

Lecture 3: Electromagnetic Spectrum, Imaging by Visible Light I

Contents

- ◆ Electromagnetic Waves and Spectrum
- ◆ Basic Visible Light Optics

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22

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Electromagnetic Waves (1)

Electromagnetic Waves

Maxwell's Equations

Electric and magnetic fields are force fields in space governed and coupled by Maxwell's equations (James Clerk Maxwell 1873)

$$\begin{aligned} \operatorname{div} E &= \varepsilon \rho, & \operatorname{rot} E + \frac{\partial B}{\partial t} &= 0, \\ \operatorname{div} B &= 0, & \operatorname{rot} B - \frac{\mu}{\varepsilon} \frac{\partial E}{\partial t} &= \mu j. \end{aligned}$$

E	electric field	B	magnetic field
ρ	charge density	j	current density
μ, ε	material-dependent constants,		t time



James Clerk Maxwell (1831–1879)
 (image: www.wolfram.com)

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5	6
7	8
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Electromagnetic Waves

- ◆ In vacuum, one has $\rho = 0$ and $j = 0$. This gives

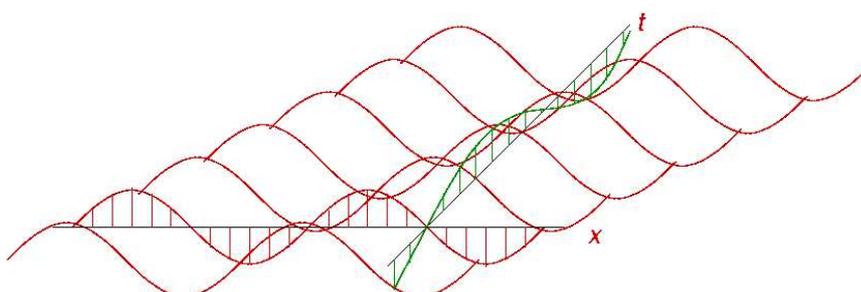
$$\Delta B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}, \quad \Delta E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}.$$

- ◆ These hyperbolic second-order differential equations are *wave equations*. They describe coupled waves in the electric and magnetic field which propagate through space at a constant speed c , and transport energy.
- ◆ These waves are called *electromagnetic waves*. One also speaks of *electromagnetic radiation*.

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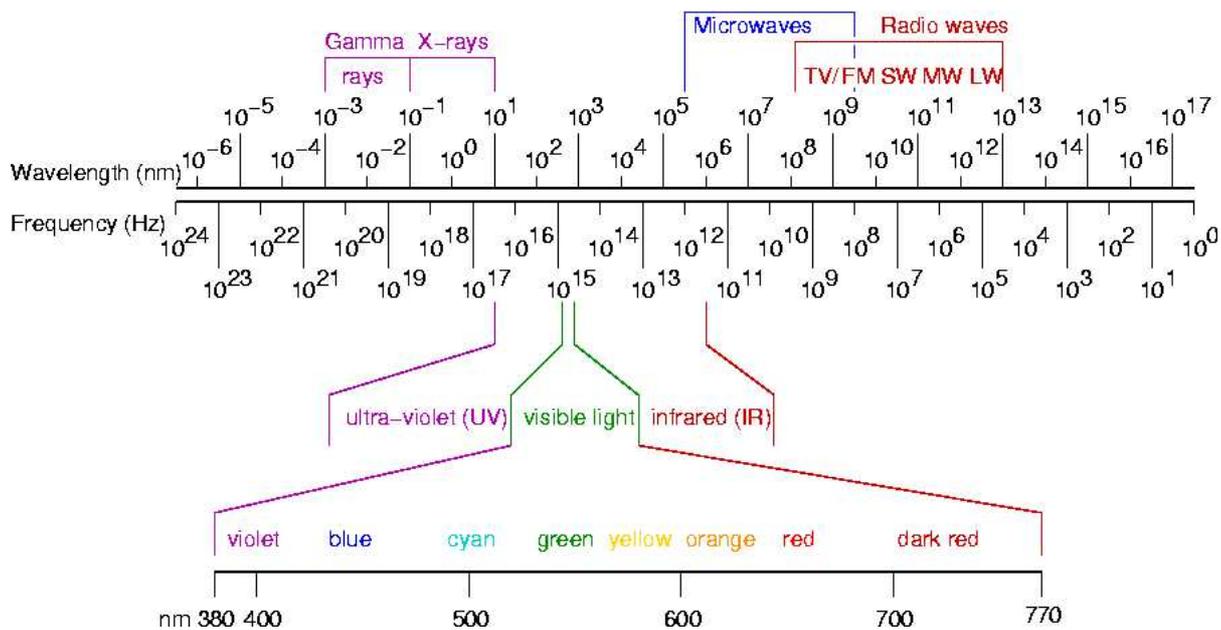
Wavelength and Frequency

- ◆ The constant c is the speed of light, $c = 299,792,458$ m/s.
- ◆ Considering a snapshot of a wave at some time, one sees that it is a periodic pattern in space, with a characteristic *wavelength* λ , measured e.g. in nm, $1 \text{ nm} = 10^{-9} \text{ m}$.
- ◆ Observing a wave passing in time through a certain point in space, it is an oscillation with some frequency ν , measured in Hz, $1 \text{ Hz} = 1 \text{ oscillation/s}$.
- ◆ The point of its oscillation cycle that a wave attains at a defined time and location is its *phase*.
- ◆ Wavelength and frequency are linked via $c = \lambda\nu$.



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The Electromagnetic Spectrum

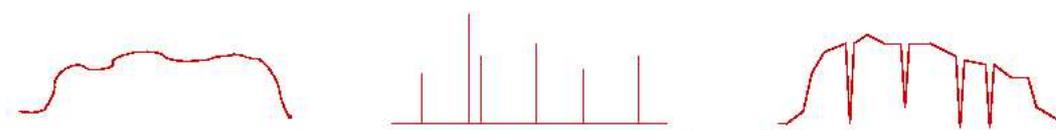


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Types of Radiation Spectra

Electromagnetic radiation can consist of a mixture of different wavelengths/frequencies, or of just one single frequency.

- ◆ Radiation consisting only of waves of a single wavelength is *monochromatic*.
- ◆ Radiation containing a broad range of wavelengths is said to have a *continuous spectrum*.
- ◆ A *line spectrum* is a mixture of isolated wavelengths. This is often the case for light emitted by hot gases (emission spectrum).
- ◆ An *absorption line spectrum* is one where certain isolated frequencies are partially deleted

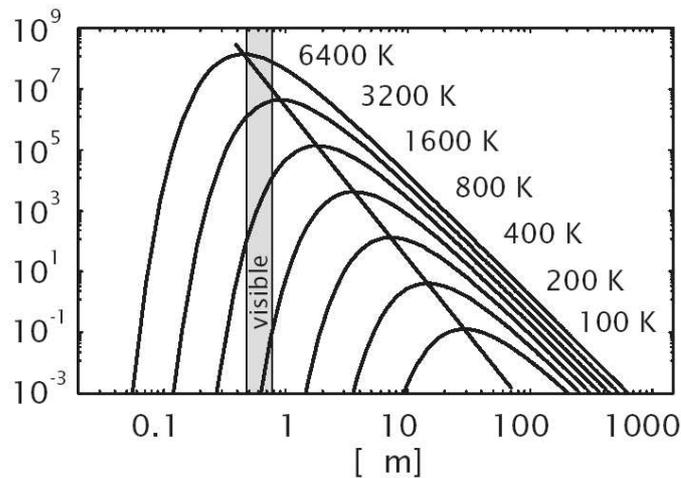


Energy–wavelength diagrams of different spectra. **Left:** A continuous spectrum. **Middle:** (Emission) line spectrum. **Right:** Absorption line spectrum.

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Black-Body Radiation: Light Temperature

A physical body that perfectly absorbs all incoming EM radiation emits a continuous spectrum of radiation with a characteristic distribution over frequencies depending on its temperature.



Spectra of black-body radiation for different temperatures. (Horizontal axis unit read [μm].) (Hau β cker 1999)

Sunlight is well approximated by black-body radiation of 6000 K, morning or evening light about 5500 K.

Superposition and Interference

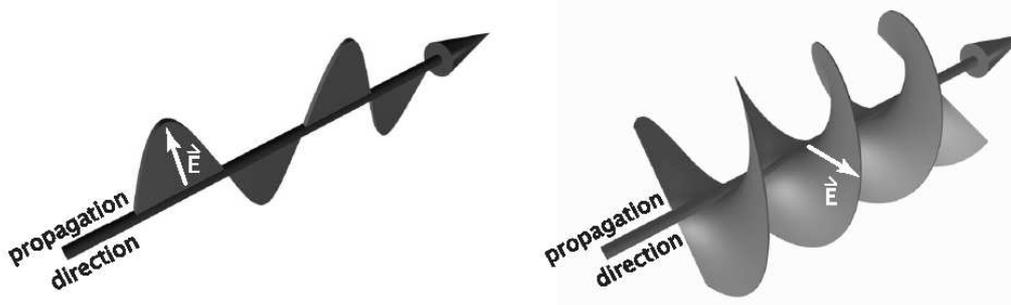
- ◆ Because of the linearity of Maxwell's equations, electromagnetic waves crossing the same region in space are *linearly superposed*.
- ◆ Typically, electromagnetic radiation is a mixture of many waves of different phase. For such radiation, the superposition implies that *crossing beams don't perturb each other*.
- ◆ If the electromagnetic waves match exactly in frequency and phase, they are called *coherent*. Superposition of coherent waves leads to *interference*, i.e. they become stronger when they are in equal phase, and they eliminate each other when they are in opposite phase.
- ◆ An outstanding source of coherent light are *lasers* (invented 1963). Before the invention of lasers, interference-capable light was generated e.g. by dividing beams using semi-reflective mirrors.

Electromagnetic Waves (8)

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Polarisation

- ◆ Each electromagnetic wave has a specific plane in which it oscillates. Radiation with all waves oscillating in the same plane is *linearly polarised*.
- ◆ Non-polarised radiation can be filtered for polarisation.
- ◆ Superposition of waves of orthogonal polarisation, with characteristic phase shifts between the components, leads to *circular polarisation* which might be left- or right-rotating.



Left: Linear polarisation. **Right:** Circular polarisation. (Garbe, 1999)

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Electromagnetic Waves (9)

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Inverse square law

- ◆ Assume we look from some distance r at a point-shaped source of radiation.
- ◆ The total energy emitted by the source distributes to a spherical surface of radius r . The area of this surface increases with r^2 . Since the total energy does not change with distance, the density of energy per surface area decreases with $1/r^2$.
- ◆ Since the observer or detector catches light on a constant part of this surface, the energy perceived or measured therefore decreases with the density, i.e. it is also proportional to $1/r^2$.

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Photons

Albert Einstein discovered in 1905 that electromagnetic waves come in packets. This was the origin of *quantum physics*.

One such packet of radiation is called a *photon*. Photons behave like particles. Electromagnetic radiation therefore has a double nature – so called *wave–particle dualism*.

Interaction of electromagnetic waves with matter relies on absorption and emission of photons by atoms.

The energy E of a photon depends on its frequency via

$$E = h\nu$$

with a universal physical constant $h \approx 6.55 \times 10^{-34}$ Js called Planck’s constant, after Max Planck.

Today it is possible to detect single photons, and quantum effects play a role for the properties of many sensors used in imaging.

(images: Spektrum-Verlag, 2000)



Albert Einstein
(1879–1955)



Max Planck
(1858–1947)

1	2
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7	8
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11	12
13	14
15	16
17	18
19	20
21	22

Basic Visible Light Optics

Interaction of Light with Matter

- ◆ **Absorption.** Light is “caught” by matter, transformed into other energy like heat. *Attenuation* takes place if only a portion of the light is absorbed while the remaining portion is transmitted.
- ◆ **Reflection.** At the interface between two media (two materials) light abruptly changes its direction and is bent back to the original medium.
- ◆ **Refraction.** At the interface between two media light changes its direction and enters the second medium.
- ◆ **Diffraction.** Near an obstruction, light changes its direction due to wave interference phenomena. *Scattering* results from multiple diffraction caused by very small obstructing particles.

These phenomena play a double role in imaging:

- They govern how information is impressed on light.
- They can be used to lead light to the sensors.

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Attenuation

Partial absorption in matter *attenuates* electromagnetic radiation.

Attenuation follows an exponential law also known as *Beer's law*:

The *attenuated energy density* (i.e. energy per area of cross-section) is

$$I(x, y) = N(x, y) E \exp \left(- \int \mu(x, y, z) dz \right)$$

where

- ◆ N is the density of incident photons, which have to be monochromatic with energy E ,
- ◆ μ a linear attenuation coefficient, expressing absorption in matter,
- ◆ it is assumed that the radiation propagates in z direction, and the integral runs over the path on which the radiation is attenuated.

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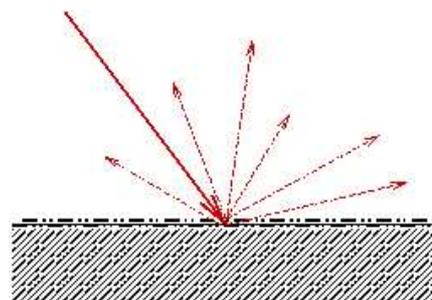
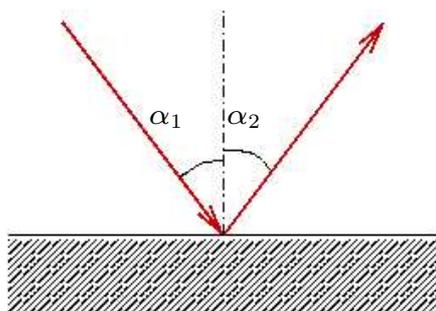
Reflection

◆ Specular Reflection.

- Occurs at smooth interfaces (typically surfaces of objects).
- Incoming and outgoing light beams and surface normal are in one plane.
- The angle α_1 between incoming beam and surface normal equals the angle α_2 between outgoing beam and normal.

◆ Diffuse Reflection.

Outgoing beams are widely spread in directions and intensities.



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Basic Visible Light Optics (4)

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- ◆ Reflections at most surfaces are a mixture between specular and diffuse reflection.
- ◆ The phase of reflected light depends on the material (particularly, its electric properties).
- ◆ Reflected light can be polarised.

Lambert's Cosine Law

- ◆ Consider a small piece A of a surface which is illuminated by parallel light.
- ◆ Let ϑ be the angle between the surface normal and the direction of incoming light.
- ◆ Then the surface patch effectively catches the light energy from a cross-section $A \cos \vartheta$ of the light bundle.
- ◆ Provided the surface is perfectly diffuse reflecting, then its brightness (measured by the energy of outgoing beams per area) is proportional to $\cos \vartheta$.

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Basic Visible Light Optics (5)

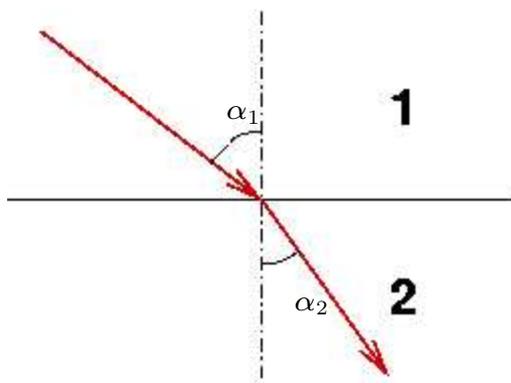
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Refraction

- ◆ Occurs when light passes between media in which it propagates at different speeds, c_1 and c_2 .
- ◆ Incoming and outgoing light beams and surface normal are in one plane.
- ◆ The angles α_1 between incoming beam and surface normal, and α_2 between outgoing beam and normal fulfil *Snell's law* (Willebrord Snell 1618)

$$\frac{\sin \alpha_2}{\sin \alpha_1} = \frac{c_2}{c_1} = \frac{n_1}{n_2}$$

($n_i := c_{\text{vacuum}}/c_i$ "refraction index").



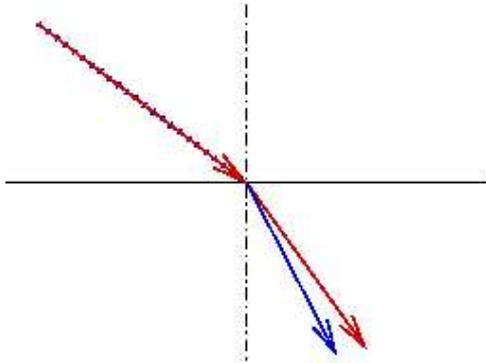
Willebrord Snell (1580 (?)-1626)
(image: www.wolfram.com)

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Basic Visible Light Optics (6)

- ◆ At interfaces between transparent media, typically part of the light is reflected, part is refracted. Both partial beams can be differently polarised.
- ◆ In most transparent media, the speed of light depends on the wavelength, thus colour. Light of different wavelengths is therefore separated by different refraction angles. This phenomenon is called *dispersion*.
- ◆ There exist *anisotropic optic materials* (certain crystals) in which the speed of light, and thus the refraction angle, depends on the polarisation.

Such materials can be used as polarising filters.

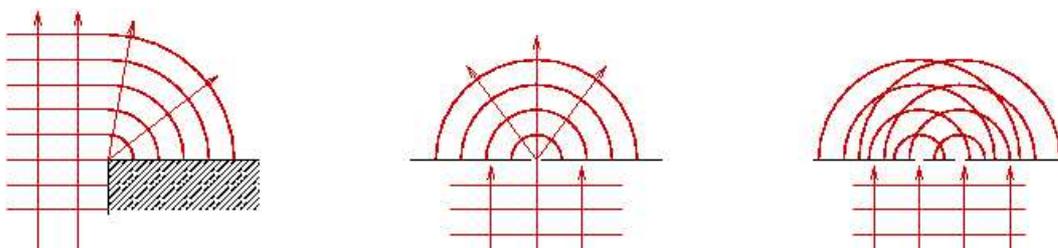


Left: Principle of dispersion. – **Right:** Prism dispersing light (Wikipedia, "Prism")

Basic Visible Light Optics (7)

Diffraction

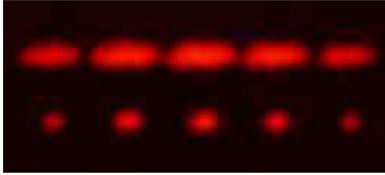
- ◆ *Diffraction* is caused by wave interferences in the surrounding of obstructions, like edges of objects.
- ◆ The propagation of waves in the presence of obstructions follows *Huygens' Principle*:
Each point of a wavefront is the starting point of a spherical wave, and all these *elementary waves* superpose/interfere to give the observed macroscopic wave.
- ◆ Due to multiple paths to the same destination point, interference patterns can occur, often in the shape of stripes. Examples are diffraction at slits or double-slits.



Diffraction at an edge, a slit and a double-slit.

Basic Visible Light Optics (8)

- ◆ An arrangement of multiple parallel equidistant slits is called a *grating*. It leads to a series of sharp interference maxima.



Diffraction patterns resulting from 2-slit and 5-slit diffraction of a laser beam. (Wikipedia, "Diffraction")

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Basic Visible Light Optics (9)

Scattering

- ◆ Multiple diffraction of light at many small particles, e.g. molecules and dust in the atmosphere, leads to a diffuse distribution of light across directions called *scattering*.
- ◆ Scattering is wavelength-dependent and is responsible for the blue appearance of the sky.
- ◆ Scattered light can be partially or fully polarised. In fact, in a cloud-free blue sky there is a band of polarised light (used by some birds and insects for orientation).

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Summary

- ◆ Electromagnetic (EM) waves can be described by wave equations for the electric and magnetic fields.
- ◆ The electromagnetic spectrum contains gamma rays, X rays, UV rays, visible light, infrared waves, microwaves and radio waves.
- ◆ EM waves are linearly superimposed.
- ◆ Electromagnetic radiation has a wave-particle dualism. Its particle nature is described by photons.
- ◆ Light may interact with matter in four ways: absorption, reflection, refraction, diffraction.
- ◆ Absorption follows Beer's law.
- ◆ Reflection is often a mixture of specular and diffuse reflection.
- ◆ Refraction is described by Snell's law.
- ◆ Diffraction follows Huygen's principle.

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