Image Processing and Computer Vision 2005/06

Example Solutions for Practical Assignments 4 (P4)

Problem 1

Let f denote an image which has been degraded by convolution with the kernel h. We want to apply Wiener filtering to obtain a filtered version u by

$$\hat{u} = \left(\frac{1}{\hat{h}} \frac{|\hat{h}|^2}{|\hat{h}|^2 + K} \right) \hat{f} \quad , \tag{1}$$

where $K > 0$ is a real positive number.

For an implementation of this formula we have to keep in mind that the symbols $\hat{f}, \hat{h}$, and $\hat{u}$ stand for Fourier coefficients, which are in general complex numbers. We use the facts

$$\frac{1}{\hat{h}} = \frac{\overline{\hat{h}}}{|\hat{h}|^2} \quad \text{and} \quad |\hat{h}|^2 = \hat{h}\overline{\hat{h}} = \Big(\mathrm{Re}(\hat{h})\Big)^2 + \Big(\mathrm{Im}(\hat{h})\Big)^2$$

to rewrite the formula

$$\frac{1}{\hat{h}} \frac{|\hat{h}|^2}{|\hat{h}|^2 + K} \quad = \quad \frac{\overline{\hat{h}}}{|\hat{h}|^2 + K} \quad . \tag{2}$$

This can simplify the implementation of the complex arithmetics:

```
void filter
  (float    **ur,    /* real part of Fourier coeffs, changed */
   float    **ui,    /* imag. part of Fourier coeffs, changed */
   float    **hr,    /* real part of Fourier kernel, unchanged */
   float    **hi,    /* imag. part of Fourier kernel, unchanged */
   float    param,   /* filter parameter */
   long     nx,      /* pixel number in x direction */
   long     ny)      /* pixel number in y direction */

/* Performs Wiener Filtering in the Fourier domain. */

{
  long   i, j;      /* loop variables */
  float  N;         /* denominator */
  float  vr,vi;     /* auxiliary variables for cplx arithm. */
```

```
/* ---- compute filtered coefficients ---- */

for (i=0; i<=nx-1; i++)
  for (j=0; j<=ny-1; j++)
    {
        /* compute the denominator */
        N = hr[i][j] * hr[i][j] + hi[i][j] * hi[i][j] + param;

        /* numerator for the real part */
        vr = hr[i][j] * ur[i][j] + hi[i][j] * ui[i][j];

        /* numerator for the imaginary part */
        vi = hr[i][j] * ui[i][j] - hi[i][j] * ur[i][j];

        ur[i][j] = vr / N;
        ui[i][j] = vi / N;
    }

return;
}
```

We now show the influence of the parameter K on the filtering results. First we take a look at the image `bus1.pgm`. For this image, a parameter near $K = 0.01$ yields relatively good results. We see that too small parameters tend to result in artifacts near the boundary of the image as well as small high-frequent artifacts all over the image. If K tends to zero, Wiener Filtering suffers from the same problems as Inverse Filtering. On the other hand, choosing the parameter K too large reduces the contrast of the image. This can be seen from formula (1): A large K results in damping all Fourier coefficients, and the image becomes darker.

Initial image (`bus1.pgm`)

$K = 0.01$

$K = 0.00001$

$K = 1.0$

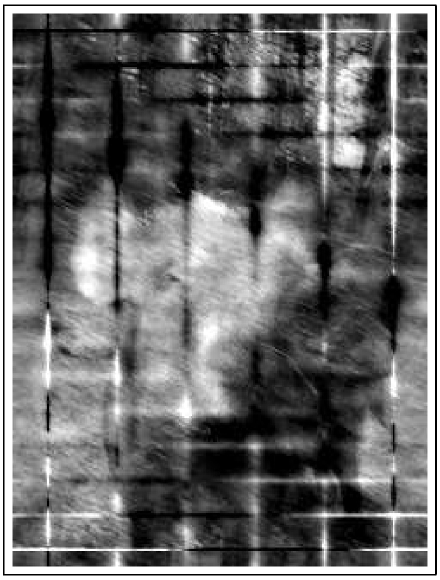
The degradations are much stronger for the second image `bus2.pgm`. Here we have to choose a smaller K to obtain a good reconstruction.

Initial image (`bus2.pgm`)

$K = 0.001$

The trade-off between sharpness and artifacts can be seen for the third image, `hogblur.pgm`.

Initial image (`hogblur.pgm`)



$K = 0.001$

$K = 0.01$

$K = 0.1$