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Sample Solution for Problem Set 6
Due: June 19, 2007, 9:14 am

Embedded Systems

Problem 1 (Reliability)

(a) With $R(t) = e^{-\lambda t^\alpha}$ we get $R'(t) = -\lambda t^{\alpha-1}\alpha e^{-\lambda t^\alpha}$. So, $r(t) = \frac{-R'(t)}{R(t)} = \lambda t^{\alpha-1}\alpha$.
(b) The question is equivalent to asking: What is the probability that the system works at $t = 1$ year. So, the answere is $R(1 \text{ year}) = R(8760 \text{ hour}) = e^{-10^{-6}\frac{1}{\text{hour}}\cdot 8760 \text{ hour}} \approx 0.991$.
(c) $R(1y) \approx 2.323 \cdot 10^{-167}$

Problem 2 (Reliability Analysis)

(a) The first steps of the algorithm are given in figure 1.

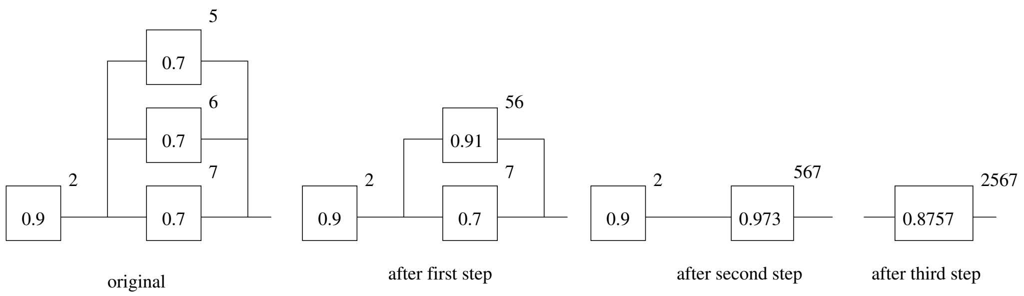

Figure 1: Illustrating the first steps of Problem 2 (a)

- replace 5 and 6 by a new component 56 with reliability $1 - (1 - 0.7)(1 - 0.7) = 0.91$
- replace 56 and 7 by a new component 567 with reliability $1 - (1 - 0.91)(1 - 0.7) = 0.973$
- replace 567 and 2 by a new component 2567 with reliability $0.973 \cdot 0.9 = 0.8757$
- replace 2567 and 3 by a new component 23567 with reliability $1 - (1 - 0.8757)(1 - 0.8) = 0.97514$
- replace 23567 and 1 by a new component 123567 with reliability $0.95 \cdot 0.97514 = 0.926383$
- replace 123567 and 4 by a new component 1234567 with reliability $1 - (1 - 0.85)(1 - 0.926383) = 0.98895745$

Finally, only one component 1234567 is left. The reliability of the system equals the reliability of this component.
(b)

- $\{1, 4\}$
- $\{2, 3, 4\}$
- $\{3, 4, 5, 6, 7\}$

(c)

$$R(t) > 1 - ((1 - 0.95)(1 - 0.85) + (1 - 0.9)(1 - 0.8)(1 - 0.85) + (1 - 0.8)(1 - 0.85)(1 - 0.7)(1 - 0.7)(1 - 0.7)) = 0.98869$$

The calculations differ by about 0.00027.

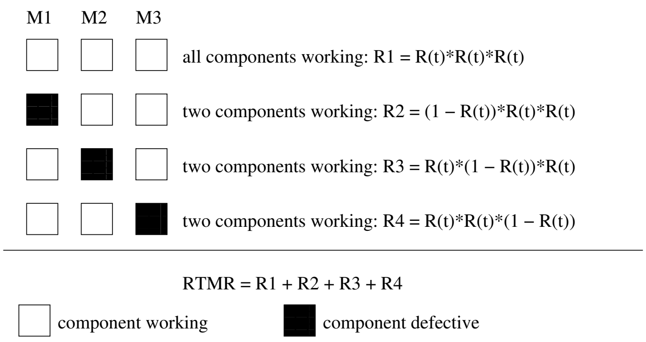

Figure 2: Illustrating static redundancy

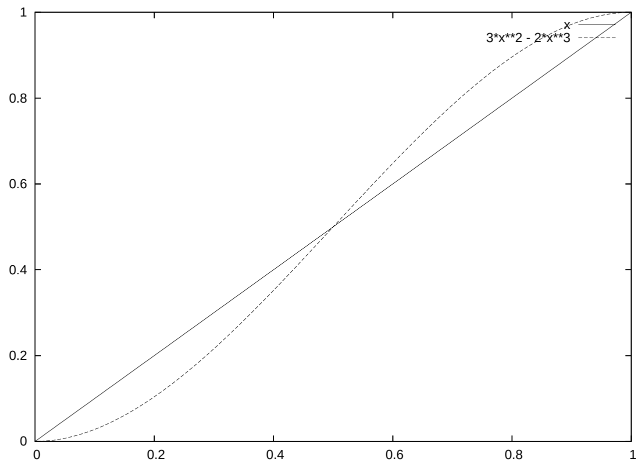

Figure 3: Reliability using MDR vs. single module ($x = R(t)$)

Problem 3 (Static Redundancy)

(a) To have the system running, either three or two modules must be working, see figure 2. There is one possibility of three running modules, but three possibilities of two running modules. So, the reliability is

$$3R^2(t)(1 - R(t)) + R^3(t) = 3R^2(t) - 2R^3(t)$$

(b) This is not necessarily the case. Using TMR is only useful if $R(t) > 0.5$, as illustrated in figure 3. If for example, we have $R(t) = 0.2$, then the probability that the system works if just one module is used is 0.2, but if three modules are used it is $3 \cdot 0.2^2 - 2 \cdot 0.2^3 = 0.104$.

However, if all $R(t)$ are greater than 0.5, then the reliability of TMR is always higher as using just a single module. This also holds if the reliability of the different modules is different (but > 0.5).

(c) As seen in the previous exercise, MTR is justified if $R(t) > 0.5$, so we have to solve the equation $e^{-\lambda t} = 0.5$. We get that $t = -\frac{\ln 0.5}{\lambda}$. So, because both $e^{-\lambda t}$ as well as $3(e^{-\lambda t})^2 - 2(e^{-\lambda t})^3$ are strictly falling with increasing t, after this time the deployment of the TMR is no longer justified.

(d) We do the calculation like in 2a:

- replace upper 1 and 2 by a new component $12u$ with reliability $R^2(t)$
- replace middle 1 and 3 by a new component $13m$ with reliability $R^2(t)$
- replace lower 2 and 3 by a new component $23l$ with reliability $R^2(t)$
- replace $12u$ and $13m$ by a new component $123u$ with reliability $1 - (1 - R^2(t))(1 - R^2(t)) = 2R^2(t) - R^4(t)$
- replace $123u$ and $23l$ by a new component 123 with reliability $1 - (1 - R^2(t))(1 - (2R^2(t) - R^4(t))) = 3R^2(t) - 3R^4(t) + R^6(t)$

The result is different, because in the method of 2a we assume that the reliability of all the components is stochastically independent. Here, this assumption is wrong, because in figure 3 of the exercise sheet the different modules named with the same numbers represent actually the same component.