after giving a flourescent contrast agent
- difference image makes active vessels visible

MIA
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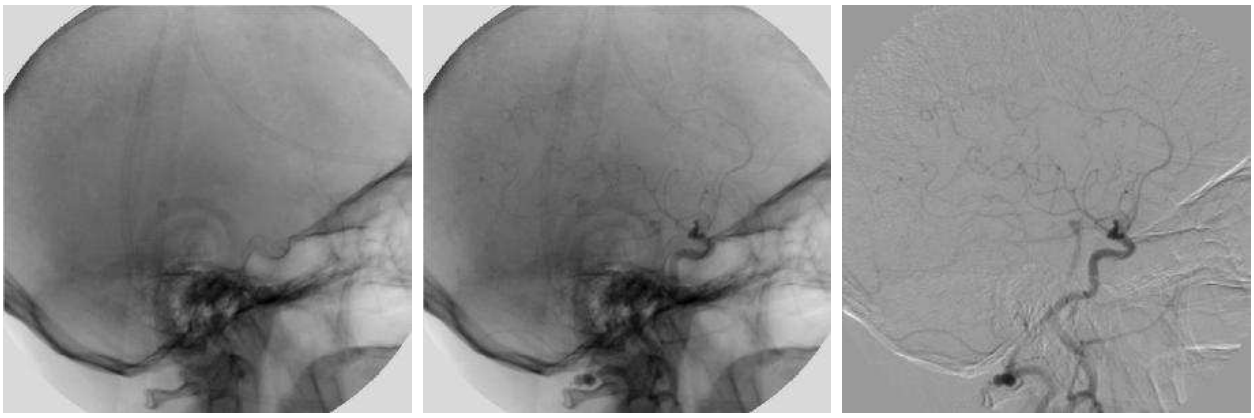

Digital subtraction angiography. **(a) Left:** Initial image (so-called mask). **(b) Middle:** After giving a contrast agent. **(c) Right:** The difference image removes the background and visualises vessel structures with blood flow. Source: `http://www.isi.uu.nl/Research/Gallery/DSA/`

Summary

- Point operations perform a global transformation of the greyscales.
- Typical application: new representation of the grey values with better visibility for humans
- The most important point transforms include:
  – affine rescaling
  – thresholding
  – logarithmic dynamics compression
  – gamma correction
  – histogram equalisation
- Pseudo- and false colour representations further improve the visible information content for humans.
- Pixelwise averaging of images reduces noise.
- Subtraction of images allows background elimination.

Literature

- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Pearson, Upper Saddle River, Third Edition, 2008.
  (Sections 3.2 to 3.3 describe point transformations.)
- R. Jain, R. Kasturi, B. G. Schunck: *Machine Vision*. McGraw-Hill, New York, 1995.
  (see in particular Chapter 4)

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Assignment C3 – Classroom Work

Problem 1 (Histogram Equalisation)

Consider an image with grey values that are quantised to 3 bits per pixel. It has the following histogram:

Grey value index	1	2	3	4	5	6	7	8
Grey value	0	1	2	3	4	5	6	7
Frequency	3	1	1	4	11	9	13	8

Apply the discrete histogram equalisation. Write down the mapping from initial to resulting grey values and the histogram of the resulting image.

(*This problem helps you to understand the idea behind histogram transformations in general.*)

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Assignment T3 – Theoretical Homework

Problem 1 (Interpolation) (8 points)

(a) Consider the 1-D case with finitely many equidistant interpolation data $f_1,...,f_n$ given at the points $x_1 = 1,...,x_n = N$. Set up the linear system of equations that has to be solved in order to determine the interpolation coefficients for *quintic B-spline interpolation*. Quintic B-splines are splines of degree 5 which are defined as follows:

$$\beta_5 = \begin{cases} -\frac{1}{120}|x|^5 + \frac{1}{8}x^4 - \frac{3}{4}|x|^3 + \frac{9}{4}x^2 - \frac{27}{8}|x| + \frac{81}{40}\,, & 2 \le |x| \le 3 \\ \frac{1}{24}|x|^5 - \frac{3}{8}x^4 + \frac{5}{4}|x|^3 - \frac{7}{4}x^2 + \frac{5}{8}|x| + \frac{17}{40}\,, & 1 \le |x| \le 2 \\ -\frac{1}{12}|x|^5 + \frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{11}{20}\,, & -1 \le x < 1 \\ 0 & \text{else.} \end{cases}$$

(b) Instead of using quintic B-splines, let us now now consider the corresponding equation system for *cubic B-spline interpolation*. This system has already been defined in the lecture. Given the signal

x	1	2	3
$f(x)$	4	28	24

use this equation system and compute the corresponding interpolation coefficients. Finally, determine the interpolation values at the points $\frac{1}{2}$ and $\frac{3}{2}$.

(*This tasks actually shows how the entries of the linear equation systems are determined and how the resulting coefficients are used to interpolate a given signal.*)

Assignment T3 (2)

Problem 2 (Multiple Choice) (6 points)

Mark which of the following statements are true and which ones are false. Justify your answers with not more than one sentence per statement.

You receive 1 *points for each correct answer but lose* 1 *points for each incorrect answer. Negative total scores are replaced by zero.*

(a) The discrete Wavelet transform has only optimal (linear) complexity in 1-D.

(b) For the German word “Lagerregal” the Huffman coding does not allow a better compression that any other coding scheme that assigns the codes 00, 01, 10, 110 and 111 arbitrarily to the letters.

(c) The compression ratio for an alternating signal $f = (7, 8, 7, 8, \ldots, 7, 8)$ can be improved significantly, if one uses bitplane coding instead of the ordinary run length encoding.

(d) Quadratic B-splines require less computational effort for determining the coefficients than cubic B-splines.

(e) Histogram equalisation is a point operation that is invertible by construction.

(f) The subsequent application of a lowpass and highpass filter always yields the original signal.

Assignment T3 (3)

Problem 3 (Linear Filters) (6 points)

Determine if the following 2-D stencils are lowpass, bandpass, or highpass filters. To this end, take a closer look at the design principles for linear filters in Lecture 11 and try to derive these stencils. Please note that these filters can have different properties in x- and y-direction.

(a) $\frac{1}{12}$

1	2	1
1	2	1
1	2	1

(b) $\frac{1}{12}$

-1	-2	-1
2	4	2
-1	-2	-1

(d) $\frac{1}{256}$

-1	-4	-6	-4	-1
-4	0	8	0	-4
-6	8	28	8	-6
-4	0	8	0	-4
-1	-4	-6	-4	-1

(*This problem gives you insights in the design of linear filters.*)

Deadline for submission: Tuesday, December 4, 10 am (before the lecture).