

### Problem 1 (Corner Detection)

- (a) To complete the implementation of the function `struct_tensor` we have to add the computation of the approximated partial derivatives. Here we have chosen to use Sobel operators for an approximation of  $v_x$  and  $v_y$ . We have to take care of boundary pixels: Before we compute the derivatives we mirror the boundaries (by calling the function `dummies`) to make sure that all pixels used during computation are filled with sensible values.

```
/* ----- */
void struct_tensor
(float    **v,    /* image !! gets smoothed on exit !! */
 long    nx,    /* image dimension in x direction */
 long    ny,    /* image dimension in y direction */
 float    hx,    /* pixel size in x direction */
 float    hy,    /* pixel size in y direction */
 float    sigma, /* noise scale */
 float    rho,   /* integration scale */
 float    **dxx, /* element of structure tensor, output */
 float    **dxy, /* element of structure tensor, output */
 float    **dyy) /* element of structure tensor, output */

/* Calculates the structure tensor. */

{
long    i, j;          /* loop variables */
float    dv_dx, dv_dy; /* derivatives of v */
float    w1, w2, w3, w4; /* time savers */

/* ---- smoothing at noise scale, reflecting b.c. ---- */

if (sigma > 0.0)
    gauss_conv (sigma, nx, ny, hx, hy, 5.0, 1, v);

/* ---- calculate gradient and its tensor product ---- */
dummies(v, nx, ny);
```

```

for(i=1; i<=nx; i++)
  for(j=1; j<= ny; j++)
    {
      /* compute the derivatives using Sobel operators */
      w1 = 1.0 / (8.0 * hx);
      w2 = 1.0 / (4.0 * hx);

      dv_dx = w1 * ( v[i+1][j+1] - v[i-1][j+1]
                    + v[i+1][j-1] - v[i-1][j-1])
              + w2 * ( v[i+1][j ] - v[i-1][j ] );

      w3 = 1.0 / (8.0 * hy);
      w4 = 1.0 / (4.0 * hy);

      dv_dy = w3 * ( v[i+1][j+1] - v[i+1][j-1]
                    + v[i-1][j+1] - v[i-1][j-1])
              + w4 * ( v[i ][j+1] - v[i ][j-1]);

      dxx[i][j] = dv_dx * dv_dx;
      dxy[i][j] = dv_dx * dv_dy;
      dyy[i][j] = dv_dy * dv_dy;
    }

/* ---- smoothing at integration scale, Dirichlet b.c. ---- */
if (rho > 0.0)
  {
    gauss_conv (rho, nx, ny, hx, hy, 5.0, 0, dxx);
    gauss_conv (rho, nx, ny, hx, hy, 5.0, 0, dxy);
    gauss_conv (rho, nx, ny, hx, hy, 5.0, 0, dyy);
  }

return;
}
/* ----- */

```

- (b) The cornerness can for instance be implemented as the determinant of the structure tensor here. Then the code looks as follows:

```

/* ----- */
void cornerness
(float    **u,    /* image !! gets smoothed on exit !! */
 long    nx,     /* image dimension in x direction */
 long    ny,     /* image dimension in y direction */
 float   hx,     /* pixel size in x direction */
 float   hy,     /* pixel size in y direction */
 float   sigma,  /* noise scale */
 float   rho,    /* integration scale */
 float   **v)   /* output */

/*
 calculates cornerness in each pixel;
 it is evaluated as the determinant of the structure tensor;
 */

{
 long    i, j;           /* loop variables */
 float   **dxx, **dxy, **dyy; /* tensor components */

/* allocate storage */
 alloc_matrix (&dxx, nx+2, ny+2);
 alloc_matrix (&dxy, nx+2, ny+2);
 alloc_matrix (&dyy, nx+2, ny+2);

/* calculate structure tensor */
 struct_tensor (u, nx, ny, hx, hy, sigma, rho, dxx, dxy, dyy);

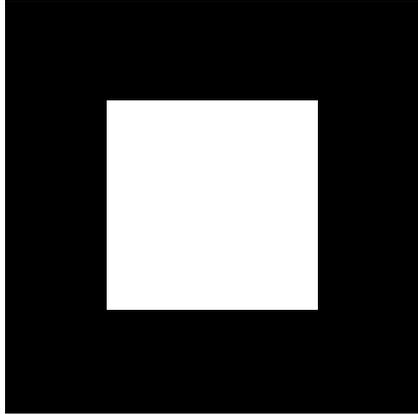
/* cornerness */
 for(i=1; i <=nx; i++)
   for(j=1; j <=ny; j++)
     v[i][j] = dxx[i][j] * dyy[i][j] - dxy[i][j] * dxy[i][j];

/* free storage */
 disalloc_matrix (dxx, nx+2, ny+2);
 disalloc_matrix (dxy, nx+2, ny+2);
 disalloc_matrix (dyy, nx+2, ny+2);

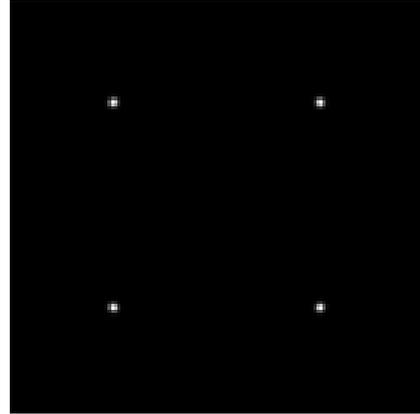
return;
}
/* ----- */

```

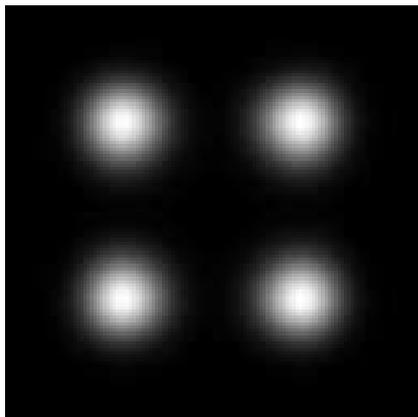
- (c) We demonstrate the influence of the parameters  $\sigma$  and  $\rho$  at the first example `square.pgm`. Higher values of both parameters lead to fuzzy edge detections.



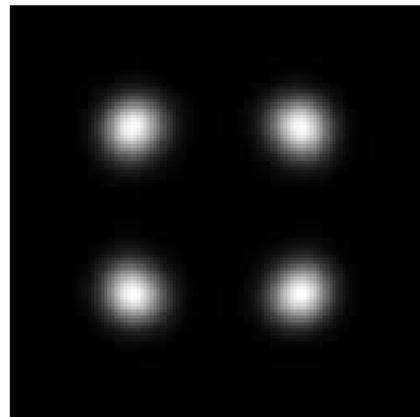
Original image `square.pgm`



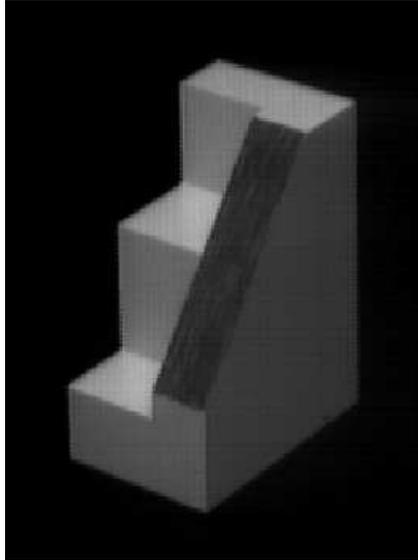
Corner detection,  $\sigma = 0.0$ .  $\rho = 1.0$



$\sigma = 0.0$ ,  $\rho = 10.0$



$\sigma = 10.0$ ,  $\rho = 1.0$

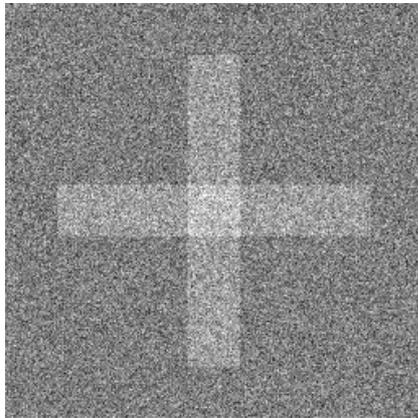


Original image stairs.pgm

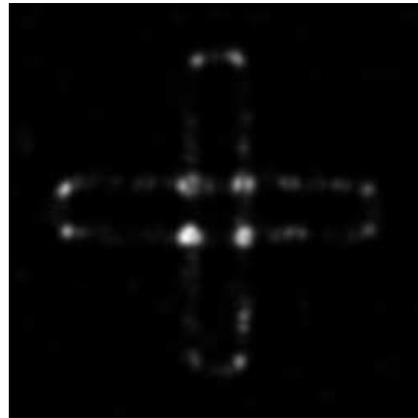


Corner detection,  $\sigma = 1.0$ ,  $\rho = 2.0$

The example `acros.pgm` is degraded with noise. We can reduce the influence of this noise by choosing a higher value  $\sigma$ .



Original image acros.pgm



Corner detection,  $\sigma = 3.0$ ,  $\rho = 3.0$

## Problem 2 (Double Thresholding)

- (a) For the double threshold method we start with determining which pixel values exceed  $t_1$ , and  $t_2$ , respectively. The values higher than  $t_2$  serve as seed points and are iteratively expanded to the final segmentation. A dilation is used to mark the current region and all its direct neighbours. All pixels of this region that exceed  $t_1$  are joined with the current region. If nothing has happened in the last step, the iterative procedure stops.

```
/* ----- */
void double_thresholding

    (double  **u,      /* image, changed on output */
     long    nx,      /* size in x direction */
     long    ny,      /* size in y direction */
     double  t1,      /* smaller threshold */
     double  t2)      /* larger threshold */

/*
  double thresholding with the threshold pair (t1,t2).
*/

{
  long    i, j;      /* loop variables */
  long    stop;      /* stop iterations? */
  double  **uold;    /* previous iteration value of growing image u */
  double  **v;       /* image thresholded at t2 */

  /* allocate storage for v and uold */
  alloc_matrix (&v, nx+2, ny+2);
  alloc_matrix (&uold, nx+2, ny+2);

  /* copy u into v */
  for (i=1; i<=nx; i++)
    for (j=1; j<=ny; j++)
      v[i][j] = u[i][j];

  /* threshold u at t2, and v at t1 */
  for (i=1; i<=nx; i++)
    for (j=1; j<=ny; j++)
      {
        if (u[i][j] <= t2) u[i][j] = 0.0; else u[i][j] = 255.0;
      }
}
```

```

        if (v[i][j] <= t1) v[i][j] = 0.0; else v[i][j] = 255.0;
    }

/* expand seed objects in u until they reach the object */
/* boundaries of v                                     */
do
{
    /* copy u into uold */
    for(i=1; i<=nx; i++)
        for(j=1; j<=ny; j++)
            uold[i][j] = u[i][j];

    /* add direct neighbours with dilation */
    dilation(uold, nx, ny, u);

    /* remove pixels in neighbourhood which are smaller than t1 */
    for(i=1; i<=nx; i++)
        for(j=1; j<=ny; j++)
            if(v[i][j] < u[i][j]) u[i][j] = v[i][j];

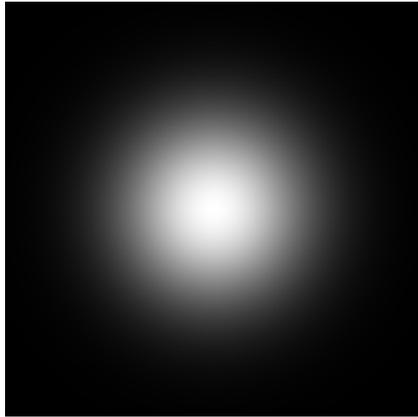
    /* check if something has changed */
    stop = 1;
    for(i=1; i<=nx; i++)
        for(j=1; j<=ny; j++)
            if(u[i][j] > uold[i][j]) stop = 0;
}
while (stop == 0);

/* free storage */
dealloc_matrix (v, nx+2, ny+2);
dealloc_matrix (uold, nx+2, ny+2);

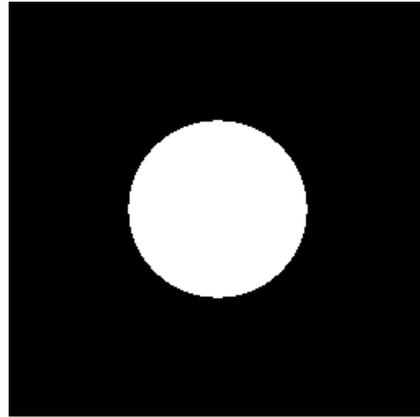
return;
}
/* ----- */

```

- (b) The first image `gauss.pgm` shows a symmetric Gaussian. The level sets of this image are concentric circles. Thus the double threshold method yields the same result as a simple threshold method with threshold  $t_1$ .

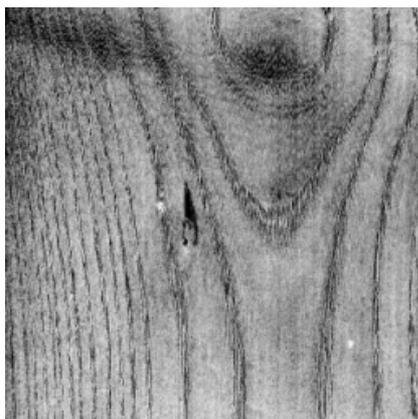


Original image `gauss.pgm`



Double thresholding with  
 $t_2 = 200, t_1 = 100$

- (c) With the double threshold technique as implemented in our program we are only able to detect bright structures. In the third example we want to find a dark defect in a wood surface. To this end in a first step we invert the image such that dark structures turn into bright ones. Grey value images can be inverted either in the code directly or using `xv`'s color editor (which can be opened by pressing 'e' with focus on the image window). The button `RevVid` inverts the image. We then apply double thresholding to the inverted image.



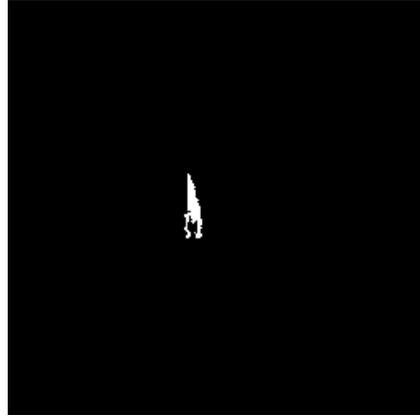
Original image `wood.pgm`



Single threshold  $t = 24$



Inverted image



Double threshold of the inverted image with  $t_2 = 254$ ,  $t_1 = 150$ .